12.7 In a Δ-Δ system, a phase voltage of 100 V produces a line voltage of:
(a) 58 V (b) 71 V (c) 100 V
(d) 173 V (e) 141 V

12.8 When a Y-connected load is supplied by voltages in *abc* phase sequence, the line voltages lag the corresponding phase voltages by 30°.
(a) True (b) False

12.9 In a balanced three-phase circuit, the total instantaneous power is equal to the average power.
(a) True (b) False

12.10 The total power supplied to a balanced Δ-load is found in the same way as for a balanced Y-load.
(a) True (b) False

Answers: 12.1a, 12.2a, 12.3c, 12.4a, 12.5b, 12.6e, 12.7c, 12.8b, 12.9a, 12.10a.

PROBLEMS[1]

Section 12.2 Balanced Three-Phase Voltages

12.1 If $\mathbf{V}_{ab} = 400$ V in a balanced Y-connected three-phase generator, find the phase voltages, assuming the phase sequence is:
(a) *abc* (b) *acb*

12.2 What is the phase sequence of a balanced three-phase circuit for which $\mathbf{V}_{an} = 160\angle 30^\circ$ V and $\mathbf{V}_{cn} = 160\angle -90^\circ$ V? Find $\mathbf{V}_{bn}$.

12.3 Determine the phase sequence of a balanced three-phase circuit in which $\mathbf{V}_{bn} = 208\angle 130^\circ$ V and $\mathbf{V}_{cn} = 208\angle 10^\circ$ V. Obtain $\mathbf{V}_{an}$.

12.4 Assuming the *abc* sequence, if $\mathbf{V}_{ca} = 208\angle 20^\circ$ V in a balanced three-phase circuit, find $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, $\mathbf{V}_{an}$, and $\mathbf{V}_{bn}$.

12.5 Given that the line voltages of a three-phase circuit are

$$\mathbf{V}_{ab} = 420\angle 0^\circ, \quad \mathbf{V}_{bc} = 420\angle -120^\circ$$
$$\mathbf{V}_{ac} = 420\angle 120^\circ \text{ V}$$

find the phase voltages $\mathbf{V}_{an}$, $\mathbf{V}_{bn}$, and $\mathbf{V}_{cn}$.

Section 12.3 Balanced Wye-Wye Connection

12.6 For the Y-Y circuit of Fig. 12.41, find the line currents, the line voltages, and the load voltages.

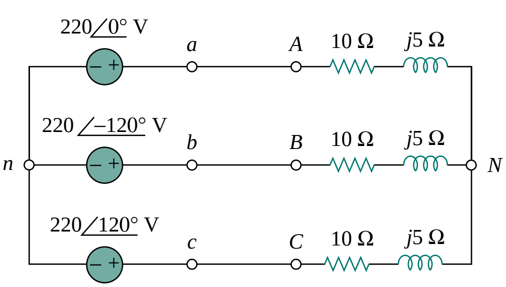

Figure 12.41 For Prob. 12.6.

12.7 Obtain the line currents in the three-phase circuit of Fig. 12.42 below.

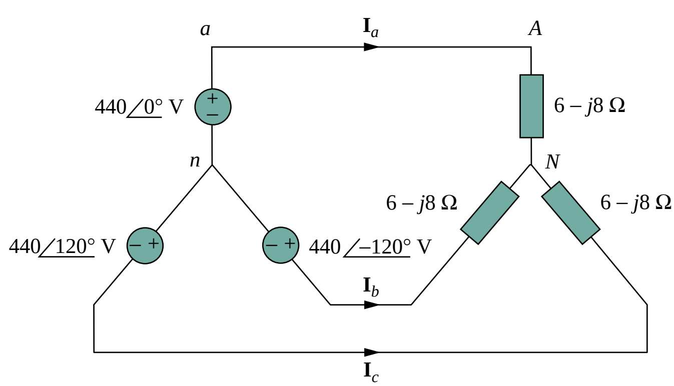

Figure 12.42 For Prob. 12.7.

[1] Remember that unless stated otherwise, all given voltages and currents are rms values.

12.8 A balanced Y-connected load with a phase impedance of $16 + j9\ \Omega$ is connected to a balanced three-phase source with a line voltage of 220 V. Calculate the line current I_L.

12.9 A balanced Y-Y four-wire system has phase voltages

$$\mathbf{V}_{an} = 120\angle 0^\circ, \qquad \mathbf{V}_{bn} = 120\angle -120^\circ$$
$$\mathbf{V}_{cn} = 120\angle 120^\circ \text{ V}$$

The load impedance per phase is $19 + j13\ \Omega$, and the line impedance per phase is $1 + j2\ \Omega$. Solve for the line currents and neutral current.

12.10 For the circuit in Fig. 12.43, determine the current in the neutral line.

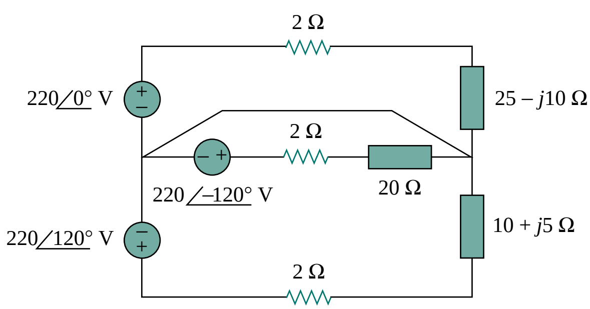

Figure 12.43 For Prob. 12.10.

Section 12.4 Balanced Wye-Delta Connection

12.11 For the three-phase circuit of Fig. 12.44, $\mathbf{I}_{bB} = 30\angle 60^\circ$ A and $\mathbf{V}_{BC} = 220\angle 10^\circ$ V. Find $\mathbf{V}_{an}$, $\mathbf{V}_{AB}$, $\mathbf{I}_{AC}$, and $\mathbf{Z}$.

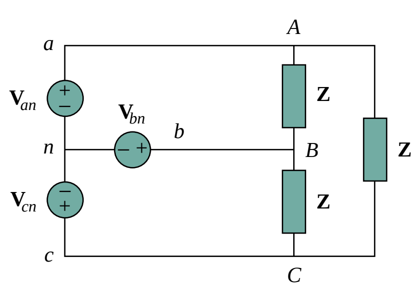

Figure 12.44 For Prob. 12.11.

12.12 Solve for the line currents in the Y-Δ circuit of Fig. 12.45. Take $\mathbf{Z}_\Delta = 60\angle 45^\circ\ \Omega$.

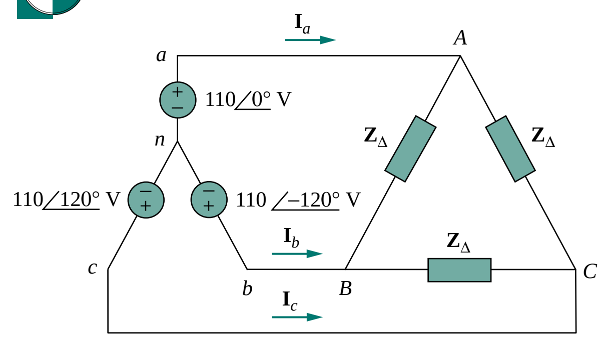

Figure 12.45 For Prob. 12.12.

12.13 The circuit in Fig. 12.46 is excited by a balanced three-phase source with a line voltage of 210 V. If $\mathbf{Z}_\ell = 1 + j1\ \Omega$, $\mathbf{Z}_\Delta = 24 - j30\ \Omega$, and $\mathbf{Z}_Y = 12 + j5\ \Omega$, determine the magnitude of the line current of the combined loads.

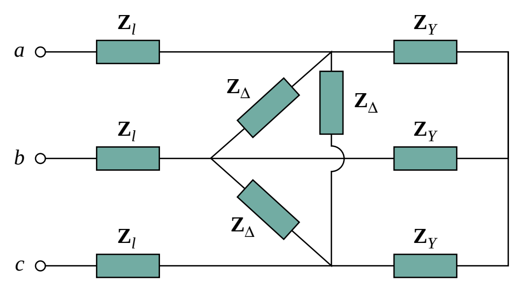

Figure 12.46 For Prob. 12.13.

12.14 A balanced delta-connected load has a phase current $\mathbf{I}_{AC} = 10\angle -30^\circ$ A.

(a) Determine the three line currents assuming that the circuit operates in the positive phase sequence.

(b) Calculate the load impedance if the line voltage is $\mathbf{V}_{AB} = 110\angle 0^\circ$ V.

12.15 In a wye-delta three-phase circuit, the source is a balanced, positive phase sequence with $\mathbf{V}_{an} = 120\angle 0^\circ$ V. It feeds a balanced load with $\mathbf{Z}_\Delta = 9 + j12\ \Omega$ per phase through a balanced line with $\mathbf{Z}_\ell = 1 + j0.5\ \Omega$ per phase. Calculate the phase voltages and currents in the load.

12.16 If $\mathbf{V}_{an} = 440\angle 60^\circ$ V in the network of Fig. 12.47, find the load phase currents $\mathbf{I}_{AB}$, $\mathbf{I}_{BC}$, and $\mathbf{I}_{CA}$.

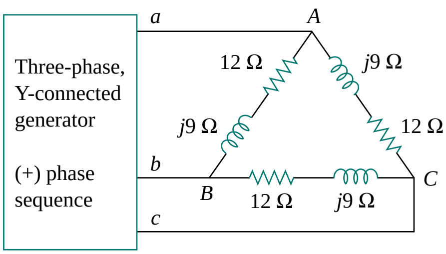

Figure 12.47 For Prob. 12.16.

Section 12.5 Balanced Delta-Delta Connection

12.17 For the Δ-Δ circuit of Fig. 12.48, calculate the phase and line currents.

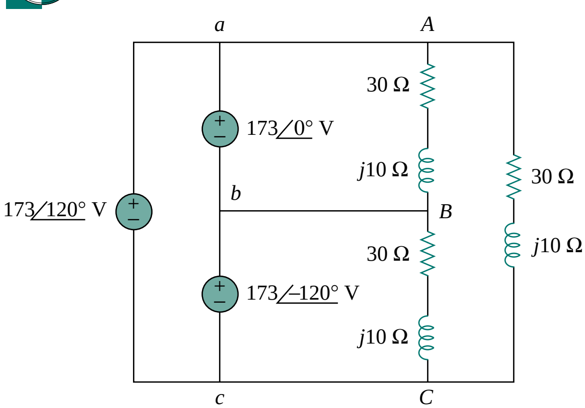

Figure 12.48 For Prob. 12.17.

12.18 Refer to the Δ-Δ circuit in Fig. 12.49. Find the line and phase currents. Assume that the load impedance is $12 + j9\ \Omega$ per phase.

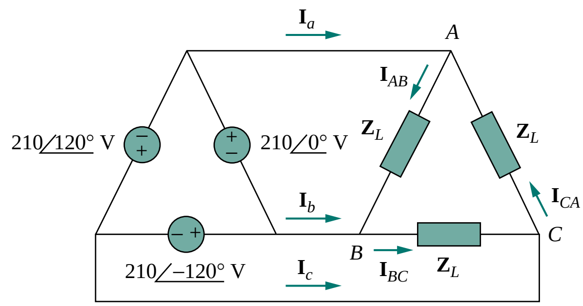

Figure 12.49 For Prob. 12.18.

12.19 Find the line currents $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$ in the three-phase network of Fig. 12.50 below. Take $\mathbf{Z}_\Delta = 12 - j15\ \Omega$, $\mathbf{Z}_Y = 4 + j6\ \Omega$, and $\mathbf{Z}_\ell = 2\ \Omega$.

12.20 A balanced delta-connected source has phase voltage $\mathbf{V}_{ab} = 416\angle 30^\circ$ V and a positive phase sequence. If this is connected to a balanced delta-connected load, find the line and phase currents. Take the load impedance per phase as $60\angle 30^\circ\ \Omega$ and line impedance per phase as $1 + j1\ \Omega$.

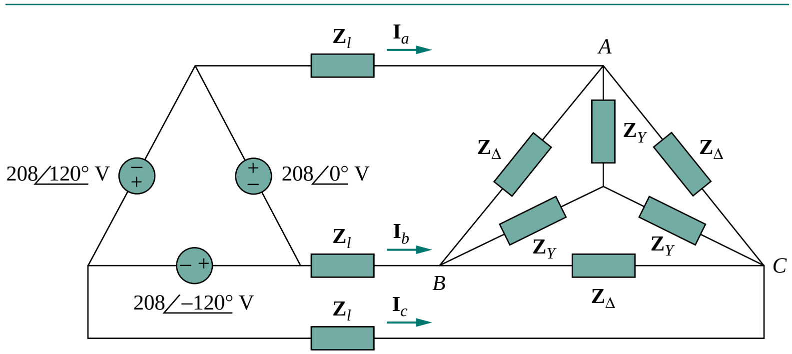

Figure 12.50 For Prob. 12.19.

Section 12.6 Balanced Delta-Wye Connection

12.21 In the circuit of Fig. 12.51, if $\mathbf{V}_{ab} = 440\angle 10°$, $\mathbf{V}_{bc} = 440\angle 250°$, $\mathbf{V}_{ca} = 440\angle 130°$ V, find the line currents.

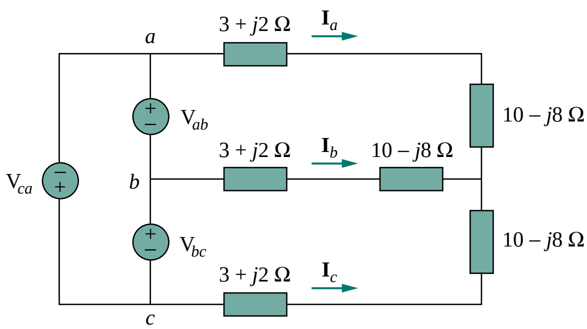

Figure 12.51 For Prob. 12.21.

12.22 For the balanced circuit in Fig. 12.52, $\mathbf{V}_{ab} = 125\angle 0°$ V. Find the line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$.

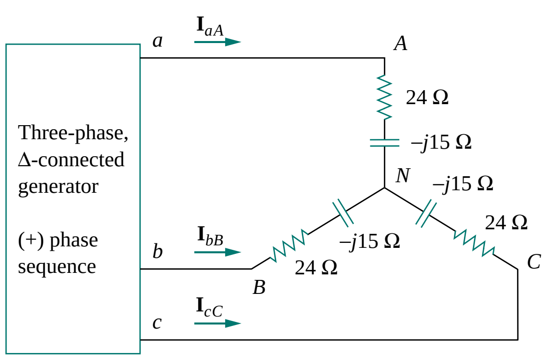

Figure 12.52 For Prob. 12.22.

12.23 In a balanced three-phase Δ-Y circuit, the source is connected in the positive sequence, with $\mathbf{V}_{ab} = 220\angle 20°$ V and $\mathbf{Z}_Y = 20 + j15\ \Omega$. Find the line currents.

12.24 A delta-connected generator supplies a balanced wye-connected load with an impedance of $30\angle -60°\ \Omega$. If the line voltages of the generator have a magnitude of 400 V and are in the positive phase sequence, find the line current I_L and phase voltage V_p at the load.

Section 12.7 Power in a Balanced System

12.25 A balanced wye-connected load absorbs a total power of 5 kW at a leading power factor of 0.6 when connected to a line voltage of 240 V. Find the impedance of each phase and the total complex power of the load.

12.26 A balanced wye-connected load absorbs 50 kVA at a 0.6 lagging power factor when the line voltage is 440 V. Find the line current and the phase impedance.

12.27 A three-phase source delivers 4800 VA to a wye-connected load with a phase voltage of 208 V and a power factor of 0.9 lagging. Calculate the source line current and the source line voltage.

12.28 A balanced wye-connected load with a phase impedance of $10 - j16\ \Omega$ is connected to a balanced three-phase generator with a line voltage of 220 V. Determine the line current and the complex power absorbed by the load.

12.29 The total power measured in a three-phase system feeding a balanced wye-connected load is 12 kW at a power factor of 0.6 leading. If the line voltage is 208 V, calculate the line current I_L and the load impedance $\mathbf{Z}_Y$.

12.30 Given the circuit in Fig. 12.53 below, find the total complex power absorbed by the load.

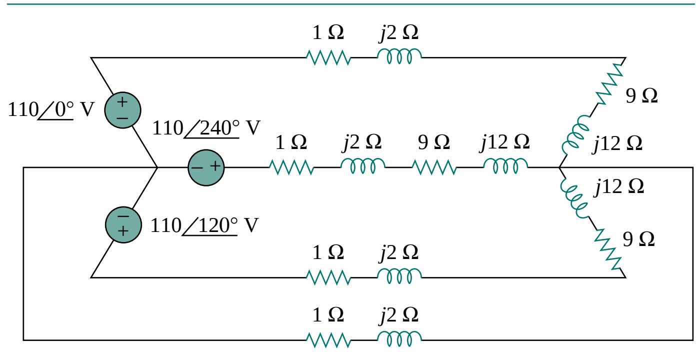

Figure 12.53 For Prob. 12.30.

12.31 Find the real power absorbed by the load in Fig. 12.54.

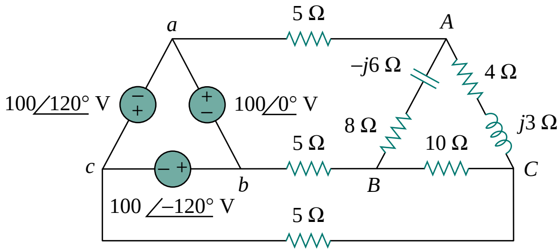

Figure 12.54 For Prob. 12.31.

12.32 For the three-phase circuit in Fig. 12.55, find the average power absorbed by the delta-connected load with $\mathbf{Z}_\Delta = 21 + j24\ \Omega$.

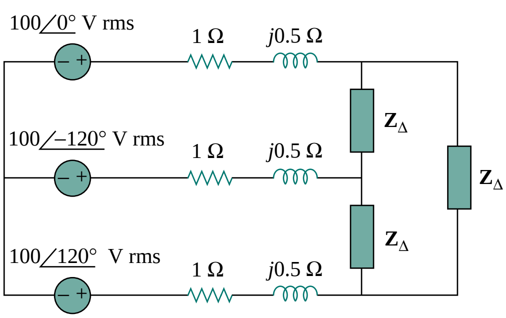

Figure 12.55 For Prob. 12.32.

12.33 A balanced delta-connected load draws 5 kW at a power factor of 0.8 lagging. If the three-phase system has an effective line voltage of 400 V, find the line current.

12.34 A balanced three-phase generator delivers 7.2 kW to a wye-connected load with impedance $30 - j40\ \Omega$ per phase. Find the line current I_L and the line voltage V_L.

12.35 Refer to Fig. 12.46. Obtain the complex power absorbed by the combined loads.

12.36 A three-phase line has an impedance of $1 + j3\ \Omega$ per phase. The line feeds a balanced delta-connected load, which absorbs a total complex power of $12 + j5$ kVA. If the line voltage at the load end has a magnitude of 240 V, calculate the magnitude of the line voltage at the source end and the source power factor.

12.37 A balanced wye-connected load is connected to the generator by a balanced transmission line with an impedance of $0.5 + j2\ \Omega$ per phase. If the load is rated at 450 kW, 0.708 power factor lagging, 440-V line voltage, find the line voltage at the generator.

12.38 A three-phase load consists of three 100-Ω resistors that can be wye- or delta-connected. Determine which connection will absorb the most average power from a three-phase source with a line voltage of 110 V. Assume zero line impedance.

12.39 The following three parallel-connected three-phase loads are fed by a balanced three-phase source.

Load 1: 250 kVA, 0.8 pf lagging
Load 2: 300 kVA, 0.95 pf leading
Load 3: 450 kVA, unity pf

If the line voltage is 13.8 kV, calculate the line current and the power factor of the source. Assume that the line impedance is zero.

Section 12.8 Unbalanced Three-Phase Systems

12.40 For the circuit in Fig. 12.56, $\mathbf{Z}_a = 6 - j8\ \Omega$, $\mathbf{Z}_b = 12 + j9\ \Omega$, and $\mathbf{Z}_c = 15\ \Omega$. Find the line currents $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$.

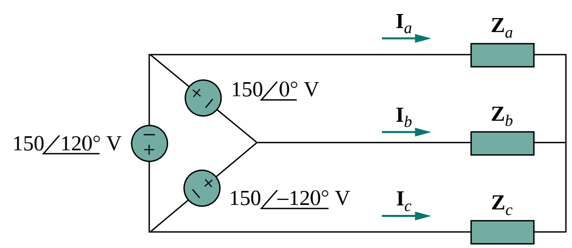

Figure 12.56 For Prob. 12.40.

12.41 A four-wire wye-wye circuit has

$$\mathbf{V}_{an} = 120\angle 120^\circ, \qquad \mathbf{V}_{bn} = 120\angle 0^\circ$$
$$\mathbf{V}_{cn} = 120\angle -120^\circ\ \text{V}$$

If the impedances are

$$\mathbf{Z}_{AN} = 20\angle 60^\circ, \qquad \mathbf{Z}_{BN} = 30\angle 0^\circ$$
$$\mathbf{Z}_{cn} = 40\angle 30^\circ\ \Omega$$

find the current in the neutral line.

12.42 For the wye-connected load of Fig. 12.57, the line voltages all have a magnitude of 250 V and are in a positive phase sequence. Calculate the line currents and the neutral current.

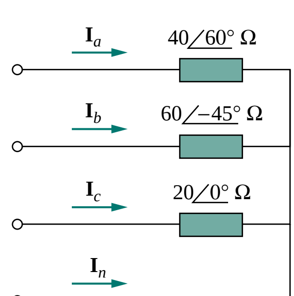

Figure 12.57 For Prob. 12.42.

12.43 A delta-connected load whose phase impedances are $\mathbf{Z}_{AB} = 50\ \Omega$, $\mathbf{Z}_{BC} = -j50\ \Omega$, and $\mathbf{Z}_{CA} = j50\ \Omega$ is fed by a balanced wye-connected three-phase source with $V_p = 100$ V. Find the phase currents.

12.44 A balanced three-phase wye-connected generator with $V_p = 220$ V supplies an unbalanced wye-connected load with $\mathbf{Z}_{AN} = 60 + j80\ \Omega$, $\mathbf{Z}_{BN} = 100 - j120\ \Omega$, and $\mathbf{Z}_{CN} = 30 + j40\ \Omega$. Find the total complex power absorbed by the load.

12.45 Refer to the unbalanced circuit of Fig. 12.58. Calculate:

(a) the line currents

(b) the real power absorbed by the load

(c) the total complex power supplied by the source

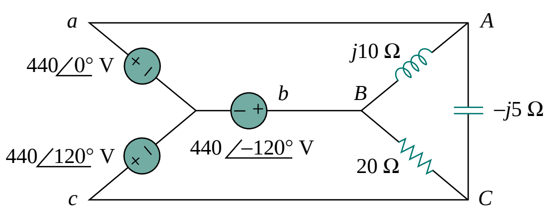

Figure 12.58 For Prob. 12.45.

Section 12.9 *PSpice* for Three-Phase Circuits

12.46 Solve Prob. 12.10 using *PSpice*.

12.47 The source in Fig. 12.59 is balanced and exhibits a positive phase sequence. If $f = 60$ Hz, use *PSpice* to find $\mathbf{V}_{AN}$, $\mathbf{V}_{BN}$, and $\mathbf{V}_{CN}$.

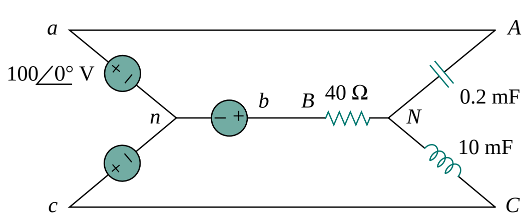

Figure 12.59 For Prob. 12.47.

12.48 Use *PSpice* to determine $\mathbf{I}_o$ in the single-phase, three-wire circuit of Fig. 12.60. Let $\mathbf{Z}_1 = 15 - j10\ \Omega$, $\mathbf{Z}_2 = 30 + j20\ \Omega$, and $\mathbf{Z}_3 = 12 + j5\ \Omega$.

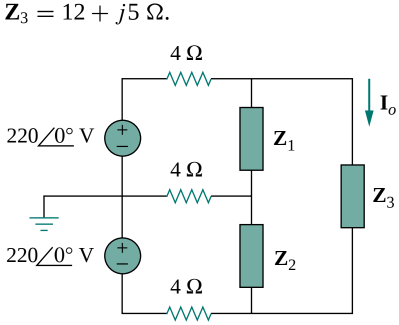

Figure 12.60 For Prob. 12.48.

12.49 Given the circuit in Fig. 12.61, use *PSpice* to determine currents $\mathbf{I}_{aA}$ and voltage $\mathbf{V}_{BN}$.

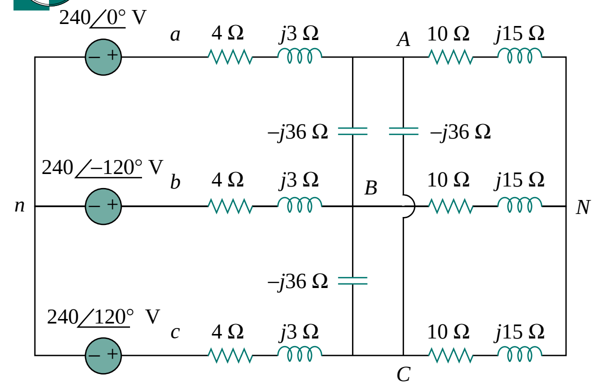

Figure 12.61 For Prob. 12.49.

12.50 The circuit in Fig. 12.62 operates at 60 Hz. Use *PSpice* to find the source current $\mathbf{I}_{ab}$ and the line current $\mathbf{I}_{bB}$.

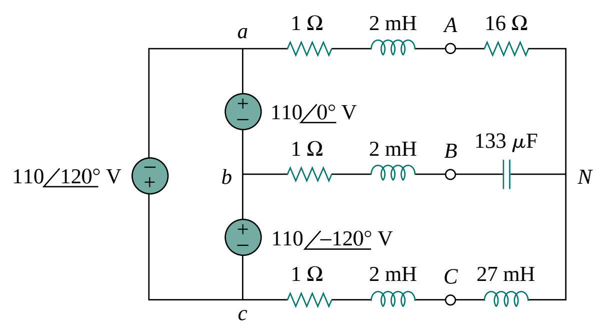

Figure 12.62 For Prob. 12.50.

12.51 For the circuit in Fig. 12.54, use *PSpice* to find the line currents and the phase currents.

12.52 A balanced three-phase circuit is shown in Fig. 12.63 on the next page. Use *PSpice* to find the line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$.

Section 12.10 Applications

12.53 A three-phase, four-wire system operating with a 208-V line voltage is shown in Fig. 12.64. The source voltages are balanced. The power absorbed by the resistive wye-connected load is measured by the three-wattmeter method. Calculate:

(a) the voltage to neutral

(b) the currents $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$, and $\mathbf{I}_n$

(c) the readings of the wattmeters

(d) the total power absorbed by the load

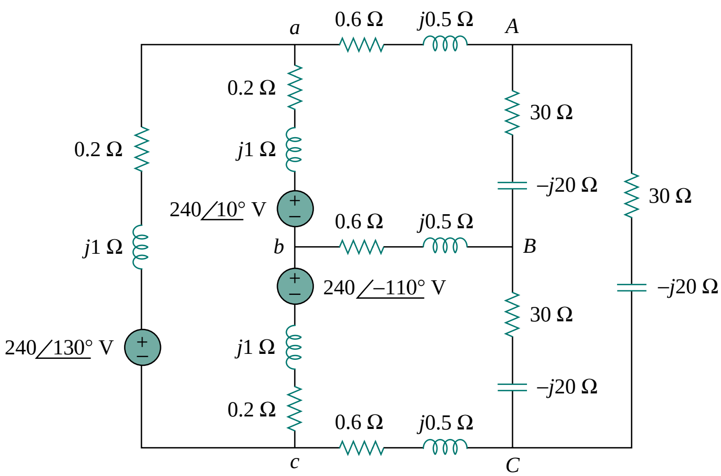

Figure 12.63 For Prob. 12.52.

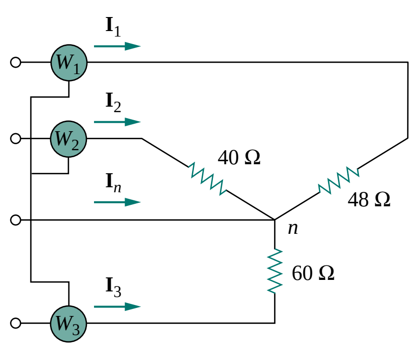

Figure 12.64 For Prob. 12.53.

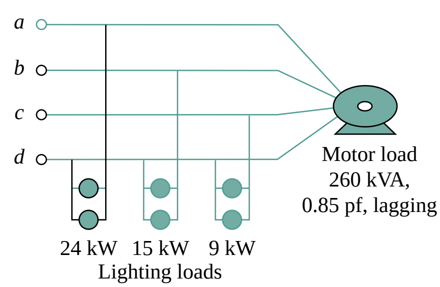

Figure 12.65 For Prob. 12.54.

*__12.54__ As shown in Fig. 12.65, a three-phase four-wire line with a phase voltage of 120 V supplies a balanced motor load at 260 kVA at 0.85 pf lagging. The motor load is connected to the three main lines marked *a*, *b*, and *c*. In addition, incandescent lamps (unity pf) are connected as follows: 24 kW from line *a* to the neutral, 15 kW from line *b* to the neutral, and 9 kW from line *a* to the neutral.

(a) If three wattmeters are arranged to measure the power in each line, calculate the reading of each meter.

(b) Find the current in the neutral line.

12.55 Meter readings for a three-phase wye-connected alternator supplying power to a motor indicate that the line voltages are 330 V, the line currents are 8.4 A, and the total line power is 4.5 kW. Find:

(a) the load in VA

(b) the load pf

(c) the phase current

(d) the phase voltage

12.56 The two-wattmeter method gives $P_1 = 1200$ W and $P_2 = -400$ W for a three-phase motor running on a 240-V line. Assume that the motor load is wye-connected and that it draws a line current of 6 A. Calculate the pf of the motor and its phase impedance.

*An asterisk indicates a challenging problem.

12.57 In Fig. 12.66, two wattmeters are properly connected to the unbalanced load supplied by a balanced source such that $\mathbf{V}_{ab} = 208\angle 0^\circ$ V with positive phase sequence.

(a) Determine the reading of each wattmeter.

(b) Calculate the total apparent power absorbed by the load.

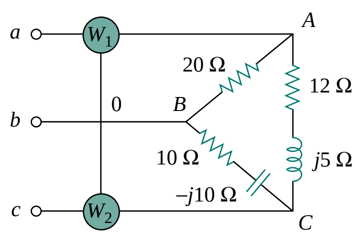

Figure 12.66 For Prob. 12.57.

12.58 If wattmeters W_1 and W_2 are properly connected respectively between lines a and b and lines b and c to measure the power absorbed by the delta-connected load in Fig. 12.44, predict their readings.

12.59 For the circuit displayed in Fig. 12.67, find the wattmeter readings.

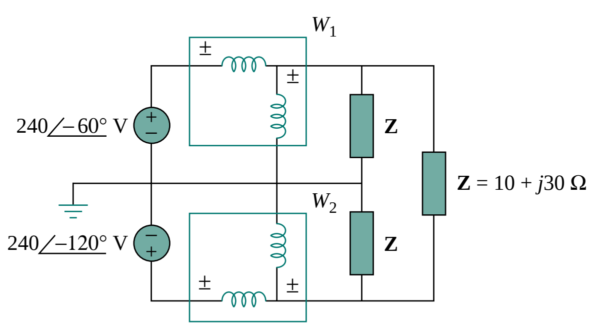

Figure 12.67 For Prob. 12.59.

12.60 Predict the wattmeter readings for the circuit in Fig. 12.68.

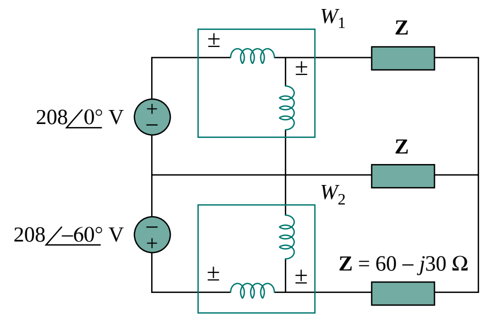

Figure 12.68 For Prob. 12.60.

12.61 A man has a body resistance of 600 Ω. How much current flows through his ungrounded body:

(a) when he touches the terminals of a 12-V autobattery?

(b) when he sticks his finger into a 120-V light socket?

12.62 Show that the I^2R losses will be higher for a 120-V appliance than for a 240-V appliance if both have the same power rating.

COMPREHENSIVE PROBLEMS

12.63 A three-phase generator supplied 3.6 kVA at a power factor of 0.85 lagging. If 2500 W are delivered to the load and line losses are 80 W per phase, what are the losses in the generator?

12.64 A three-phase 440-V, 51-kW, 60-kVA inductive load operates at 60 Hz and is wye-connected. It is desired to correct the power factor to 0.95 lagging. What value of capacitor should be placed in parallel with each load impedance?

12.65 A balanced three-phase generator has an abc phase sequence with phase voltage $\mathbf{V}_{an} = 255\angle 0^\circ$ V. The generator feeds an induction motor which may be represented by a balanced Y-connected load with an impedance of $12 + j5$ Ω per phase. Find the line currents and the load voltages. Assume a line impedance of 2 Ω per phase.

12.66 Three balanced loads are connected to a distribution line as depicted in Fig. 12.69. The loads are

Transformer: 12 kVA at 0.6 pf lagging
Motor: 16 kVA at 0.8 pf lagging
Unknown load: — — ——

If the line voltage is 220 V, the line current is 120 A, and the power factor of the combined load is 0.95 lagging, determine the unknown load.

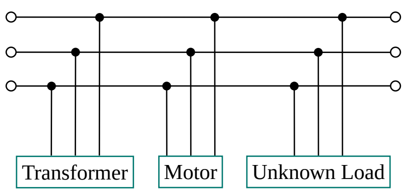

Figure 12.69 For Prob. 12.66.

12.67 A professional center is supplied by a balanced three-phase source. The center has four plants, each a balanced three-phase load as follows:

Load 1: 150 kVA at 0.8 pf leading
Load 2: 100 kW at unity pf
Load 3: 200 kVA at 0.6 pf lagging
Load 4: 80 kW and 95 kVAR (inductive)

If the line impedance is $0.02 + j0.05\ \Omega$ per phase and the line voltage at the loads is 480 V, find the magnitude of the line voltage at the source.

***12.68** Figure 12.70 displays a three-phase delta-connected motor load which is connected to a line voltage of 440 V and draws 4 kVA at a power factor of 72 percent lagging. In addition, a single 1.8 kVAR capacitor is connected between lines a and b, while a 800-W lighting load is connected between line c and neutral. Assuming the abc sequence and taking $\mathbf{V}_{an} = V_p\angle 0°$, find the magnitude and phase angle of currents $\mathbf{I}_a$, $\mathbf{I}_b$, $\mathbf{I}_c$, and $\mathbf{I}_n$.

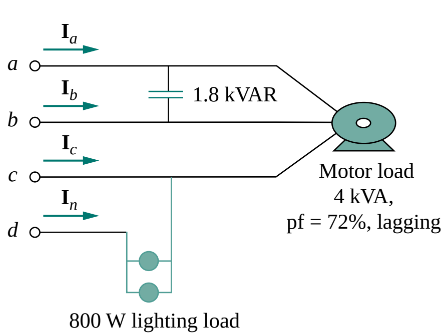

Figure 12.70 For Prob. 12.68.

12.69 Design a three-phase heater with suitable symmetric loads using wye-connected pure resistance. Assume that the heater is supplied by a 240-V line voltage and is to give 27 kW of heat.

12.70 For the single-phase three-wire system in Fig. 12.71, find currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{nN}$.

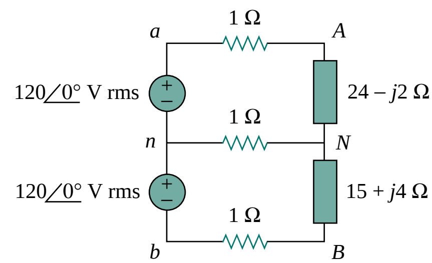

Figure 12.71 For Prob. 12.70.

12.71 Consider the single-phase three-wire system shown in Fig. 12.72. Find the current in the neutral wire and the complex power supplied by each source. Take $\mathbf{V}_s$ as a $115\angle 0°$-V, 60-Hz source.

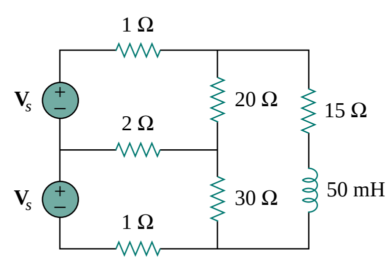

Figure 12.72 For Prob. 12.71.