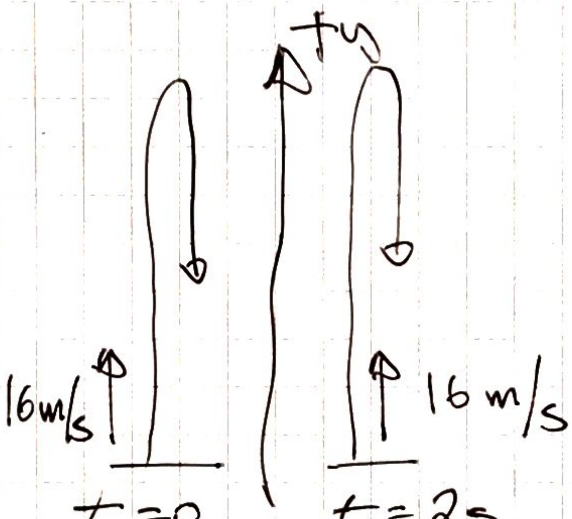
Physics Example

Framing:

Assume: no air drag
$g = 10\ m/s^2$



$\Rightarrow \bar{F}_{net} = m\bar{a}$

$-mg = m\bar{a}$

$\Rightarrow \bar{a} = -g$

Questions: How high?
How long to top or round trip?
Where do they pass each other?

Estimate $a = -10\ m/s^2 \Rightarrow (-10\ m/s)/s \Rightarrow 10\ m/s$ is take from speed each s $\Rightarrow$ 1.6 sec to stop (at top) $\Rightarrow$ 3.2 s RTrip.

$\Rightarrow$ 1st one is coming down when 2nd one passes on way up

starts @ 16 m/s & finishes @ 0 m/s $\Rightarrow V_{ave} = \dfrac{16\ m/s + 0\ m/s}{2} = 8\ m/s$

$y_f \simeq 8\ m/s \cdot 1.6\ s = \underline{12\ m}$

Strategy: use eqns for a = cnst, count unknowns

Data:

$y_0 = 0\,m$

$y_f = ??$

$v_{y_0} = 16\,m/s$

$v_{y_f} = 0\,m/s$

$a_y = -g$

$t_0 = 0,\ t_f = ?$

to find top

① $v_f = v_0 + a_y \Delta t$ ($v_f \to 0$)

② $y_f = y_0 + v_0 \Delta t + \frac{1}{2} a t^2$ ($y_0 \to 0$)

③ $v_f^2 = v_0^2 + 2a_y(y_f - y_0)$ ($v_f \to 0$, $y_0 \to 0$)

1 & 3 look good

$0 = v_0 + a_y \Delta t$

$\frac{-v_0}{a_y} = \Delta t = \frac{+16\,m/s}{+10\,m/s^2} = \underline{1.6\,s}$

$y_{1_0} = 0$ $\quad$ $y_{2_0} = 0$

$y_{1f} = ??$ $\overset{\text{same}}{\Longleftrightarrow}$ $y_{2f} = ??$

$v_{y_{1_0}} = 16\,m/s$ $\quad$ $v_{y_{2_0}} = 16\,m/s$

$v_{y_{1f}} = ??$ $\quad$ $v_{y_{2f}} = ??$

$a_y = -g$ $\quad$ $a_y = -g$

$t_0 = 0,\ t_f = ?$ $\quad$ $t_0 = 2s,\ t_f = ?$

passing ↳ implies same y

$v_{1f} = v_{1_0} + a t$ $\quad$ $v_{2f} = v_{2_0} + a(t - t_0)$

$y_{1f} = y_{1_0} + v_{1_0} \Delta t + \frac{1}{2} a \Delta t^2$ $\quad$ $y_{2f} = y_{2_0} + v_{2_0} \Delta t + \frac{1}{2} a (\Delta t)^2$

$v_{1f}^2 = v_{1_0}^2 + 2a(y_f - y_0)$ $\quad$ $v_{2f}^2 = v_{2_0}^2 + 2a(y_f - y_0)$

all eqns have 2 unknowns

I need 2 eqns w/ same 2 unknown.

yikes — until I notice that

$y_{1f} = y_{2f}$ <u>if</u> they are passing!

t to top $= 1.6\,s$

$\Rightarrow t$ round trip $= 3.2\,s$

now that I have t eqn #2 only has 1 unknown or I can use #3

#3 $0 = V_0^2 + 2a(y_f)$

$$-\frac{V_0^2}{2a} = y_f = \frac{-(16\,m/s)^2}{2(-10\,m/s^2)}$$

$$y_f = \frac{+256\,m^2/s^2}{+20\,m/s^2}$$

$y_f = 12.8\,m$ @ top!

Discussion – matches estimate and units are right – feels good.

$y_{1f} = y_{2f} \Rightarrow$

$$y_{1_0} + V_{1_0}\Delta t_1 + \tfrac{1}{2}a\Delta t_1^2 = y_{2_0} + V_{2_0}\Delta t_2 + \tfrac{1}{2}a\Delta t_2^2$$

($y_{1_0} \to 0$, $y_{2_0} \to 0$)

$\Delta t_1 = \Delta t_2 + 2s$ ← various way to do this

$$V_{1_0}(\Delta t_2 + 2s) + \tfrac{1}{2}a(\Delta t_2 + 2s)^2 = V_{2_0}\Delta t_2 + \tfrac{1}{2}a\Delta t_2^2$$

remembering $V_{1_0} = V_{2_0} = V_0$

$$V_0(\Delta t_2) + V_0(2s) + \tfrac{1}{2}a(\Delta t_2^2 + 4s\Delta t_2 + 4s^2) = V_0\Delta t_2 + \tfrac{1}{2}a\Delta t_2^2$$

cancel! also cancel!

$$V_0(2s) + \tfrac{1}{2}a(4s\Delta t_2 + 4s^2) = 0$$

$$\tfrac{1}{2}a(4s\Delta t_2 + 4s^2) = -V_0(2s)$$

$$4s\Delta t_2 + 4s^2 = \frac{-V_0(2s)}{\tfrac{1}{2}a}$$

$$\Delta t_2 = \frac{\left(\frac{-V_0(2s)}{\tfrac{1}{2}a} - 4s^2\right)}{4s}$$

$$\Delta t_2 = \frac{\left[\frac{+(16\,m/s)\,2s}{-5\,m/s^2} - 4s^2\right]}{4s}$$

has units of s!! yay!

$$\Delta t_2 = \frac{\frac{32s^2}{5} - 4s^2}{4s} = \frac{5.4 - 4}{4}\,s$$

$\Delta t_2 = .35\,s$ after 2nd ball is thrown

$$y_f = y_0^{\nearrow 0} + v_0 \Delta t + \frac{1}{2} a \Delta t^2$$

$$y_f = 16\,m/s\,(.35\,s) + \frac{1}{2}(-10\,m/s^2)(.35\,s)^2$$

$$= 5.6\,m - .61\,m = 4.99\,m$$ on the way up for 2nd ball

Discussion: Give 12 m is peak passing 5 m above starting point feels reasonable given that 1st ball is past peak before 2nd Ball is thrown.