15. The Touch-Tone telephone and the crossover network are two typical applications of filters. The Touch-Tone telephone system employs filters to separate tones of different frequencies to activate electronic switches. The crossover network separates signals in different frequency ranges so that they can be delivered to different devices such as tweeters and woofers in a loudspeaker system.

REVIEW QUESTIONS

14.1 A zero of the transfer function

$$H(s) = \frac{10(s+1)}{(s+2)(s+3)}$$

is at

(a) 10 (b) -1 (c) -2 (d) -3

14.2 On the Bode magnitude plot, the slope of the pole $1/(5+j\omega)^2$ is

(a) 20 dB/decade (b) 40 dB/decade
(c) -40 dB/decade (d) -20 dB/decade

14.3 On the Bode phase plot, the slope of $[1 + j10\omega - \omega^2/25]^2$ is

(a) 45°/decade (b) 90°/decade
(c) 135°/decade (d) 180°/decade

14.4 How much inductance is needed to resonate at 5 kHz with a capacitance of 12 nF?

(a) 2652 H (b) 11.844 H
(c) 3.333 H (d) 84.43 mH

14.5 The difference between the half-power frequencies is called the:

(a) quality factor (b) resonant frequency
(c) bandwidth (d) cutoff frequency

14.6 In a series RLC circuit, which of these quality factors has the steepest curve at resonance?

(a) $Q = 20$ (b) $Q = 12$
(c) $Q = 8$ (d) $Q = 4$

14.7 In a parallel RLC circuit, the bandwidth B is directly proportional to R.

(a) True (b) False

14.8 When the elements of an RLC circuit are both magnitude-scaled and frequency-scaled, which quality is unaffected?

(a) resistor (b) resonant frequency
(c) bandwidth (d) quality factor

14.9 What kind of filter can be used to select a signal of one particular radio station?

(a) lowpass (b) highpass
(c) bandpass (d) bandstop

14.10 A voltage source supplies a signal of constant amplitude, from 0 to 40 kHz, to an RC lowpass filter. The load resistor experiences the maximum voltage at:

(a) dc (b) 10 kHz
(c) 20 kHz (d) 40 kHz

Answers: 14.1b, 14.2c, 14.3d, 14.4d, 14.5c, 14.6a, 14.7b, 14.8d, 14.9c, 14.10a.

PROBLEMS

Section 14.2 Transfer Function

14.1 Find the transfer function $\mathbf{V}_o/\mathbf{V}_i$ of the RC circuit in Fig. 14.65.

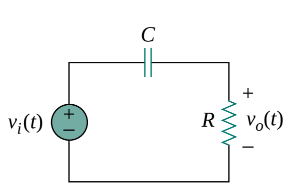

Figure 14.65 For Prob. 14.1.

14.2 Obtain the transfer function $\mathbf{V}_o/\mathbf{V}_i$ of the RL circuit of Fig. 14.66.

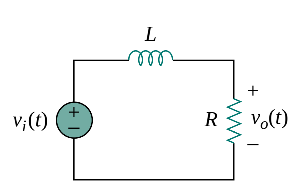

Figure 14.66 For Probs. 14.2 and 14.36.

14.3 (a) Given the circuit in Fig. 14.67, determine the transfer function $H(s) = V_o(s)/V_i(s)$.
(b) If $R = 40$ kΩ and $C = 2\ \mu$F, specify the locations of the poles and zeros of $H(s)$.

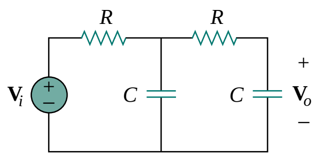

Figure 14.67 For Prob. 14.3.

14.4 Find the transfer function $\mathbf{H}(\omega) = \mathbf{V}_o/\mathbf{V}_i$ of the circuits shown in Fig. 14.68.

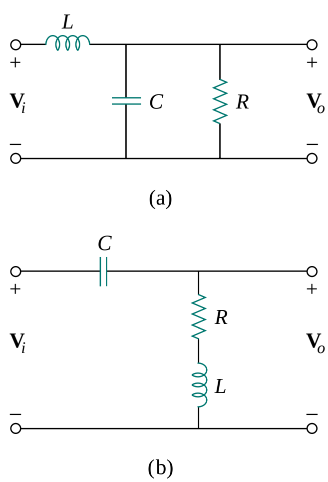

Figure 14.68 For Prob. 14.4.

14.5 Repeat Prob. 14.4 for the circuits in Fig. 14.69.

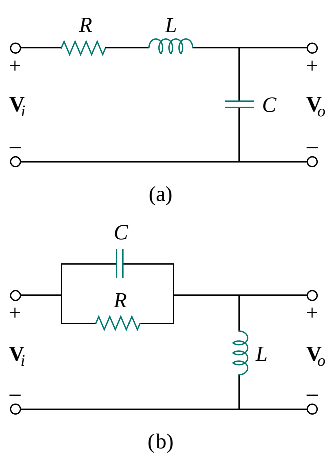

Figure 14.69 For Prob. 14.5.

14.6 Obtain the transfer function $\mathbf{H}(\omega) = \mathbf{I}_o/\mathbf{I}_s$ of the circuits shown in Fig. 14.70.

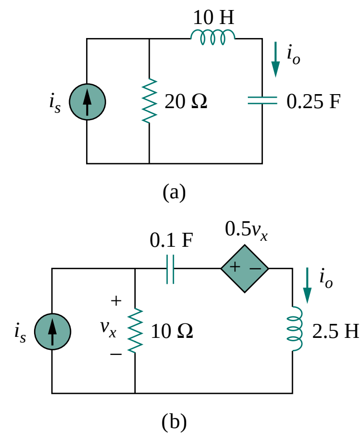

Figure 14.70 For Prob. 14.6.

Section 14.3 The Decibel Scale

14.7 Calculate $|\mathbf{H}(\omega)|$ if H_{dB} equals
(a) 0.05 dB (b) -6.2 dB (c) 104.7 dB

14.8 Determine the magnitude (in dB) and the phase (in degrees) of $\mathbf{H}(\omega)$ at $\omega = 1$ if $\mathbf{H}(\omega)$ equals
(a) 0.05 (b) 125
(c) $\dfrac{10j\omega}{2 + j\omega}$ (d) $\dfrac{3}{1 + j\omega} + \dfrac{6}{2 + j\omega}$

Section 14.4 Bode Plots

14.9 A ladder network has a voltage gain of

$$\mathbf{H}(\omega) = \frac{10}{(1 + j\omega)(10 + j\omega)}$$

Sketch the Bode plots for the gain.

14.10 Sketch the Bode plots for

$$\mathbf{H}(\omega) = \frac{10 + j\omega}{j\omega(2 + j\omega)}$$

14.11 Construct the Bode plots for

$$G(s) = \frac{s + 1}{s^2(s + 10)}, \qquad s = j\omega$$

14.12 Draw the Bode plots for

$$\mathbf{H}(\omega) = \frac{50(j\omega + 1)}{j\omega(-\omega^2 + 10j\omega + 25)}$$

14.13 Construct the Bode magnitude and phase plots for

$$H(s) = \frac{40(s + 1)}{(s + 2)(s + 10)}, \qquad s = j\omega$$

14.14 Sketch the Bode plots for

$$G(s) = \frac{s}{(s + 2)^2(s + 1)}, \qquad s = j\omega$$

14.15 Draw Bode plots for

$$G(s) = \frac{(s+2)^2}{s(s+5)(s+10)}, \qquad s = j\omega$$

14.16 A filter has

$$H(s) = \frac{s}{s^2 + 10s + 100}$$

Sketch the filter's Bode magnitude and phase plots.

14.17 Sketch Bode magnitude and phase plots for

$$N(s) = \frac{100(s^2 + s + 1)}{(s+1)(s+10)}, \qquad s = j\omega$$

Construct the straight-line approximate plots and the exact plots.

14.18 Construct Bode plots for

$$\mathbf{T}(\omega) = \frac{10j\omega(1 + j\omega)}{(10 + j\omega)(100 + 10j\omega - \omega^2)}$$

14.19 Find the transfer function $\mathbf{H}(\omega)$ with the Bode magnitude plot shown in Fig. 14.71.

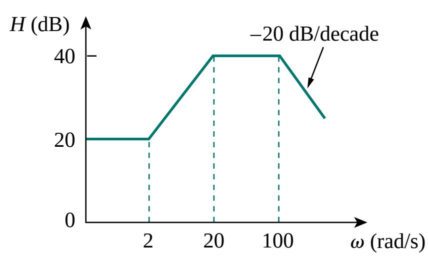

Figure 14.71 For Prob. 14.19.

14.20 The Bode magnitude plot of $\mathbf{H}(\omega)$ is shown in Fig. 14.72. Find $\mathbf{H}(\omega)$.

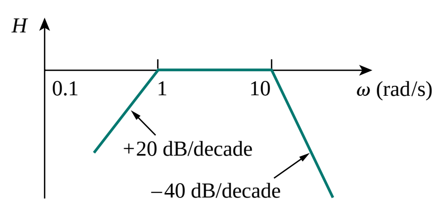

Figure 14.72 For Prob. 14.20.

14.21 The Bode phase plot of $\mathbf{G}(\omega)$ of a network is depicted in Fig. 14.73. Find $\mathbf{G}(\omega)$.

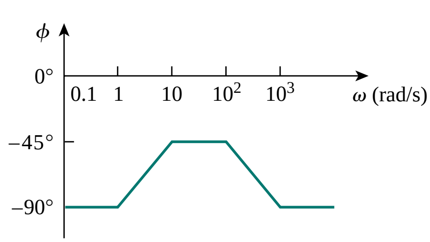

Figure 14.73 For Prob. 14.21.

Section 14.5 Series Resonance

14.22 A series RLC network has $R = 2\ \text{k}\Omega$, $L = 40$ mH, and $C = 1\ \mu$F. Calculate the impedance at resonance and at one-fourth, one-half, twice, and four times the resonant frequency.

14.23 Design a series RLC circuit that will have an impedance of 10 Ω at the resonant frequency of $\omega_0 = 50$ rad/s and a quality factor of 80. Find the bandwidth.

14.24 Design a series RLC circuit with $B = 20$ rad/s and $\omega_0 = 1000$ rad/s. Find the circuit's Q.

14.25 For the circuit in Fig. 14.74, find the frequency ω for which $v(t)$ and $i(t)$ are in phase.

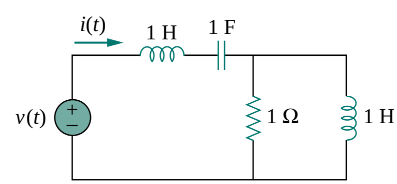

Figure 14.74 For Prob. 14.25.

Section 14.6 Parallel Resonance

14.26 Design a parallel resonant RLC circuit with $\omega_0 = 10$ rad/s and $Q = 20$. Calculate the bandwidth of the circuit.

14.27 A parallel resonant circuit with quality factor 120 has a resonant frequency of 6×10^6 rad/s. Calculate the bandwidth and half-power frequencies.

14.28 It is expected that a parallel RLC resonant circuit has a midband admittance of 25×10^3 S, quality factor of 80, and a resonant frequency of 200 krad/s. Calculate the values of R, L, and C. Find the bandwidth and the half-power frequencies.

14.29 Rework Prob. 14.22 if the elements are connected in parallel.

14.30 For the "tank" circuit in Fig. 14.75, find the resonant frequency.

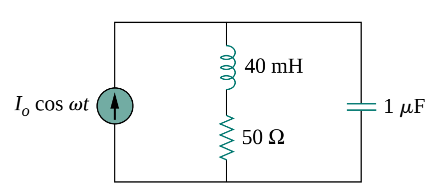

Figure 14.75 For Probs. 14.30 and 14.71.

14.31 For the circuits in Fig. 14.76, find the resonant frequency ω_0, the quality factor Q, and the bandwidth B.

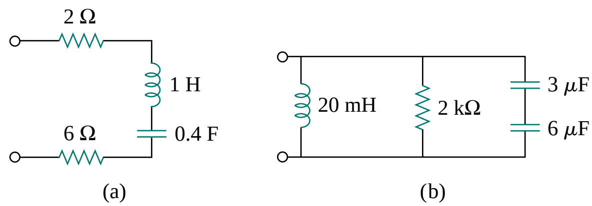

Figure 14.76 For Prob. 14.31.

14.32 Calculate the resonant frequency of each of the circuits in Fig. 14.77.

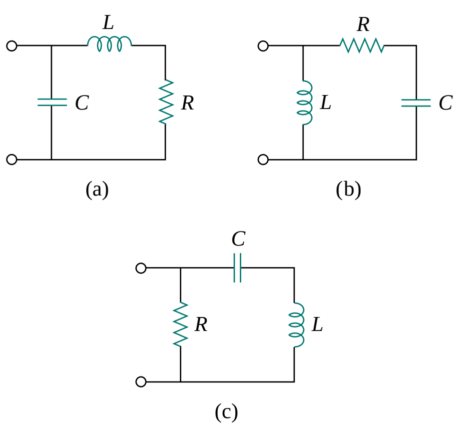

Figure 14.77 For Prob. 14.32.

*__14.33__ For the circuit in Fig. 14.78, find:
(a) the resonant frequency ω_0
(b) $\mathbf{Z}_{\text{in}}(\omega)$

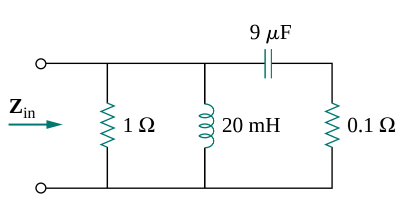

Figure 14.78 For Prob. 14.33.

14.34 In the circuit of Fig. 14.79, $i(t) = 10 \sin t$. Calculate the value of C such that $v(t) = V_o \sin t$ V. Find V_o.

*An asterisk indicates a challenging problem.

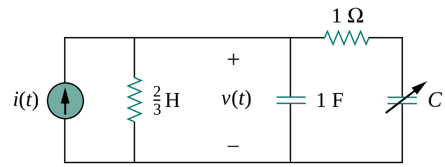

Figure 14.79 For Prob. 14.34.

14.35 For the network illustrated in Fig. 14.80, find
(a) the transfer function $\mathbf{H}(\omega) = \mathbf{V}_o(\omega)/\mathbf{I}(\omega)$,
(b) the magnitude of $\mathbf{H}$ at $\omega_0 = 1$ rad/s.

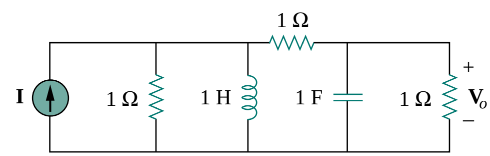

Figure 14.80 For Probs. 14.35, 14.61, and 14.72.

Section 14.7 Passive Filters

14.36 Show that the circuit in Fig. 14.66 is a lowpass filter. Calculate the corner frequency f_c if $L = 2$ mH and $R = 10$ kΩ.

14.37 Find the transfer function $\mathbf{V}_o/\mathbf{V}_s$ of the circuit in Fig. 14.81. Show that the circuit is a lowpass filter.

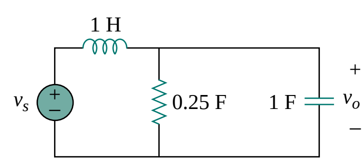

Figure 14.81 For Prob. 14.37.

14.38 Determine the cutoff frequency of the lowpass filter described by

$$\mathbf{H}(\omega) = \frac{4}{2 + j\omega 10}$$

Find the gain in dB and phase of $\mathbf{H}(\omega)$ at $\omega = 2$ rad/s.

14.39 Determine what type of filter is in Fig. 14.82. Calculate the corner frequency f_c.

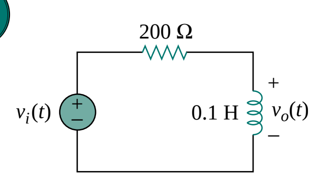

Figure 14.82 For Prob. 14.39.

14.40 Obtain the transfer function of a highpass filter with a passband gain of 10 and a cutoff frequency of 50 rad/s.

14.41 In a highpass RL filter with a cutoff frequency of 100 kHz, $L = 40$ mH. Find R.

14.42 Design a series RLC type bandpass filter with cutoff frequencies of 10 kHz and 11 kHz. Assuming $C = 80$ pF, find R, L, and Q.

14.43 Determine the range of frequencies that will be passed by a series RLC bandpass filter with $R = 10\ \Omega$, $L = 25$ mH, and $C = 0.4\ \mu$F. Find the quality factor.

14.44 (a) Show that for a bandpass filter,

$$\mathbf{H}(s) = \frac{sB}{s^2 + sB + \omega_0^2}$$

where B = bandwidth of the filter and ω_0 is the center frequency.

(b) Similarly, show that for a bandstop filter,

$$\mathbf{H}(s) = \frac{s^2 + \omega_0^2}{s^2 + sB + \omega_0^2}$$

14.45 Determine the center frequency and bandwidth of the bandpass filters in Fig. 14.83.

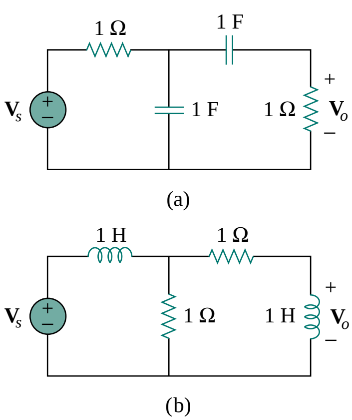

Figure 14.83 For Prob. 14.45.

14.46 The circuit parameters for a series RLC bandstop filter are $R = 2\ \text{k}\Omega$, $L = 0.1$ H, $C = 40$ pF. Calculate:

(a) the center frequency

(b) the half-power frequencies

(c) the quality factor

14.47 Find the bandwidth and center frequency of the bandstop filter of Fig. 14.84.

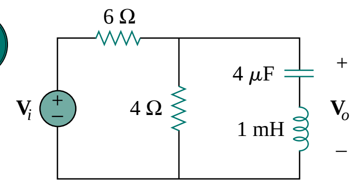

Figure 14.84 For Prob. 14.47.

Section 14.8 Active Filters

14.48 Find the transfer function for each of the active filters in Fig. 14.85.

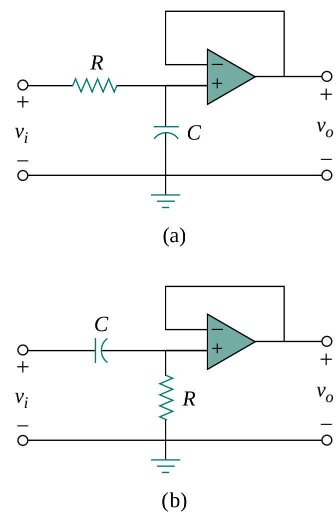

Figure 14.85 For Probs. 14.48 and 14.49.

14.49 The filter in Fig. 14.85(b) has a 3-dB cutoff frequency at 1 kHz. If its input is connected to a 120-mV variable frequency signal, find the output voltage at:

(a) 200 Hz (b) 2 kHz (c) 10 kHz

14.50 Obtain the transfer function of the active filter in Fig. 14.86. What kind of filter is it?

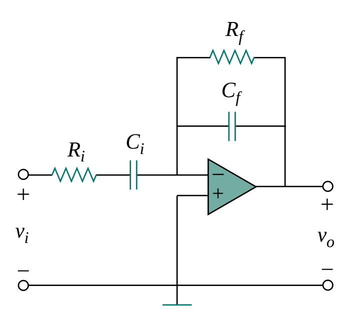

Figure 14.86 For Prob. 14.50.

14.51 A highpass filter is shown in Fig. 14.87. Show that the transfer function is

$$\mathbf{H}(\omega) = \left(1 + \frac{R_f}{R_i}\right) \frac{j\omega RC}{1 + j\omega RC}$$

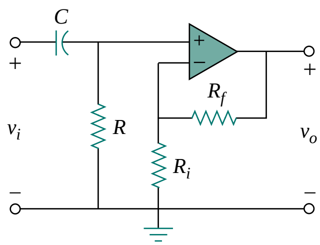

Figure 14.87 For Prob. 14.51.

14.52 A "general" first-order filter is shown in Fig. 14.88.

(a) Show that the transfer function is

$$\mathbf{H}(s) = \frac{R_4}{R_3 + R_4} \times \frac{s + (1/R_1C)[R_1/R_2 - R_3/R_4]}{s + 1/R_2C},$$

$$s = j\omega$$

(b) What condition must be satisfied for the circuit to operate as a highpass filter?

(c) What condition must be satisfied for the circuit to operate as a lowpass filter?

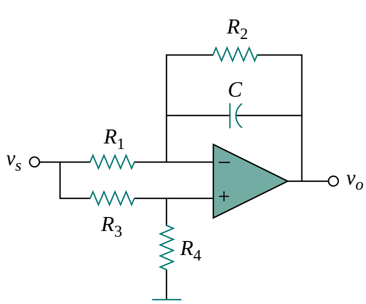

Figure 14.88 For Prob. 14.52.

14.53 Design an active lowpass filter with dc gain of 0.25 and a corner frequency of 500 Hz.

14.54 Design an active highpass filter with a high-frequency gain of 5 and a corner frequency of 200 Hz.

14.55 Design the filter in Fig. 14.89 to meet the following requirements:

(a) It must attenuate a signal at 2 kHz by 3 dB compared with its value at 10 MHz.

(b) It must provide a steady-state output of $v_o(t) = 10\sin(2\pi \times 10^8 t + 180^\circ)$ V for an input $v_s(t) = 4\sin(2\pi \times 10^8 t)$ V.

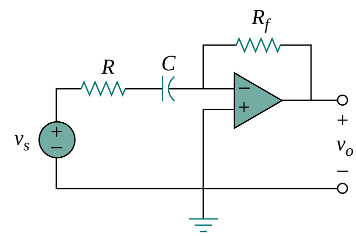

Figure 14.89 For Prob. 14.55.

*__14.56__ A second-order active filter known as a Butterworth filter is shown in Fig. 14.90.

(a) Find the transfer function $\mathbf{V}_o/\mathbf{V}_i$.

(b) Show that it is a lowpass filter.

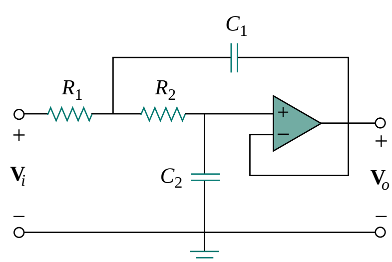

Figure 14.90 For Prob. 14.56.

Section 14.9 Scaling

14.57 Use magnitude and frequency scaling on the circuit of Fig. 14.75 to obtain an equivalent circuit in which the inductor and capacitor have magnitude 1 H and 1 C respectively.

14.58 What values of K_m and K_f will scale a 4-mH inductor and a 20-μF capacitor to 1 H and 2 F respectively?

14.59 Calculate the values of R, L, and C that will result in $R = 12$ kΩ, $L = 40$ μH, and $C = 300$ nF respectively when magnitude-scaled by 800 and frequency-scaled by 1000.

14.60 A series RLC circuit has $R = 10\ \Omega$, $\omega_0 = 40$ rad/s, and $B = 5$ rad/s. Find L and C when the circuit is scaled:

(a) in magnitude by a factor of 600,

(b) in frequency by a factor of 1000,

(c) in magnitude by a factor of 400 and in frequency by a factor of 10^5.

14.61 Redesign the circuit in Fig. 14.80 so that all resistive elements are scaled by a factor of 1000 and all frequency-sensitive elements are frequency-scaled by a factor of 10^4.

*14.62 Refer to the network in Fig. 14.91.

(a) Find $\mathbf{Z}_{in}(s)$.

(b) Scale the elements by $K_m = 10$ and $K_f = 100$. Find $\mathbf{Z}_{in}(s)$ and ω_0.

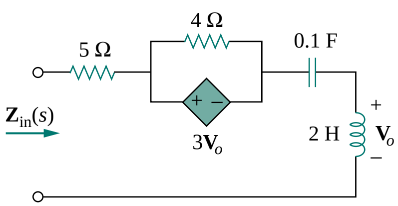

Figure 14.91 For Prob. 14.62.

14.63 (a) For the circuit in Fig. 14.92, draw the new circuit after it has been scaled by $K_m = 200$ and $K_f = 10^4$.

(b) Obtain the Thevenin equivalent impedance at terminals *a*-*b* of the scaled circuit at $\omega = 10^4$ rad/s.

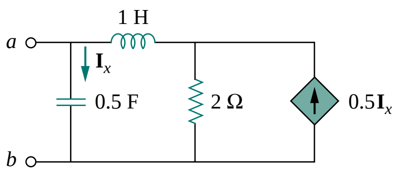

Figure 14.92 For Prob. 14.63.

14.64 Scale the lowpass active filter in Fig. 14.93 so that its corner frequency increases from 1 rad/s to 200 rad/s. Use a 1-μF capacitor.

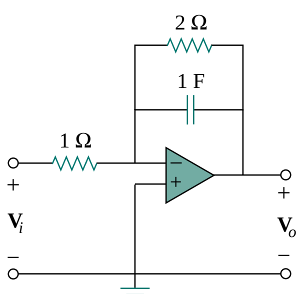

Figure 14.93 For Prob. 14.64.

Section 14.10 Frequency Response Using *PSpice*

14.65 Obtain the frequency response of the circuit in Fig. 14.94 using *PSpice*.

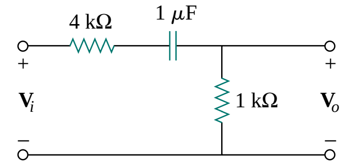

Figure 14.94 For Prob. 14.65.

14.66 Use *PSpice* to provide the frequency response (magnitude and phase of i) of the circuit in Fig. 14.95. Use linear frequency sweep from 1 to 10,000 Hz.

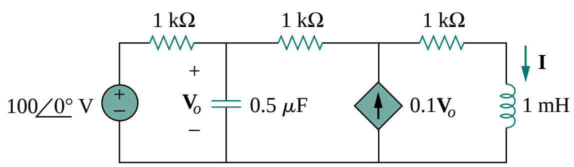

Figure 14.95 For Prob. 14.66.

14.67 In the interval $0.1 < f < 100$ Hz, plot the response of the network in Fig. 14.96. Classify this filter and obtain ω_0.

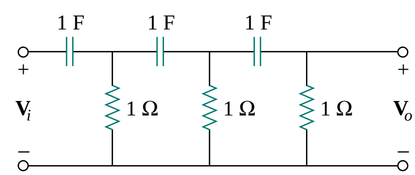

Figure 14.96 For Prob. 14.67.

14.68 Use *PSpice* to generate the magnitude and phase Bode plots of $\mathbf{V}_o$ in the circuit of Fig. 14.97.

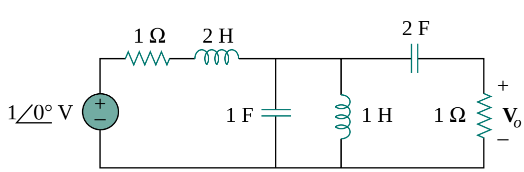

Figure 14.97 For Prob. 14.68.

14.69 Obtain the magnitude plot of the response $\mathbf{V}_o$ in the network of Fig. 14.98 for the frequency interval $100 < f < 1000$ Hz.

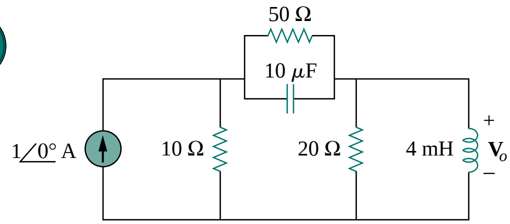

Figure 14.98 For Prob. 14.69.

14.70 Obtain the frequency response of the circuit in Fig. 14.40 (see Practice Problem 14.10). Take $R_1 = R_2 = 100\ \Omega$, $L = 2$ mH. Use $1 < f < 100{,}000$ Hz.

14.71 For the "tank" circuit of Fig. 14.75, obtain the frequency response (voltage across the capacitor) using *PSpice*. Determine the resonant frequency of the circuit.

14.72 Using *PSpice*, plot the magnitude of the frequency response of the circuit in Fig. 14.80.

Section 14.11 Applications

14.73 The resonant circuit for a radio broadcast consists of a 120-pF capacitor in parallel with a 240-μH inductor. If the inductor has an internal resistance of 400 Ω, what is the resonant frequency of the circuit? What would be the resonant frequency if the inductor resistance were reduced to 40 Ω?

14.74 A series-tuned antenna circuit consists of a variable capacitor (40 pF to 360 pF) and a 240-μH antenna coil which has a dc resistance of 12 Ω.

(a) Find the frequency range of radio signals to which the radio is tunable.

(b) Determine the value of Q at each end of the frequency range.

14.75 The crossover circuit in Fig. 14.99 is a lowpass filter that is connected to a woofer. Find the transfer function $\mathbf{H}(\omega) = \mathbf{V}_o(\omega)/\mathbf{V}_i(\omega)$.

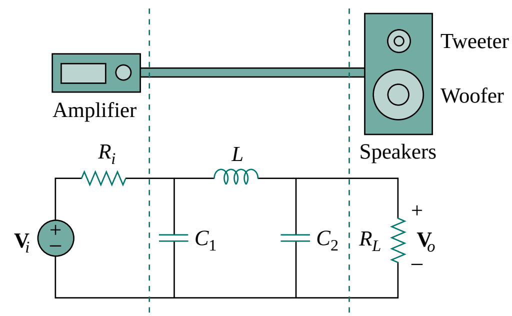

Figure 14.99 For Prob. 14.75.

14.76 The crossover circuit in Fig. 14.100 is a highpass filter that is connected to a tweeter. Determine the transfer function $\mathbf{H}(\omega) = \mathbf{V}_o(\omega)/\mathbf{V}_i(\omega)$.

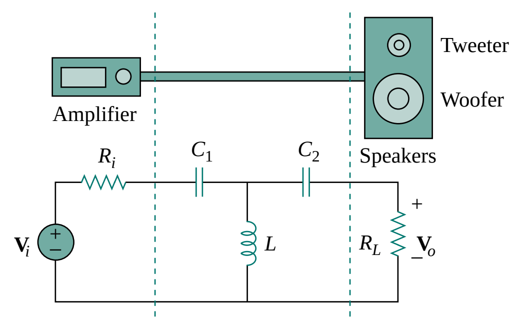

Figure 14.100 For Prob. 14.76.

COMPREHENSIVE PROBLEMS

14.77 A certain electronic test circuit produced a resonant curve with half-power points at 432 Hz and 454 Hz. If $Q = 20$, what is the resonant frequency of the circuit?

14.78 In an electronic device, a series circuit is employed that has a resistance of 100 Ω, a capacitive reactance of 5 kΩ, and an inductive reactance of 300 Ω when used at 2 MHz. Find the resonant frequency and bandwidth of the circuit.

14.79 In a certain application, a simple RC lowpass filter is designed to reduce high frequency noise. If the desired corner frequency is 20 kHz and $C = 0.5\ \mu$F, find the value of R.

14.80 In an amplifier circuit, a simple RC highpass filter is needed to block the dc component while passing the time-varying component. If the desired rolloff frequency is 15 Hz and $C = 10\ \mu$F, find the value of R.

14.81 Practical
RC filter design should allow for source and load resistances as shown in Fig. 14.101. Let $R = 4$ kΩ and $C = 40$-nF. Obtain the cutoff frequency when:

(a) $R_s = 0$, $R_L = \infty$,

(b) $R_s = 1$ kΩ, $R_L = 5$ kΩ.

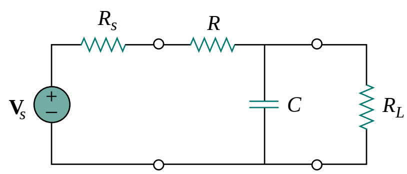

Figure 14.101 For Prob. 14.81.

14.82 The RC circuit in Fig. 14.102 is used for a lead compensator in a system design. Obtain the transfer function of the circuit.

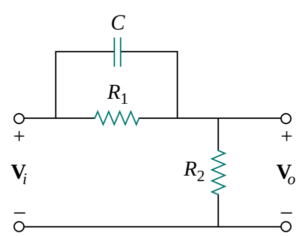

Figure 14.102 For Prob. 14.82.

14.83 A low-quality factor, double-tuned bandpass filter is shown in Fig. 14.103. Use *PSpice* to generate the magnitude plot of $\mathbf{V}_o(\omega)$.

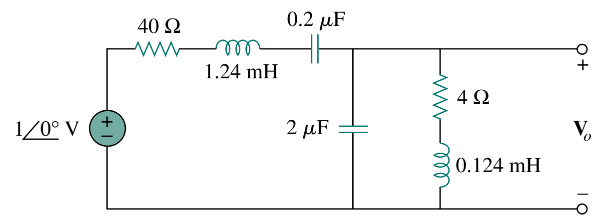

Figure 14.103 For Prob. 14.83.