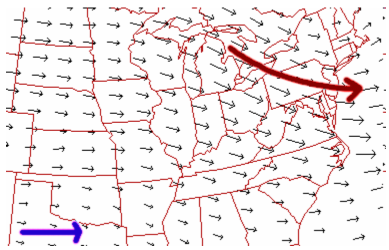
Scalars and Vectors



Scalars and Vectors

A scalar is a number which expresses quantity. Scalars may or may not have units associated with them.

Examples: mass, volume, energy, money

A vector is a quantity which has both magnitude and direction. The magnitude of a vector is a scalar.

Examples: Displacement, velocity, acceleration, electric field

Vector Notation

- Vectors are denoted as a symbol with an arrow over the top:

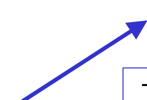
$$\vec{x}$$

- Vectors can be written as a magnitude and direction:

$$\vec{E} = 15.7\,\mathrm{N/C} @ 30^{\circ}\ \mathrm{deg}$$

Vector Representation

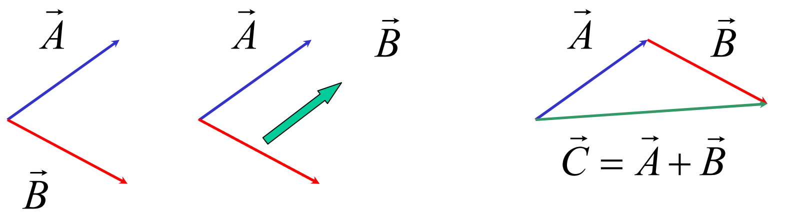
- Vectors are represented by an arrow pointing in the direction of the vector.
- The length of the vector represents the magnitude of the vector.
- WARNING!!! The length of the arrow does not necessarily represent a length.



$$\vec{A} = 2.3\,\mathrm{m/s}$$

Vector Addition

Adding Vectors Graphically.



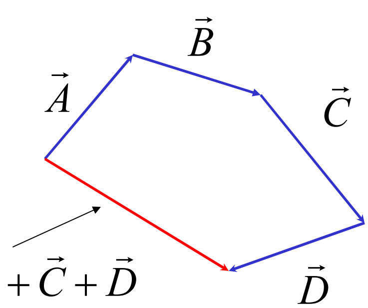
$$\vec{C} = \vec{A} + \vec{B}$$

Arrange the vectors in a head to tail fashion.

The resultant is drawn from the tail of the first to the head of the last vector.

Vector Addition

This works for any number of vectors.

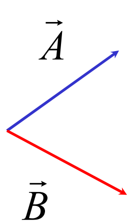


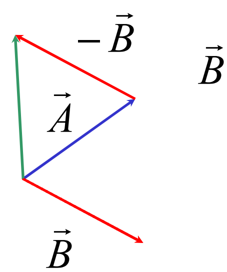
$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

Vector Addition

Vector Subtraction

Subtracting Vectors Graphically.





$$\vec{C} = \vec{A} - \vec{B} = \vec{A} + \left(-\vec{B}\right)$$

Flip one vector.

Then proceed to add the vectors

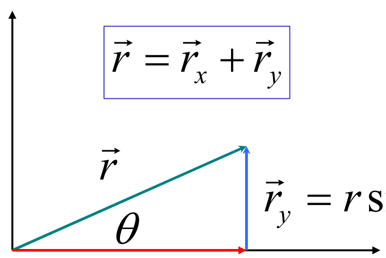
The resultant is drawn from the tail of the first to the head of the last vector.

Vector Components

Any vector can be broken down into components along the x and y axes.

Example: $\vec{r} = 5.0\text{m} @ 30^{\circ}$ from the horizontal. Find its components.

$$\vec{r} = \vec{r}_x + \vec{r}_y$$



$$\vec{r}_y = r\sin\theta\hat{j}$$

$$\vec{r}_x = r\cos\theta\hat{i}$$

$$\vec{r}_x = (5.0\text{m})\cos 30^{\circ}\,\hat{i}$$

$$\vec{r}_x = 4.3\text{m}\hat{i}$$

$$\vec{r}_y = (5.0\text{m})\sin 30^{\circ}\,\hat{j}$$

$$\vec{r}_y = 2.5\text{m}\hat{j}$$

Vector Addition by Components

You can add two vectors by adding the components of the vector along each direction. Note that you can only add components which lie along the same direction.

$$\vec{A} = 3.2\,m/s\,\hat{i} + 2.5\,m/s\,\hat{j}$$

$$+\vec{B} = 1.5\,m/s\,\hat{i} + 5.2\,m/s\,\hat{j}$$

$$\vec{A} + \vec{B} =$$

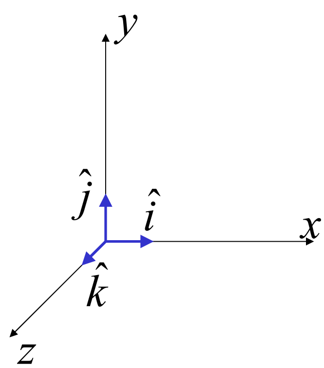
$$\vec{A} + \vec{B} = 12.4\,\text{m/s}$$

Never add the x-component and the y-component

Unit Vectors

Unit vectors have a magnitude of 1.

They only give the direction.



A displacement of 5 m in the x-direction is written as

$$\vec{d} = 5\text{m}\hat{i}$$

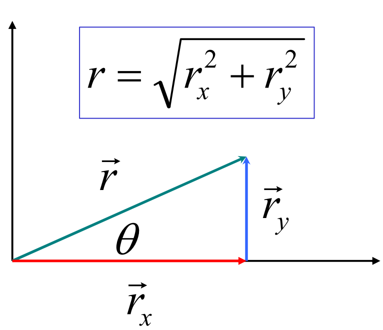
The magnitude is 5m.

The direction is the î-direction.

Finding the Magnitude and Direction

Pythagorean Theorem

$$r = \sqrt{r_x^2 + r_y^2}$$



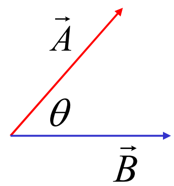
$$\tan\theta = \frac{r_y}{r_x}$$

$$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right)$$

Vector Multiplication I: The Dot Product

The result of a dot product of two vectors is a *scalar*!

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$



$$\hat{i}\cdot\hat{i} = 1 \qquad \hat{i}\cdot\hat{j} = 0$$

$$\hat{j}\cdot\hat{j} = 1 \qquad \hat{j}\cdot\hat{k} = 0$$

$$\hat{k}\cdot\hat{k} = 1 \qquad \hat{i}\cdot\hat{k} = 0$$

Vector Multiplication I: The Dot Product

$$\vec{F} = \left(2\hat{i} + 3\hat{j} - 2\hat{k}\right)\mathrm{N} \qquad \vec{s} = \left(3\hat{i} - 4\hat{j} - 6\hat{k}\right)\mathrm{m}$$

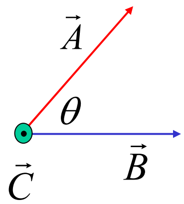
$$\vec{F}\cdot\vec{s} = 2(3)\mathrm{N}\cdot\mathrm{m} + 3(-4)\mathrm{N}\cdot\mathrm{m} + (-2)(-6)\mathrm{N}\cdot\mathrm{m}$$

$$\vec{F}\cdot\vec{s} = 6\mathrm{N}\cdot\mathrm{m}$$

Vector Multiplication II: The Cross Product

The result of a cross product of two vectors is a new *vector*!

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$



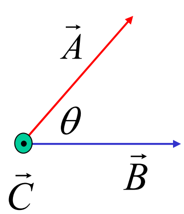
$$\hat{i}\times\hat{i}=0 \qquad \hat{i}\times\hat{j}=\hat{k}$$

$$\hat{j}\times\hat{j}=0 \qquad \hat{j}\times\hat{k}=\hat{i}$$

$$\hat{k}\times\hat{k}=0 \qquad \hat{k}\times\hat{i}=\hat{j}$$

Vector Multiplication II: The Cross Product

$$q\left(\vec{v}\times\vec{B}\right)=\left(q\vec{v}\times\vec{B}\right)=\left(\vec{v}\times q\vec{B}\right)$$



$$\vec{C}=\left(\vec{B}\times\vec{A}\right)=-\left(\vec{A}\times\vec{B}\right)$$

$$\vec{C}\perp\vec{A} \qquad \vec{C}\perp\vec{B}$$

Vector Multiplication II: The Cross Product

$$\vec{F} = \left(2\hat{i} + 3\hat{j} - 2\hat{k}\right)\mathrm{N} \qquad \vec{r} = \left(3\hat{i} - 4\hat{j} - 6\hat{k}\right)\mathrm{m}$$

$$\vec{\tau} = \left(\vec{r} \times \vec{F}\right)$$

$$\vec{\tau} = (2\mathrm{N})(3\mathrm{m})\left(\hat{i}\times\hat{i}\right) + (2\mathrm{N})(-4\mathrm{m})\left(\hat{i}\times\hat{j}\right) + (2\mathrm{N})(-6\mathrm{m})\left(\hat{i}\times\hat{k}\right)$$

$$+ (3\mathrm{N})(3\mathrm{m})\left(\hat{j}\times\hat{i}\right) + (3\mathrm{N})(-4\mathrm{m})\left(\hat{j}\times\hat{j}\right) + (3\mathrm{N})(-6\mathrm{m})\left(\hat{j}\times\hat{k}\right)$$

$$+ (-2\mathrm{N})(3\mathrm{m})\left(\hat{k}\times\hat{i}\right) + (-2\mathrm{N})(-4\mathrm{m})\left(\hat{k}\times\hat{j}\right) + (-2\mathrm{N})(-6\mathrm{m})\left(\hat{k}\times\hat{k}\right)$$

$$\vec{\tau} = \left(26\hat{i} - 6\hat{j} + 17\hat{k}\right)\mathrm{N}\cdot\mathrm{m}$$

Vector Multiplication II: The Cross Product

$$\vec{F} = \left(2\hat{i} + 3\hat{j} - 2\hat{k}\right)\mathrm{N} \qquad \vec{r} = \left(3\hat{i} - 4\hat{j} - 6\hat{k}\right)\mathrm{m}$$

$$\vec{\tau} = \left(\vec{r} \times \vec{F}\right)$$

$$\begin{matrix} \hat{j} & \hat{k} \\ \end{matrix} \quad \begin{pmatrix} -4 & -6 \\ 3 & -2 \end{pmatrix}$$

$$\begin{matrix} \hat{i} & \hat{j} \\ \end{matrix} \quad \begin{pmatrix} 3 & -4 \\ 2 & 3 \end{pmatrix}$$

$$\begin{matrix} \hat{k} & \hat{i} \\ \end{matrix} \quad \begin{pmatrix} -6 & 3 \\ -2 & 2 \end{pmatrix}$$

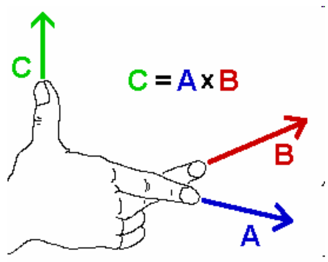
$$\vec{\tau} = \left(-4(-2) - (3)(-6)\right)\hat{i}$$

$$+ \left(-6(2) - (3)(-2)\right)\hat{j}$$

$$+ \left(3(3) - (2)(-4)\right)\hat{k}$$

$$\vec{\tau} = \left(26\hat{i} - 6\hat{j} + 17\hat{k}\right)\mathrm{N}\cdot\mathrm{m}$$

Vector Multiplication II: Right Hand Rule



Index finger in the direction of the first vector.

Middle finger in the direction of the second vector

Thumb points in the direction of the cross product.

WARNING: Make sure you are using your right hand!!!