Maple User Manual

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Preface

Maple Software

Maple™ software is a powerful system that you can use to solve mathematical problems from simple to complex. You can also create professional quality documents, presentations, and custom interactive computational tools in the Maple environment.

You can access the power of the Maple computational engine through a variety of interfaces.

Interface	Description
Standard (default)	A full-featured graphical user interface that helps you create electronic documents to show all your calculations, assumptions, and any margin of error in your results. You can also hide the computations to allow your reader to focus on the problem setup and final results. The advanced formatting features lets you create the customized document you need. Because the documents are *live*, you can edit the parameters and, with the click of a button, compute the new results. The Standard interface has two modes: *Document* mode and *Worksheet* mode. An interactive version of this manual is available in the Standard Worksheet interface. From the **Help** menu, select **Manuals, Resources, and more → Manuals → User Manual**.
Classic	A basic worksheet environment for older computers with limited memory. The Classic interface does not offer all of the graphical user interface features that are available in the Standard interface. The Classic interface has only one mode, *Worksheet* mode.
Command-line version	A command-line interface for solving very large complex problems or batch processing with scripts. No graphical user interface features are available.
Maplet™ Applications	Graphical user interfaces containing windows, textbox regions, and other visual interfaces, which gives you point-and-click access to the power of Maple. You can perform calculations and plot functions without using the worksheet.
Maplesoft™ Graphing Calculator	A graphical calculator interface to the Maple computational engine. Using it, you can perform simple computations and create customizable, zoomable graphs. This is available on Microsoft® Windows® only.

This manual describes how to use the Standard interface. As mentioned, the Standard interface offers two modes: *Document* mode and *Worksheet* mode. Using either mode, you can create high quality interactive mathematical documents. Each mode offers the same features and functionality, the only difference is the default input region of each mode.

Shortcut Keys by Platform

This manual will frequently refer to context menus and command completion when entering expressions. The keyboard keys used to invoke these features differ based on your operating system.

This manual will only refer to the keyboard keys needed for a Windows operating system. The shortcut keys for your operating system can be viewed from the **Help** menu (**Help → Manuals, Resources, and more → Shortcut Keys**).

Context Menus

- **Right-click**, Windows and UNIX®
- **Control-click**, Macintosh®

That is, place the mouse over the input or output region and press the right button on the mouse or press and hold the **Control** key and click the mouse key for Macintosh.

For more information on Context Menus, see *Context Menus (page 39)*.

Command Completion

- **Esc**, Macintosh, Windows, and UNIX
- **Ctrl + Space**, Windows
- **Ctrl** + **Shift** + **Space**, UNIX

Begin entering a command in a Maple document. Press the **Esc** key. Alternatively, use the platform-specific keys. For Windows, press and hold the **Ctrl** key and then press the **Space** bar.

For more information on Command Completion, see *Command Completion (page 48)*.

In This Manual

This manual provides an introduction to the following Maple features:

- Ease-of-use when entering and solving problems
- Point-and-click interaction with various interfaces to help you solve problems quickly
- Maple commands and standard math notation
- Clickable Calculus
- The help system
- Online resources
- Performing computations
- Creating plots and animations
- The Maple programming language
- Using and creating custom Maplet applications
- File input and output, and using Maple with third party products
- Data structures

For a complete list of manuals, study guides, toolboxes, and other resources, visit the Maplesoft web site at **http://www.maplesoft.com**

Audience

The information in this manual is intended for first-time Maple users and users looking for a little more information.

Conventions

This manual uses the following typographical conventions.

- **bold** font - Maple command, package name, option name, dialog, menu, or text field
- *italics* - new or important concept
- **Note** - additional information relevant to the section
- **Important** - information that must be read and followed

Customer Feedback

Maplesoft welcomes your feedback. For suggestions and comments related to this and other manuals, contact doc@maplesoft.com.

1 Getting Started

Don't worry about your difficulties in Mathematics. I can assure you mine are still greater.

~Albert Einstein

Mathematics touches us every day—from the simple chore of calculating the total cost of our purchases to the complex calculations used to construct the bridges we travel.

To harness the power of mathematics, Maplesoft provides a tool in an accessible and complete form. That tool is Maple.

In this chapter:

Section	Topics
Introduction to Maple (page 2) - The main features of Maple's Standard Interface	• Starting the Standard Document Interface • Entering commands and mathematical expressions • Toolbars • Context menus • Copy and drag keys • Saving Maple documents
Entering Expressions (page 19) - Methods of entering expressions in 1-D and 2-D Math	• Execution groups • Math Mode and Text Mode • Palettes • Symbol names • Toolbar icons
Point-and-Click Interaction (page 32) - An introduction to the point-and-click features in Maple	• Assistants • Tutors • Context menus • Task templates • Exploration Assistant
Commands (page 45) - An introduction to the commands of the Maple language	• Using commands from the Maple library • Entering commands • Document blocks

Section	Topics
The Maple Help System (page 54) - Accessing help on commands, packages, point-and-click features, and more	• How to access help for Maple features • Interacting with help pages • Viewing and interacting with examples
Available Resources (page 58) - Both online and from within Maple	• New user resources, including the Maple Tour and the Maple Portal • Examples • Online help • Maple web site resources

1.1 Introduction to Maple

Working in Maple

With Maple, you can create powerful interactive documents. The Maple environment lets you start solving problems right away by entering expressions in 2-D Math and solving these expressions using point-and-click interfaces. You can combine text and math in the same line, add tables to organize the content of your work, or insert images, sketch regions, and spreadsheets. You can visualize and animate problems in two and three dimensions, format text for academic papers or books, and insert hyperlinks to other Maple files, web sites, or email addresses. You can embed and program graphical user interface components, as well as devise custom solutions using the Maple programming language.

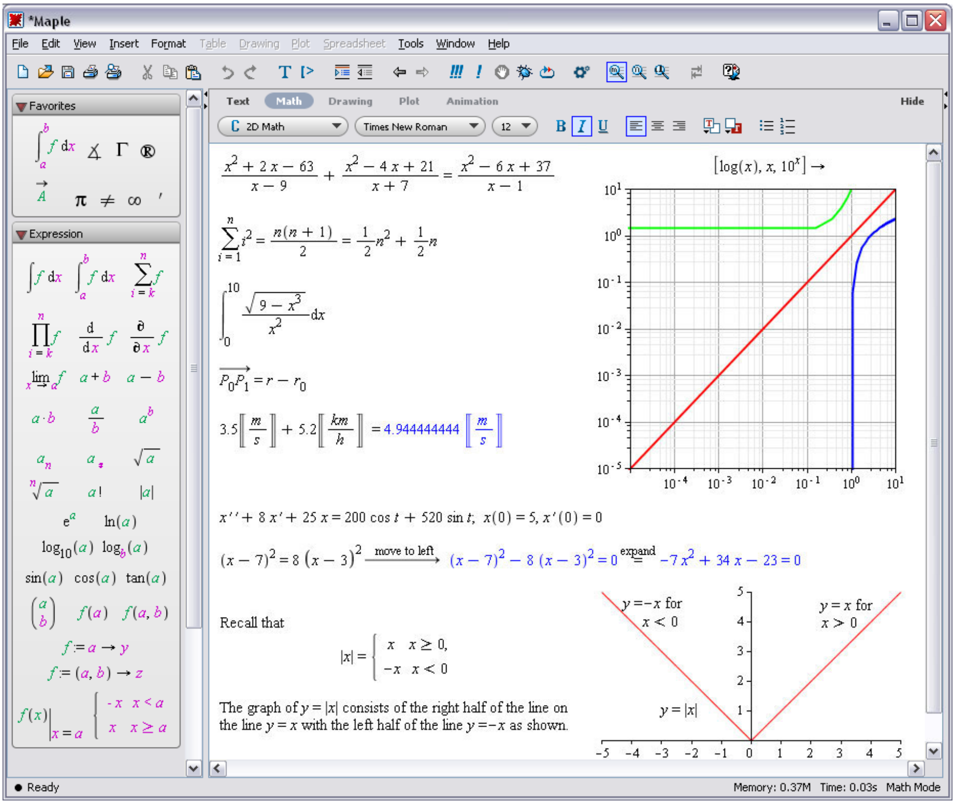


Figure 1.1: The Maple Environment

Starting the Standard Document Interface

To start Maple on:

Windows	From the **Start** menu, select **All Programs** → **Maple 14** → **Maple 14**. **Alternatively:** Double-click the **Maple 14** desktop icon.
Macintosh	1. From the Finder, select **Applications** and **Maple 14**. 2. Double-click **Maple 14**.

UNIX	Enter the full path, for example, **/usr/local/maple/bin/xmaple** **Alternatively:** 1. Add the Maple directory (for example, **/usr/local/maple/bin)** to your command search path. 2. Enter **xmaple**.

The first Maple session opens with a **Startup** dialog explaining the difference between *Document Mode* and *Worksheet Mode*. Using either mode, you can create high quality interactive mathematical documents. Each mode offers the same features and functionality; the only difference is the default input region of each mode.

Document Mode

Document mode uses *Document Blocks* as the default input region to hide Maple syntax. A Document Block region is indicated by two triangles located in the vertical Markers column along the left pane of the Maple Document, . If the Markers column is not visible, open the **View** menu and select **Markers**. This allows you to focus on the problem instead of the commands used to solve the problem. For example, when using context menus on Maple input in Document mode (invoked by right-clicking or **Control**-clicking for Macintosh), input and output are connected using an arrow or equal sign with self-documenting text indicating the calculation that had taken place. The command used to solve this expression is hidden.

$$x^2 + 7x + 10 \xrightarrow{\text{solve}} \{x = -2\}, \{x = -5\}$$

When starting Standard Maple, the default mode is Document mode.

Worksheet Mode

Worksheet mode uses a Maple prompt as the default input region. The Maple input prompt is a red angle bracket, >. When using content menus on input in Worksheet mode, all commands are displayed.

```
> x^2 + 7 x + 10
> solve( { x^2 + 7*x + 10 = 0 } )
                        {x = -2}, {x = -5}
```

To work in Worksheet mode, select **File** → **New** → **Worksheet Mode**.

Document and Worksheet Modes

Regardless of which mode you are working in, you have the opportunity to show or hide your calculations. You can hide commands in Worksheet Mode by adding a document block from the **Format** menu, **Format → Create Document Block** (see *Document Blocks (page 51)*), or you can show commands in Document mode by adding a Maple prompt from the **Insert** menu, **Insert → Execution Group → Before / After Cursor** (see *Input Prompt (page 80)*).

This chapter discusses features common to both modes. Specific aspects of Document mode are explained in *Document Mode (page 63)*, and aspects of Worksheet mode are explained in *Worksheet Mode (page 79)*.

The Startup dialog also contains links to items, such as various document options, help resources including updates and other introductory help pages, and application resources on the Maplesoft web site. Subsequent sessions display **Tip of the Day** information.

To start a Maple session:

1. In the **Startup** dialog, select **Blank Document** or **Blank Worksheet**. A blank document displays.

or

1. Close the **Startup** dialog.

2. From the **File** menu, select **New**, and then either **Document Mode** or **Worksheet Mode**. A blank document displays.

Every time you open a document, Maple displays a **Quick Help** pop-up list of important shortcut keys. To invoke **Quick Help** at any time, press the **F1** key.

Entering 2-D Math

In Maple, the default format for entering mathematical expressions is 2-D Math. This results in mathematical expressions that are equivalent to the quality of math found in textbooks. Entering 2-D Math in Maple is done using common key strokes or palette items. For more information on palettes, see *Palettes (page 22)*. An example of entering an expression using common key strokes is presented in the following section. An example of entering an expression using palette items is presented in *Example 3 - Enter an Expression Using Palettes (page 27)*.

Common Operations

Entering mathematical expressions, such as $\frac{35}{99} + \frac{1}{9}$, $x^2 + x$, and $x \cdot y$ is natural in 2-D Math.

To enter a fraction:

1. Enter the numerator.

2. Press the forward slash (/) key.

3. Enter the denominator.

4. To leave the denominator, press the right arrow key.

To enter a power:

1. Enter the base.

2. Press the caret (^) key.

3. Enter the exponent, which displays in math as a superscript.

4. To leave the exponent, press the right arrow key.

To enter a product:

1. Enter the first factor.

2. Press the asterisk (*) key, which displays in 2-D Math as a dot, $\cdot$.

3. Enter the second factor.

Implied Multiplication:

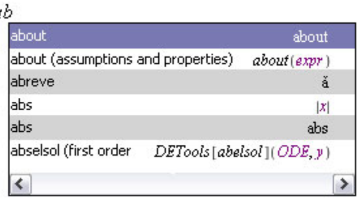
In most cases, you do not need to include the multiplication operator, $\cdot$. Insert a space character between two quantities to multiply them.

Note: In some cases, you do not need to enter the multiplication operator or a space character. For example, Maple interprets a number followed by a variable as multiplication.

Important: Maple interprets a sequence of letters, for example, *xy*, as a single variable. To specify the product of two variables, you must insert a space character (or multiplication operator), for example, $x\,y$ or $x \cdot y$. For more information, refer to the **?2DMathDetails** help page.

Shortcuts for Entering Mathematical Expressions

Table 1.1: Common Keystrokes for Entering Symbols and Formats

Symbol/Formats	Key	Example
implicit multiplication	**Space** key	$(x^2 - 7\,x\,y + 3\,y^2)\,x\,y$
explicit multiplication [1]	* (asterisk)	$2 \cdot 3$
fraction [2]	/ (forward slash)	$\frac{1}{4}$
exponent (superscript) [2]	^ (**Shift + 6** or caret key)	x^2
subscript [2]	_ (**Shift** + underscore)	x_a
navigating expressions	Arrow keys	
command / symbol completion [3]	• **Esc**, Macintosh,Windows, and UNIX • **Ctrl + Space**, Windows • **Ctrl + Shift + Space**, UNIX	*ab* 
square root	*sqrt* and then command completion	$\sqrt{25}$
exponential function [2]	*exp* and then command completion	e^x
enter / exit 2-D Math	**F5** key **Math** and **Text** icons in the toolbar	$\frac{1}{4}$ versus 1/4

[1] required for products of numbers

[2] use the right arrow key to leave a denominator, superscript, or subscript region

[3] for more information, see *Command Completion (page 48)*.

For a complete list of shortcut keys, refer to the **2-D Math Shortcut Keys and Hints** help page. To access this help page in the Maple software, in Math mode enter **?MathShortcuts** and then press **Enter**. For information on the Maple Help System, see *The Maple Help System (page 54)*.

Example 1 - Enter and Evaluate an Expression Using Keystrokes

Review the following example:

$$\frac{x^2 + y^2}{2}$$

In this example, you will enter $\frac{x^2 + y^2}{2}$ and evaluate the expression.

Action	Result in Document
To enter the expression: 1. Enter **x**.	x
2. Press **Shift + 6** (the ^ or caret key). The cursor moves to the superscript position.	$x^{\square}$
3. Enter **2**.	x^2
4. Press the right arrow key. The cursor moves right and out of the superscript position.	x^2
5. Enter the + symbol.	$x^2 +$
6. Enter **y**.	$x^2 + y$
7. Press **Shift + 6** to move to the superscript position.	$x^2 + y^{\square}$
8. Enter **2** and press the right arrow key.	$x^2 + y^2$
9. With the mouse, select the expression that will be the numerator of the fraction.	$x^2 + y^2$
10. Enter the / symbol. The cursor moves to the denominator, with the entire expression in the numerator.	$\frac{x^2 + y^2}{\square}$
11. Enter **2**.	$\frac{x^2 + y^2}{2}$

Action	Result in Document
12. Press the right arrow key to move right and out of the denominator position.	$\frac{x^2+y^2}{2}$
To evaluate the expression and display the result inline: 13. Press **Ctrl** + = (**Command** + =, Macintosh).	$\frac{x^2+y^2}{2} =$ $\frac{1}{2}x^2+\frac{1}{2}y^2$

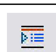
To execute 2-D Math, you can use any of the following methods.

- Pressing **Ctrl** + = (**Command** + =, for Macintosh). That is, *press and hold* the **Ctrl** (or **Command**) key, and then press the equal sign (=) key. This evaluates and displays results inline.
- Pressing the **Enter** key. This evaluates and displays results on the next line and centered.
- Right-click (**Control**-click for Macintosh) the input to invoke a context menu item. From the context menu, select **Evaluate and Display Inline**. See *Context Menus (page 39)* for more details.
- Using the context-menu item **Evaluate**.

Toolbar Options

The Maple toolbar offers several buttons to assist you when interacting with Maple. See Table 1.2.

Table 1.2: Maple Toolbar Options

Basic Usage	Icon	Equivalent Menu Option or Command
Inserts plain text after the current execution group.		From the **Insert** menu, select **Text**.
Inserts Maple Input after the current execution group. For details, refer to *Execution Groups (page 19)*.		From the **Insert** menu, select **Execution Group** and then **After Cursor**.
Encloses the selection in a subsection. For details, refer to *Sections (page 295)*.		From the **Format** menu, select **Indent**.
Removes any section enclosing the selection.		From the **Format** menu, select **Outdent**.
Executes all commands in the worksheet or document		From the **Edit** menu, select **Execute** and then **Worksheet**.

Basic Usage	Icon	Equivalent Menu Option or Command
Executes a selected area.		From the **Edit** menu, select **Execute** and then **Selection**.
Clears Maple's internal memory. For details, refer to the **?restart** help page.		Enter *restart.*
Add and edit Maple code that is executed each time the worksheet is opened. For details, refer to the **?startupcode** help page.		From the **Edit** menu, select **Startup Code**.
Adjusts the display size of document content. **Note:** plots, spreadsheets, images, and sketches remain unchanged.		From the **View** menu, select **Zoom Factor** and then a zoom size.
Opens the Maple help system. For details, refer to *The Maple Help System (page 54)*.		From the **Help** menu, select **Maple Help**.

For 1-D Math and text regions, the **Tab** icon in the toolbar allows you to set the **Tab** key to move between placeholders (or cells in a table) or to indent text.

Table 1.3: Tab Icon Description

Tab Icon	Description
	Tab icon **off**. Allows you to move between placeholders using the **Tab** key.
	Tab icon **on**. Allows you to indent in the worksheet using the **Tab** key.
	The Tab icon is disabled when using 2-D Math (**Math** mode), and as such, the **Tab** key allows you to move between placeholders.

Toolbar icons are controlled by the location of the cursor in the document. For example, place the cursor at an input region and the **Text** and **Math** icons are accessible while the others are dimmed. See Table 1.4 for a list of the tools available in each icon.

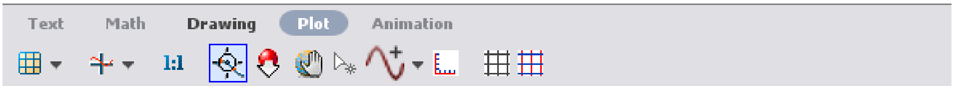
Table 1.4: Toolbar Icons and their Tools

Toolbar Icon Options
Text tools


Math tools

Drawing tools

2-D Plot tools



3-D Plot tools

Animation tools

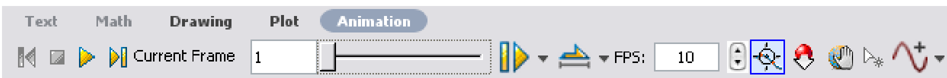


Table 1.5: Toolbar Icon Availability

Region	Available Tools
Input region	Text and Math icons
Plot region	Drawing and Plot icons
Animation region	Drawing, Plot, and Animation icons
Canvas and Image regions	Drawing icon

The **Text** and **Math** icons allow you to enter text and math in the same line by choosing the appropriate input style at each stage when entering the sentence.

The derivative of $\sin(x)$ is $\cos(x)$.

For an example, see *Example 6 - Enter Text and 2-D Math in the Same Line (page 31)*.

Using the tools available in these icons, you can customize the input style of the text and 2-D Math. For the **Text** and **Math** icons, the icon that is selected remains in that state until prompted otherwise; therefore, if the **Text** icon is selected and you press the **Enter** key, the new input region remains a Text region.

The **Text** and **Math** icons differ while at a Maple input prompt. The Math icon displays input as 2-D Math, whereas the Text icon displays Maple input. For details, refer to *Math Mode vs. Text Mode (page 20)*.

> $\frac{x^2}{2}$

```
> x^2/2;
```

To access the tools available in the **Plot** and **Drawing** icons, click a plot region. These tools allow you to manipulate the plot or draw shapes and enter text on the plot region. By clicking an animation region, you have the same features available for a plot region, in addition to tools for playing the animation in the **Animation** icon. For details on plots and animations, refer to *Plots and Animations (page 237)*.

For the remaining icons, hover the mouse over the icon to display the icon description.

Context Menus and Copy & Drag

Context Menus

Maple dynamically generates a context menu of applicable options when you right-click an object, expression, or region. The options available in the context menu depend on the selected input region. For example, you can manipulate and graph expressions, enhance plots, format text, manage palettes, structure tables, and more. When using context menus to perform an action on an expression, the input and output are connected with a self-documenting arrow or equal sign indicating the action that had taken place. For more information, see *Context Menus (page 39)*.

Copy & Drag

With Maple, you can drag input, output, or curves in a plot region into a new input region. This is done by highlighting the input or selecting the curve and dragging it with your mouse into a new input region. Dragging the highlighted region will cut or delete the original input. To prevent this, use the copy and drag feature.

- **Ctrl** + drag, Windows and UNIX
- **Command** + drag, Macintosh

That is, highlight the region you want to copy. Press and hold the **Ctrl** key while you drag the input to the new region using the mouse. The steps are the same for Macintosh with the exception of pressing the **Command** key.

Example 2 - Solve and Plot an Equation Using Context Menus and Copy & Drag

Review the following example:

$5x - 7 = 3x + 2$

In this example, we will enter the equation and then solve and plot the equation using context menus and Maple's copy & drag feature. This example will only refer to the keystrokes needed on a Windows operating system to invoke the context menus and the copy & drag feature. For your operating system, refer to section *Shortcut Keys by Platform (page xiv)* for the equivalent keystrokes.

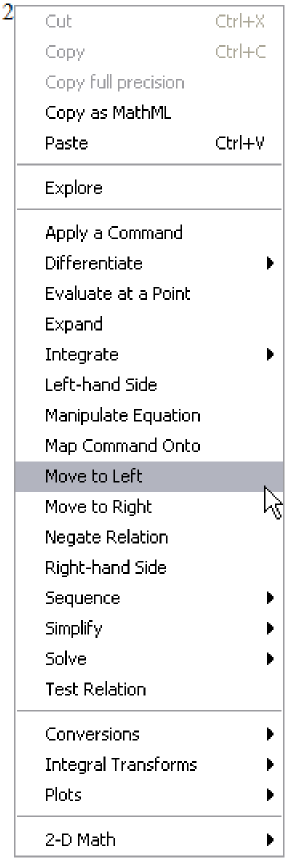
To solve the equation:

1. Enter the equation.

2. Right-click the equation and select **Move to Left**.

Input:

> $5x - 7 = 3x + 2$



Result:

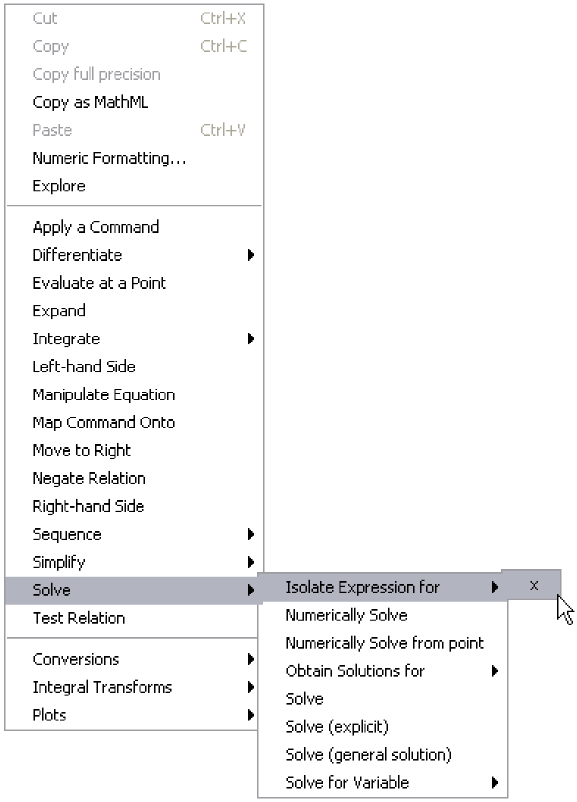
$$5x - 7 = 3x + 2 \xrightarrow{\text{move to left}} 2x - 9 = 0$$

A brief description, "move to left" is displayed above the arrow that connects the input and output.

3. Right-click the output from the previous action, $2x - 9 = 0$, and select **Solve** → **Isolate Expression for** → **x**.

Input:

$$5\,x-7=3\,x+2 \xrightarrow{\text{move to left}} 2\,x-9=0$$



Result:

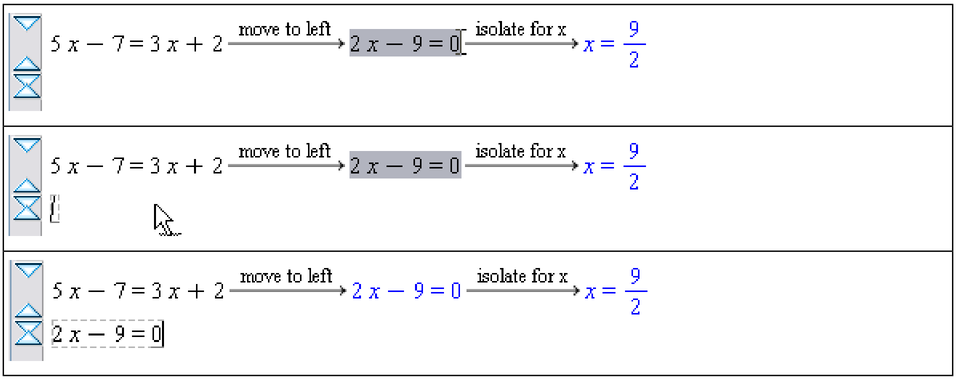
$$5\,x-7=3\,x+2 \xrightarrow{\text{move to left}} 2\,x-9=0 \xrightarrow{\text{isolate for x}} x=\frac{9}{2}$$

Now that we have solved the equation, we can plot it. To do this, we will copy the equation $2x - 9 = 0$ to a new document block and use context menus again.

4. From the **Format** menu, select **Create Document Block**.

5. To copy the expression $2x - 9 = 0$, highlight only this expression from the previous result. Press and hold the **Ctrl** key and drag the expression to the new document block region.

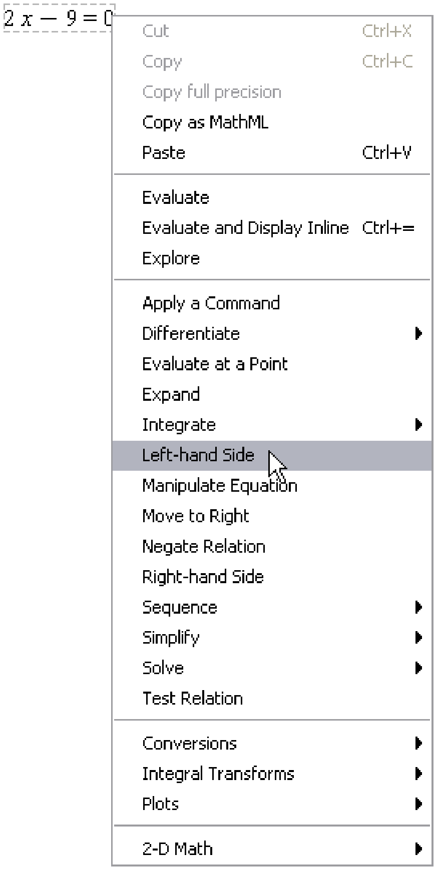
Result:



To plot the expression:

6. Right-click the equation, and select **Left-hand Side**.

Input:



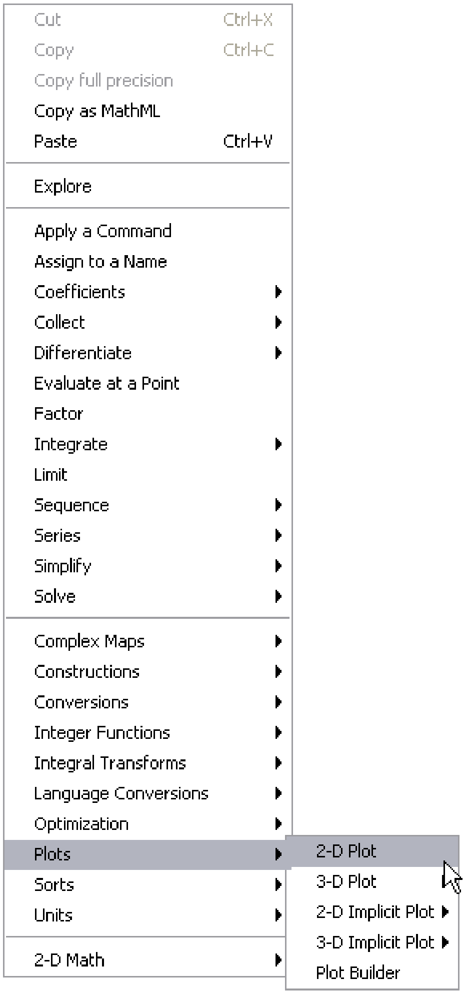
Result:

$$2x - 9 = 0 \xrightarrow{\text{left hand side}} 2x - 9$$

7. Right-click the expression and select **Plots → 2-D Plot**.

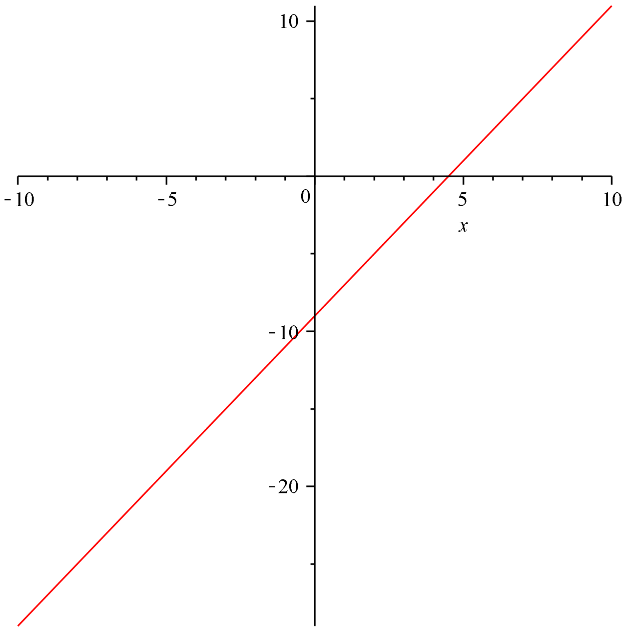
Input:

$2x-9$


Cut Ctrl+X
Copy Ctrl+C
Copy full precision
Copy as MathML
Paste Ctrl+V
Explore
Apply a Command
Assign to a Name
Coefficients
Collect
Differentiate
Evaluate at a Point
Factor
Integrate
Limit
Sequence
Series
Simplify
Solve
Complex Maps
Constructions
Conversions
Integer Functions
Integral Transforms
Language Conversions
Optimization
Plots
Sorts
Units
2-D Math
2-D Plot
3-D Plot
2-D Implicit Plot
3-D Implicit Plot
Plot Builder

Result:

$2x - 9 \quad \rightarrow$



Saving a Maple Document

To save these examples you created, from the **File** menu, select **Save**. Maple documents are saved as **.mw** files.

1.2 Entering Expressions

Execution Groups

An execution group is a grouping of Maple input with its corresponding Maple output. It is distinguished by a large square bracket, called a *group boundary*, at the left. An execution group may also contain any or all of the following: a plot, a spreadsheet, text, embedded components, and a drawing canvas.

Execution groups are the fundamental computation and documentation elements in the document. If you place the cursor in an input command and press the **Enter** or **Return** key, Maple executes all of the input commands in the current execution group.

Math Mode vs. Text Mode

The default mode of entry in Document or Worksheet mode is Math Mode, which displays input in 2-D Math. In earlier releases of Maple, commands and expressions were entered using Maple Input or 1-D Math.

Important: With Maple input, you must terminate commands with a semicolon or colon.

```
> cos(alpha)^2+sin(alpha)^2;
```

$$\cos(\alpha)^2 + \sin(\alpha)^2$$

```
> a*int(exp(sqrt(2)*x),x);
```

$$\frac{1}{2}\, a\sqrt{2}\, e^{\sqrt{2}\, x}$$

```
> limit(f(x),x=infinity);
```

$$\lim_{x \to \infty} f(x)$$

```
> sum(a[k]*x^k, k=0..m)=product(b[j]*x^j, j=0..n);
```

$$\sum_{k=0}^{m} a_k x^k = \prod_{j=0}^{n} \left(b_j x^j\right)$$

In Document Mode, to enter input using Maple Input mode, insert a Maple prompt by clicking [>] in the toolbar, and then click the **Text** button in the toolbar. In Worksheet Mode, simply click the **Text** button. See Figure 1.2.

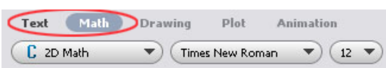
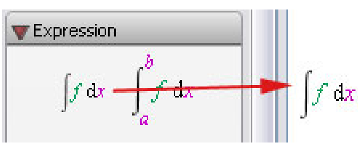
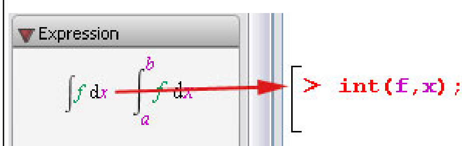


Figure 1.2: Text and Math buttons on the Toolbar

Table 1.6: Math Mode vs. Text Mode

Math Mode	**Text Mode**
Maple's default setting. Executable standard math notation. This is also referred to as **2-D Math Input**. > $\int x^2 + 2x + 1\, dx$ $\frac{1}{3}x^3 + x^2 + x$	Executable Maple notation. This is also referred to as **1-D Math Input** or **Maple Input**. `> int(x^2+2*x+1, x);` $\frac{1}{3}x^3 + x^2 + x$
Access from the **Insert** → **2-D Math** menu.	Access from the **Insert** → **Maple Input** menu.
When using 2-D Math, the **Math** mode icon is highlighted in the toolbar, Text Math.	When entering Maple Input or text in a text region, the **Text** mode icon is highlighted in the toolbar, Text Math.
In Document Mode (or a document block), input is entered in a document block with a slanted cursor, .	In Document Mode (or a document block), input is entered with a vertical cursor, as plain text, Enter some text.
In Worksheet Mode, input is made at an input prompt with a slanted cursor, [> .	In Worksheet Mode, input is made at an input prompt with a vertical cursor, [> | .
To convert a 2-D Math expression to 1-D Math, right-click the expression (**Command**-click, Macintosh) and select **2-D Math** → **Convert To** → **1-D Math Input**.	To convert a 1-D Math expression to 2-D Math, right-click the expression (**Command-click,** Macintosh) and select **Convert To** → **2-D Math Input**.
No termination symbol is required.	All input must end with a semi-colon (;) or a colon (:).
Palettes make entering expressions in familiar notation easier than entering foreign syntax and reduces the possibility of introducing typing errors. 	Using palettes while in 1-D Math teaches you the related Maple command syntax. 

If you prefer 1-D Math input, you can change the default math input notation.

To change math input notation for a session or globally across all documents:

1. From the **Tools** menu, select **Options**. The **Options Dialog** opens.

2. Click the **Display** tab.

3. In the **Input Display** drop-down list, select **Maple Notation**.

4. Click the **Apply to Session** or **Apply Globally** button.

Important: The new input display becomes the default setting *after* pressing the **Enter** key.

Palettes

Palettes are collections of related items that you can insert into a document by clicking or drag-and-dropping. The Maple environment provides access to over 20 palettes containing items such as symbols (∞), layouts (A^b), mathematical operations $\left(\int_a^b f\,\mathrm{d}x\right)$, and much more.

By default, palettes are displayed in the left pane of the Maple environment when you launch Maple. If the palettes are not displayed,

1. From the **View** menu, select **Palettes**.

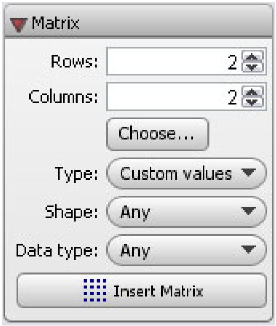
2. Select **Expand Docks**.

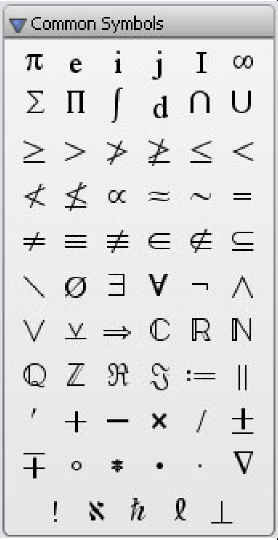
3. Right-click (**Control**-click, Macintosh) the palette dock. From the context menu, select **Show All Palettes**.

Alternatively, from the main menu, select **View** → **Palettes** → **Arrange Palettes** to display specific palettes.

You can create a **Favorites** palette of the expressions and entities you use often by right-clicking (**Control**-click, Macintosh) the palette template you want to add and selecting **Add To Favorites Palette** from the context menu.

Table 1.7: Palette Categories

Palette Category	Palette Description
Expression Palettes 	**Expression** - construct expressions such as integrals $\int_a^b f\, dx$. **Matrix** - enter the number of rows and columns required, designate type, such as zero-filled, and designate shape, such as diagonal. **Layout** - add math content that has specific layout, such as expressions with one or more superscripts and subscripts A^b. **Components** - embed graphical interface components such as a button into your document or worksheet. Components can be programmed to perform an action when selected such as executing a command when a button is clicked Toggle Button. **Handwriting** - an easy way to find a desired symbol. **Units (SI)** - insert a unit from the International System of Units (SI), or any general unit $[[kg]]$. **Units (FPS)** - insert a unit from the Foot-Pound-Second System (FPS), or any general unit $[[ft]]$. **Accents** - insert decorated names, such as an x with an arrow over it to denote a vector $\vec{A}$. **Favorites** - add templates that you use most often from other palettes.

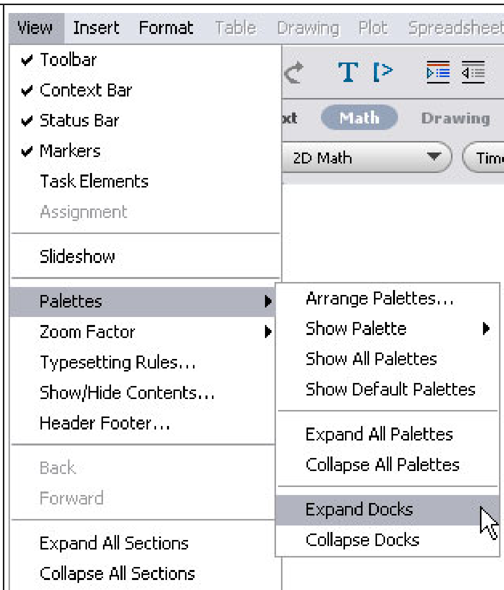
Palette Category	Palette Description
Mathematical Palettes 	Palettes for constructing expressions **Common Symbols**, **Relational** ≥, **Relational Round** ≥, **Operators** ÷, **Large Operators**, **Negated** ≠, **Fenced**, **Arrows**, **Constants and Symbols** ∞. **Punctuation** - insert punctuation symbols, such as inserting the registered trademark and copyright symbols © into text regions **Miscellaneous** - insert miscellaneous math and other symbols outside the above categories.
Alphabetical Palettes 	**Greek**, **Script**, **Fraktur**, **Open Face**, **Cyrillic**, **Diacritical Marks**, **Roman Extended Upper Case** Æ, **Roman Extended Lower Case** æ.

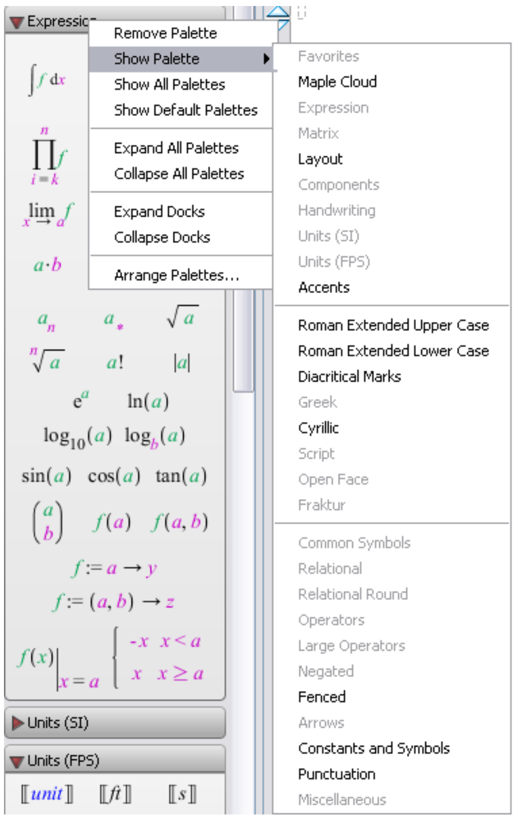
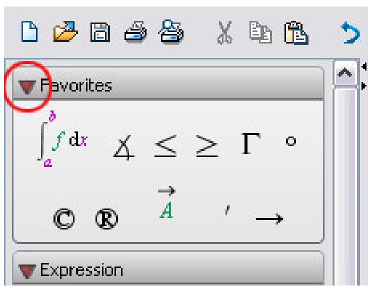
Viewing and Arranging Palettes

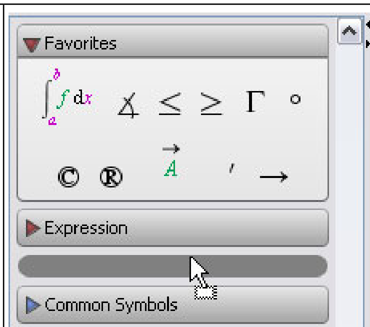
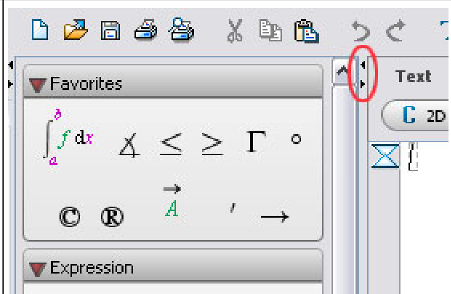
By default, palettes display in palette docks at the right and left sides of the Maple window. To view and manage palettes and palette docks, see Table 1.8.

Table 1.8: Managing Palettes

<table>
<tr>
<td>To view palette docks:

From the View menu, select Palettes, and then Expand Docks. There are docks on the far right and left of the window.</td>
<td>
</td>
</tr>
</table>

To add a palette: 1. Right-click the palette dock. Maple displays a context menu near the palette. 2. From the context menu, select **Show Palette** and then select the palette.	
To expand or collapse a palette in the palette dock: Click the triangle at the left of the palette title.	

To move a palette in the palette dock: Move the palette by clicking the title and dragging the palette to the new location.	
To expand or collapse the palette docks: Select the appropriate triangle at the top right or top left side of the palette region.	

Example 3 - Enter an Expression Using Palettes

Review the following example:

$$\sum_{i=1}^{10} \left(7\, i^2 - 5\, i\right) \;=\; 2420$$

In this example, we will enter $\sum_{i=0}^{10} \left(7\, i^2 - 5\, i\right)$ and evaluate the expression.

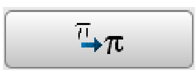
Action	Result in Document
1. Place the cursor in a new document block. In the **Expression** palette, click the summation template $\sum_{i=k}^{n} f$. Maple inserts the summation symbol with the range variable placeholder highlighted.	$\sum_{i=k}^{n} f$

Action	**Result in Document**
2. Enter **i** and then press **Tab**. The left endpoint placeholder is selected. Notice that the color of the range placeholder has changed to black. Each placeholder must have an assigned value before you execute the expression. The **Tab** key advances you through the placeholders of an inserted palette item.	$\sum_{i=k}^{n} f$
3. Enter **1** and then press **Tab**. The right endpoint placeholder is selected.	$\sum_{i=1}^{n} f$
4. Enter **10** and then press **Tab**. The expression placeholder is selected.	$\sum_{i=1}^{10} f$
5. Enter $(7i^2 - 5i)$. For instructions on entering this type of expression, see *Example 1 - Enter and Evaluate an Expression (page 8)*.	$\sum_{i=1}^{10} (7i^2 - 5i)$
6. Press **Ctrl** + = (**Command** + = for Macintosh) to evaluate the summation.	$\sum_{i=1}^{10} (7i^2 - 5i) = 2420$

Handwriting Palette

The **Handwriting** palette provides another way to find and insert desired symbols easily.

1. Draw the symbol with your mouse in the space provided.

2. Click the **recognize** button, . Maple matches your input against symbols available in the system. See Figure 1.3.

3. To view more symbols (where indicated with a box around the result), click the displayed symbol and choose one of the selections from the drop-down menu.

4. To insert a symbol, click the displayed symbol.

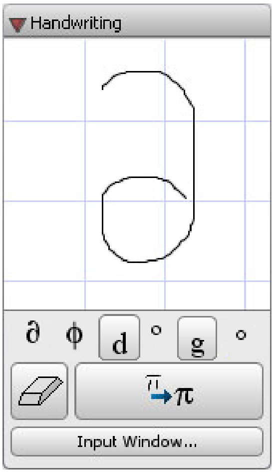


Figure 1.3: Handwriting Palette

For more information, refer to the **?handwritingpalette** help page.

Symbol Names

Each symbol has a name, and some have aliases. By entering its name (or an alias) in Math mode, you can insert the symbol.

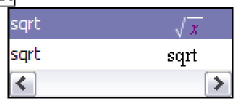
Note: If you hover the mouse pointer over a palette item, a tooltip displays the symbol's name.

To enter a symbol quickly, you can enter the first few characters of its name and then press the completion shortcut key, **Esc** (see *Shortcut Keys by Platform (page xiv)*). Symbol completion works in the same way as command completion (see *Command Completion (page 48)*).

- If a unique symbol name matches the characters entered, Maple inserts the corresponding symbol.
- If multiple symbol names match the characters entered, Maple displays the completion list, which lists all matches, including commands. To select an item, click its name or symbol.

Example 4 - Square Root

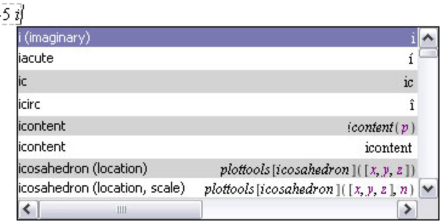
To find the square root of 603729 :

Action	Result in Document
1. In a new document block, enter *sqrt*.	sqrt
2. Press the symbol completion shortcut key, **Esc**. Maple displays a pop-up list of exact matches.	sqrt 
3. In the completion list, select sqrt $\sqrt{x}$. Maple inserts the symbol with the x placeholder selected.	$\sqrt{x}$
4. Enter 603729.	$\sqrt{603729}$
5. Press **Ctrl** + = (**Command** + =, Macintosh).	$\sqrt{603729} = 777$

Example 5 - Complex Numbers

When you simply type the letter *i* in Math mode, it is in italics. This letter is just a variable, and is not the same as the imaginary unit $\sqrt{-1}$, denoted by I or i in Maple.

Multiply two complex numbers, $-0.123 + 0.745\,\mathrm{i}$ **and** $4.2 - \mathrm{i}$ **:**

Action	Result in Document
1. In a new document block, enter $(-0.123 + 0.745\,i$.	$(-0.123 + 0.745\,i$
2. Press the symbol completion shortcut key, **Esc**. Maple displays a pop-up list of partial and exact matches, including symbols and commands.	$(-0.123 + 0.745\,i$ 
3. Select the imaginary unit, i (imaginary) i.	$(-0.123 + 0.745\,\mathrm{i}$
4. Close the parentheses, enter a space (for implicit multiplication), and type the second expression in parentheses, using symbol completion for the second imaginary number.	$(-0.123 + 0.745\,\mathrm{i})\,(4.2 - \mathrm{i})$
5. Press **Ctrl** + = (**Command** + =, Macintosh) to evaluate the product.	$(-0.123 + 0.745\,\mathrm{i})\,(4.2 - \mathrm{i}) = 0.2284 + 3.2520\,\mathrm{I}$

For more information on entering complex numbers, refer to the **?HowDoI** help page.

Toolbar Icons

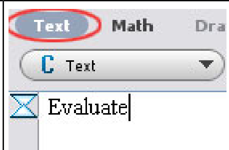
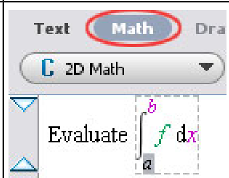
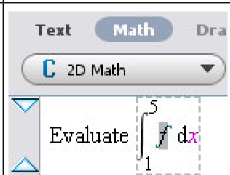
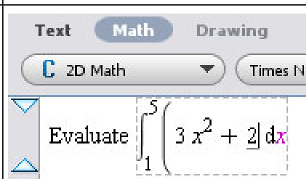
In the introduction section, you learned about the toolbar icons and context toolbars available in Maple (see *Toolbar Options (page 9)*). The toolbar can be used to format your document, alter plots and animations, draw in a canvas, write in both Math and Text modes in one line and much more. The last of these is demonstrated in the next example.

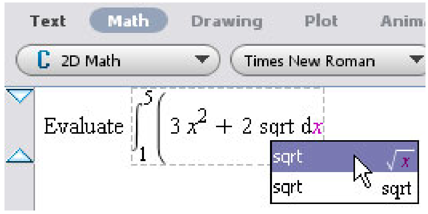
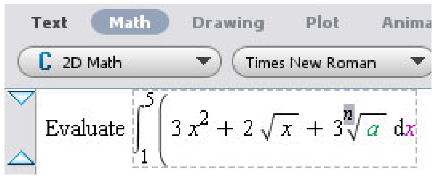
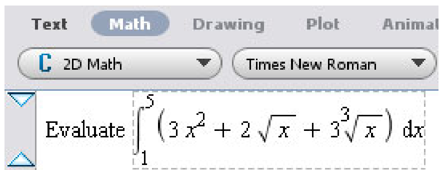
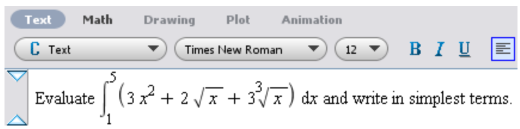
Example 6 - Enter Text and 2-D Math in the Same Line Using Toolbar Icons

Enter the following sentence:

Evaluate

$\int_1^5 \left(3x^2 + 2\sqrt{x} + 3\sqrt[3]{x}\right) dx$ and write in simplest terms.

Action	**Result in Document**
To enter this sentence: 1. Select the **Text** icon and enter **Evaluate**.	
2. Select the **Math** icon. 3. From the **Expression** palette, select the definite integration template, $\int_a^b f\,dx$. The expression is displayed with the first placeholder highlighted.	
4. With the first placeholder highlighted, enter **1**, then press **Tab**. 5. Enter **5** and press **Tab** to highlight the integrand region.	
6. Enter **(3x^2** and press the right arrow to leave the superscript position. 7. Enter + **2**.	

Action	**Result in Document**
8. Press the **Space** bar for implicit multiplication. Enter **sqrt** and press **Esc** to show the command completion options. Maple displays a pop-up list of exact matches. Select the square root symbol, $\sqrt{x}$. Maple inserts the symbol with the x placeholder selected. Alternatively, select the square root symbol from the **Expression** palette.	
9. Enter **x**, then press the right arrow to leave the square root region. 10. Enter + **3**, and then press the **Space** bar. 11. Select the **n-th root** symbol from the Expression palette, $\sqrt[n]{a}$.	
12. Enter **3**, then press **Tab**. 13. Enter **x)**, then press **Tab**. 14. Enter **x** for the integration variable.	
15. Click the **Text** icon in the toolbar, then enter the rest of the sentence: "and write in simplest terms."	

1.3 Point-and-Click Interaction

Maple contains many built-in features that allow you to solve problems quickly without having to know any commands.

Assistants

Maple offers a set of assistants in the form of graphical user interfaces to perform many tasks without the need to use any syntax. An example of an assistant is shown in Figure 1.4.

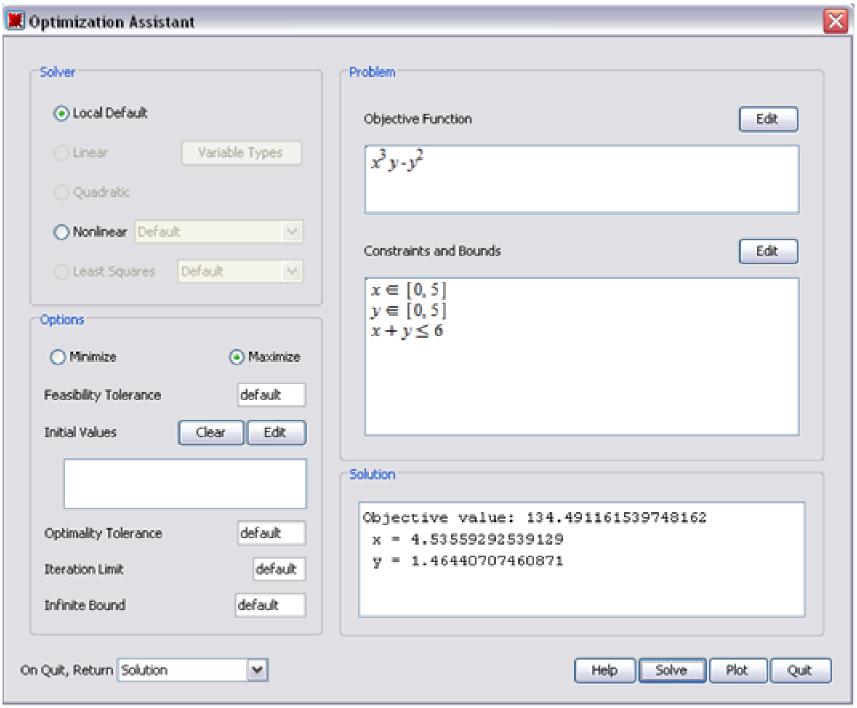


Figure 1.4: Optimization Assistant

Using the **Tools** → **Assistants** menu, you can access tools to help you accomplish various tasks. See Figure 1.5. In some cases, you can launch an assistant by entering an expression and selecting the assistant from the context menu that displays.

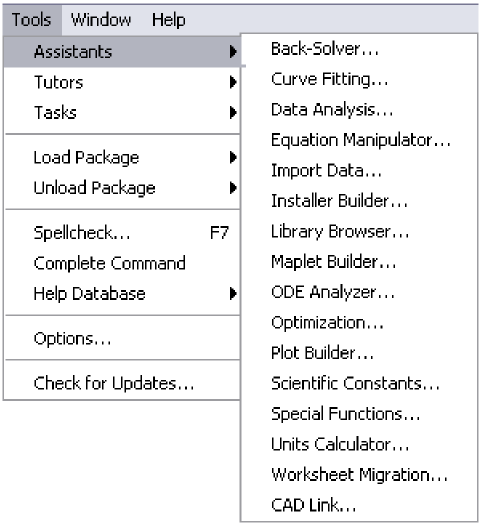
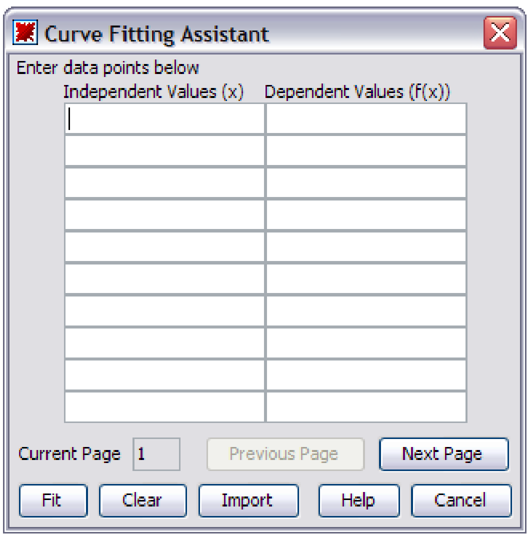
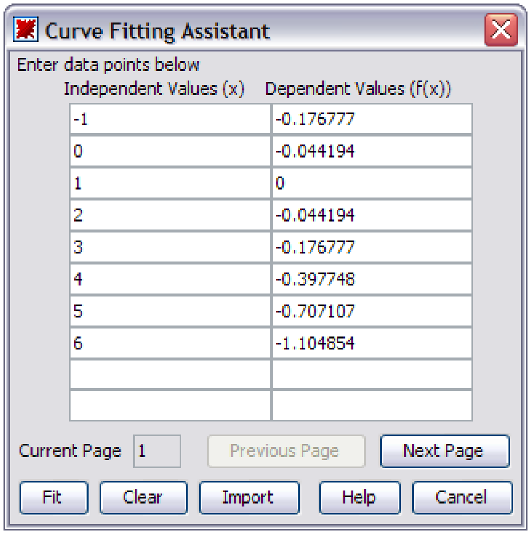
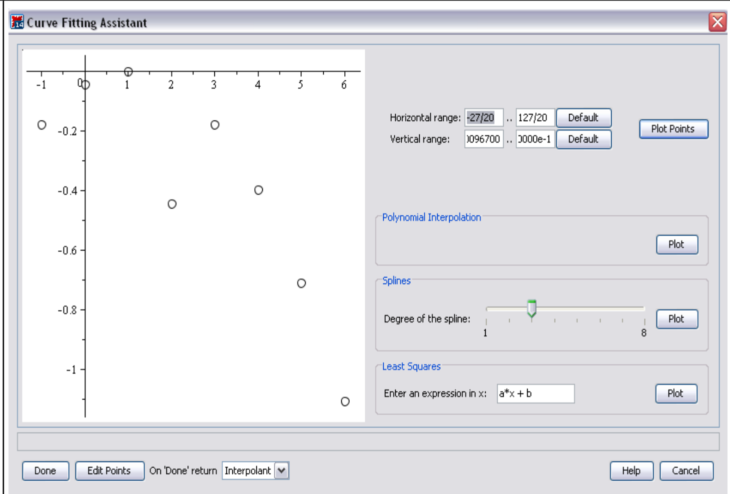


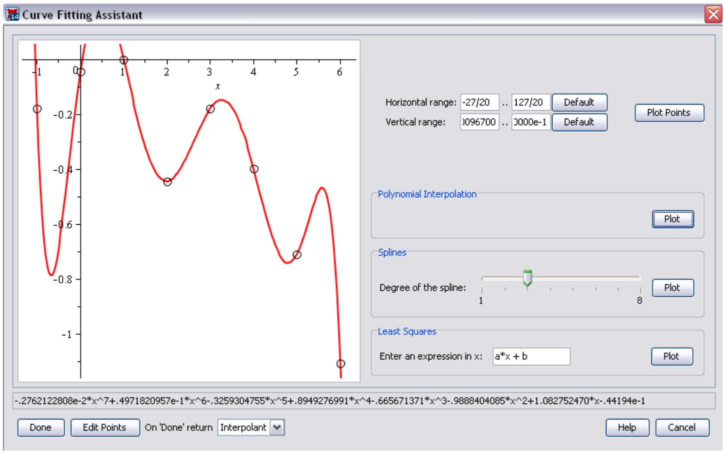
Figure 1.5: Accessing the Assistants from the Tools Menu

Example 7 - Curve Fitting Assistant

Enter a data sample and use the **Curve Fitting Assistant** to find the best approximation of a function to fit the data.

Action	**Result in Document**
1. From the **Tools** menu, select **Assistants** → **Curve Fitting**. The first dialog in the **Curve Fitting Assistant** appears.	

<table>
<tr><th>Action</th><th>Result in Document</th></tr>
<tr><td>2. Enter data as Independent Values and Dependent Values. Alternatively, you could import a file containing data. If you have more data than the space provided, click the Next Page button for more space. For this example, enter the data as shown.</td><td>
</td></tr>
<tr><td>3. Once you have entered the data, click the Fit button. The second dialog of the Curve Fitting Assistant appears.</td><td>
</td></tr>
</table>

<table>
<tr><th>Action</th><th>Result in Document</th></tr>
<tr><td>4. In this dialog, you can plot the data and several types of interpolations, including Polynomial, Spline, and Least Squares. For example, click the Plot button in the Polynomial Interpolation section. The polynomial is plotted with the data, and the interpolating function is displayed below.</td><td>
</td></tr>
<tr><td>5. You can choose to return either the interpolating function or the plot to your document. When finished, click Done.</td><td>$2.182341270\,10^{-9}x^7 - 4.027388889\,10^{-8}x^6 + 2.735838887\,10^{-7}x^5 - 7.985305551\,10^{-7}x^4 + 7.52701389\,10^{-7}x^3 - 0.04419366118\,x^2 + 0.08838747155\,x - 0.0441940000$</td></tr>
</table>

Descriptions of Assistants

The remaining assistants are described below. Some of the assistants are interfaces to package commands. For more information on package commands, see *Package Commands (page 47)*.

- **Back-Solver** - an interface that allows you to take a mathematical formula, involving multiple parameters, enter values for all but one of the parameters and solve for the remaining value. You can also plot the behavior of the formula as one of the parameters change.
- **Curve Fitting** - an interface to commands in the **CurveFitting** package. Data points can be entered as independent and dependent values, and interpolated with polynomials, rational functions, or splines.
- **Data Analysis** - an interface to the data analysis commands in the **Statistics** package.
- **Equation Manipulator** - an interface for interactively performing a sequence of operations on an equation. You can group terms, apply an operation to both sides of the equation, complete the square, and so on.
- **Import Data** - an interface to read data from an external file into Maple.

- **Installer Builder** - an interface to the **InstallerBuilder** package in which you can create installers for your Maple toolboxes.

For information on toolboxes, go to **http://www.maplesoft.com/developers/index.aspx**.

- **Library Browser** - an interface to manipulate the libraries in a specified directory.
- **Maplet Builder** - an interface to the **Maplets** package. The **Maplets** package contains commands for creating and displaying Maplet applications (point-and-click interfaces). Using the Maplet Builder, you can define the layout of a Maplet, drag-and-drop elements (visual and functional components of Maplets), set actions associated with elements, and directly run a Maplet application. The Maplet Builder is available in the Standard interface only.
- **ODE Analyzer** - an interface to obtain numeric or symbolic solutions to a single ordinary differential equation (ODE) or a system of ODEs and plot a solution of the result.
- **Optimization** - an interface to the solver commands in the **Optimization** package. The **Optimization** package is a collection of commands for numerically solving optimization problems, which involves finding the minimum or maximum of an objective function possibly subject to constraints.
- **Plot Builder** - an interface for creating two and three-dimensional plots, animations, and interactive plots.
- **Scientific Constants** - an interface to over 20 000 values of physical constants and properties of chemical elements. All of these constants come with the corresponding unit and, if applicable, with the uncertainty or error, that is, how precisely the value of this constant is known.
- **Special Functions** - an interface to the properties of over 200 special functions, including the Hypergeometric, Bessel, Mathieu, Heun and Legendre families of functions.
- **Units Calculator** - an interface to convert between 500 units of measurement.
- **Worksheet Migration** - an interface to convert worksheets from Classic Maple (.mws files) to Standard Maple (.mw files)
- **CAD Link** - an interface to explore the properties of models from supported CAD applications (available on Microsoft Windows only)

Tutors

Maple provides over 40 interactive tutors to aid in the learning of

- Precalculus
- Calculus
- Multivariate Calculus

- Vector Calculus
- Differential Equations
- Linear Algebra
- Complex Variables

These tutors are easily accessible in the **Tools** menu by selecting **Tutors**. See Figure 1.6.

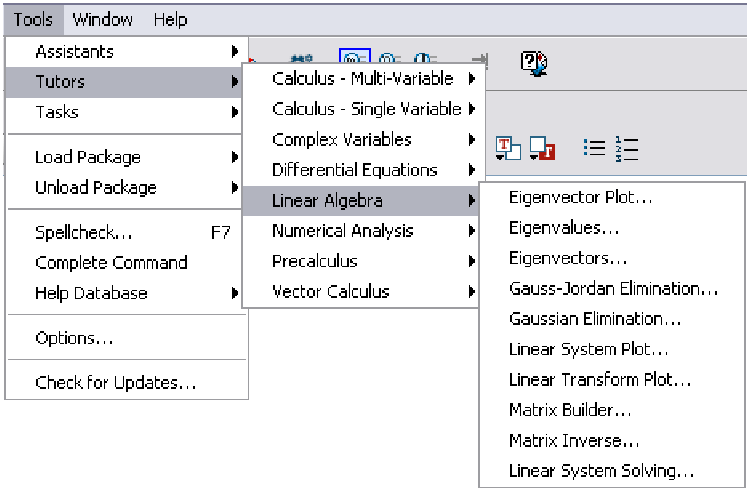


Figure 1.6: Accessing Tutors from the Tools Menu

Some of the tutors can also be accessed through the **Student** package. The Differential Equations tutor, **DE Plots**, is accessible through the **DEtools** package. For a definition of the term *package*, see *Package Commands (page 47)*.

The **Student** package is a collection of subpackages designed to assist with the teaching and learning of standard undergraduate mathematics. The subpackages contain many commands for displaying functions, computations, and theorems in various ways, and include support for stepping through important computations.

The **interactive** commands help you explore concepts and solve problems using a point-and-click interface. These commands launch tutors that provide a graphical interface to some of the visualization and computation commands described above. See Figure 1.7 for an example of one of the tutors.

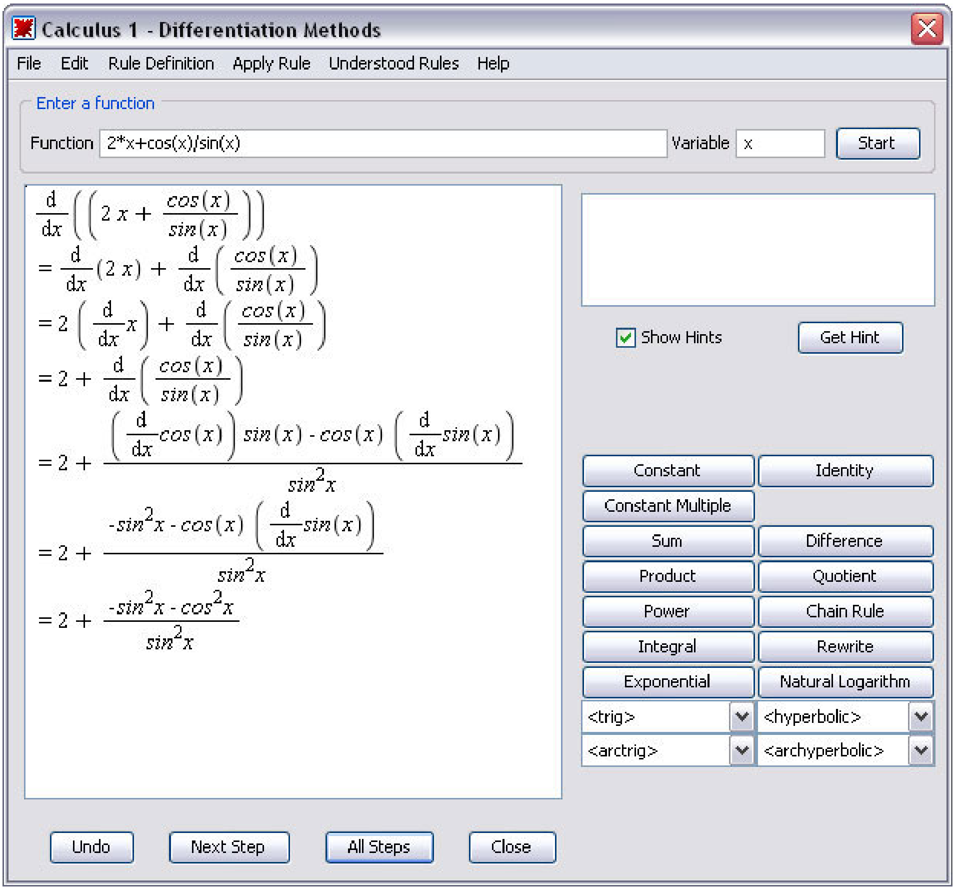


Figure 1.7: Calculus - Single Variable → Differentiation Methods Tutor

Context Menus

A context menu is a dynamically generated menu of actions that are applicable for the region upon which it is invoked. Context menus allow you to perform calculations and manipulations on expressions without using Maple syntax. To display a context menu, right-click an object, expression, or region. Context menus are available for many input regions, including:

- **expressions** to perform calculations, manipulations, or plotting
- **plot regions** to apply plot options and manipulate the plot
- **tables** to modify the table properties
- **palette regions** to add or remove palettes and palette regions
- **text regions** to add annotations and format text

- **spreadsheets** to manipulate the spreadsheet

When performing calculations or manipulations on an expression, a self-documenting arrow or equal sign connects the input and output, indicating the action that took place. See Figures 1.8 and 1.9 for two examples of context menus.

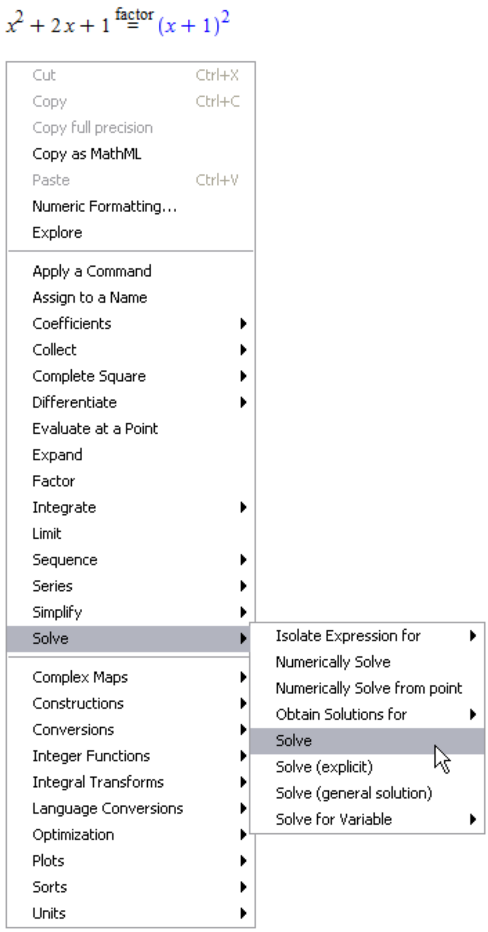


Figure 1.8: Right-click the expression to see a menu of applicable operations

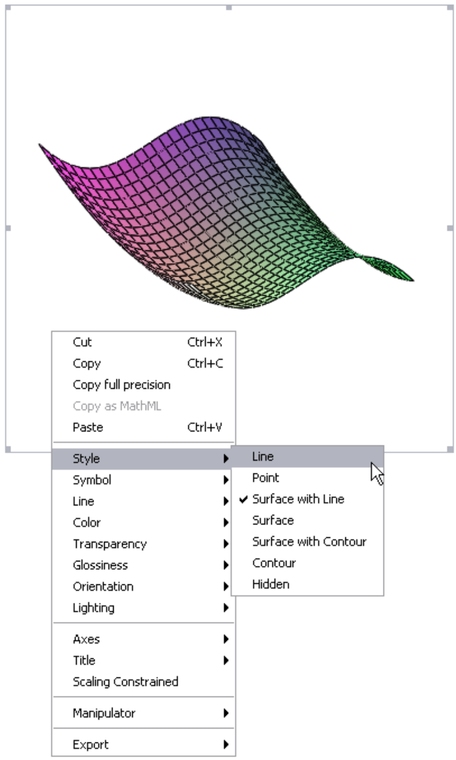


Figure 1.9: Right-click the plot to see a menu of plot options

Task Templates

Task templates help you perform specific tasks in Maple, such as:

- performing a mathematical computation such as solving an equation symbolically or numerically, or determining the Taylor approximation of a function of one variable
- constructing a Maple object such as a function
- creating a document such as an application

Each task contains a description along with a collection of content that you can insert directly into your document. Content consists of 2-D mathematics, commands, embedded components (for example, buttons), and plots. You specify the parameters of your problem and then execute the commands in the document. See Figure 1.10 for an example of a Task Template.

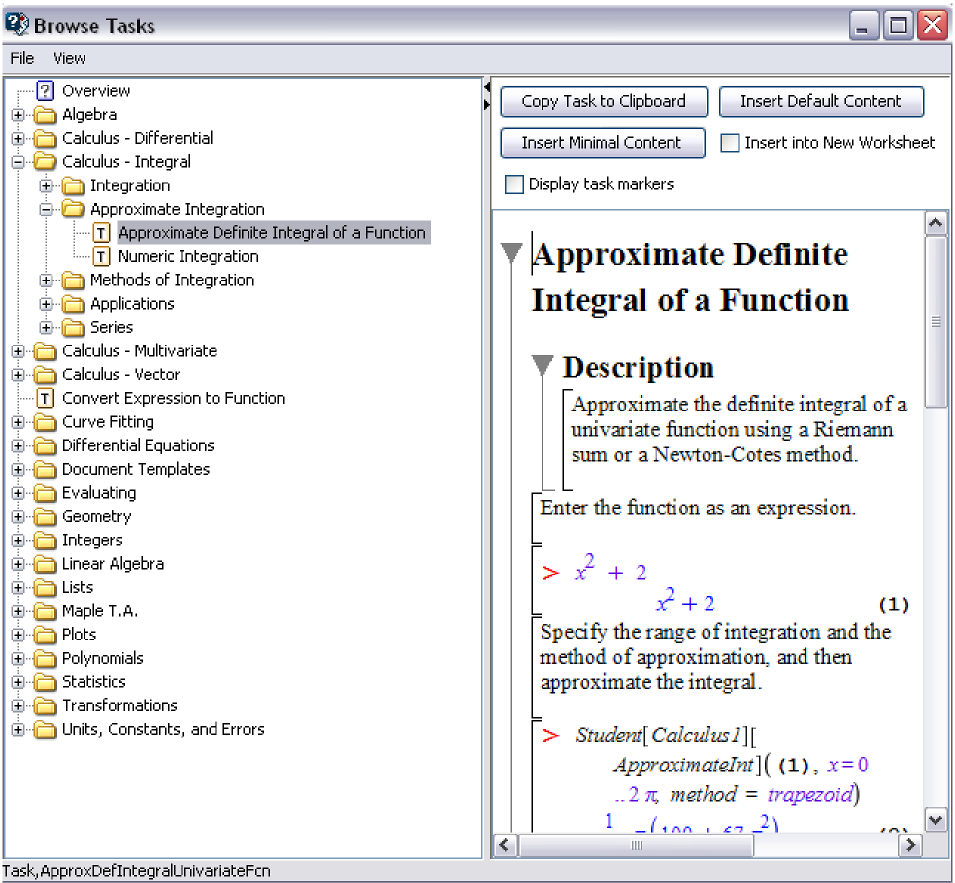


Figure 1.10 Browse Tasks Dialog

Previewing Tasks

To preview Maple tasks,

- From the **Tools** menu, select **Tasks**, and then **Browse**. The **Browse Tasks** dialog opens and displays the list of tasks.

The tasks are sorted by subject to help you quickly find the desired task. In the **Browse Tasks** dialog, you can view tasks without inserting them into your document.

Inserting a Task into the Document

To insert a task into your document,

1. Select the **Insert into New Worksheet** check box to insert the task into a new document.

2. Click one of the insert buttons.

- Click the **Insert Default Content** button. Maple inserts the *default content*. The default content level is set using the **Options** dialog. For instructions, see the **?usingtasks** help page.
- Click the **Insert Minimal Content** button. Maple inserts only the commands and embedded components, for example, a button to launch the related assistant or tutor.
- Click the **Copy Task to Clipboard** button. Place the cursor where you want to insert the task, and then paste the task. Maple inserts the default content. Use this method to quickly insert a task multiple times.

Note: You can view the history of previously inserted tasks. From the **Tools** menu, select **Tasks.** Previously selected task names are displayed below the **Browse** menu item.

Before inserting a task, Maple checks whether the task variables have assigned values in your document. If any task variable is assigned, the **Task Variables** dialog opens to allow you to modify the names. Maple uses the edited variable names for all variable instances in the inserted task.

By default, the **Task Variables** dialog is displayed only if there is a naming conflict. You can set it to display every time you insert a task.

To specify that the Task Variables dialog be displayed every time you insert a task:

1. From the **Tools** menu, select **Options**.

2. Click the **Display** tab.

3. In the **Show task variables on insert** drop-down list, select **Always**.

4. Click **Apply to Session** or **Apply Globally**, as necessary.

Updating Parameters and Executing the Commands

In inserted Task Templates, parameters are marked as placeholders (in purple text) or specified using sliders or other embedded components.

1. Specify values for the parameters in placeholders or using graphical interface components. You can move to the next placeholder by pressing **Tab**.

2. Execute all commands in the task by:

- Placing the cursor in the first task command, and then pressing **Enter** repeatedly to execute each command.
- Selecting all the template commands, and then clicking the execute toolbar icon **!**.

3. If the template contains a button that computes the result, click it.

For more information on task templates, refer to the **?tasks** help page.

Exploration Assistant

The Exploration Assistant allows you to interactively make parameter changes to expressions and view the result. The assistant can be used with almost any Maple expression or command that has at least one variable or parameter.

To launch the Exploration Assistant:

1. Enter an expression or command.

2. Right-click (**Control**-click, Macintosh) the expression or command. From the context menu, select **Explore**.

3. The **Explore** parameter selection dialog appears, where you can select the parameters to explore and the range for each parameter.

If you enter integer ranges, only integer values are allowed for parameters. To allow floating-point values, enter floating-point ranges.

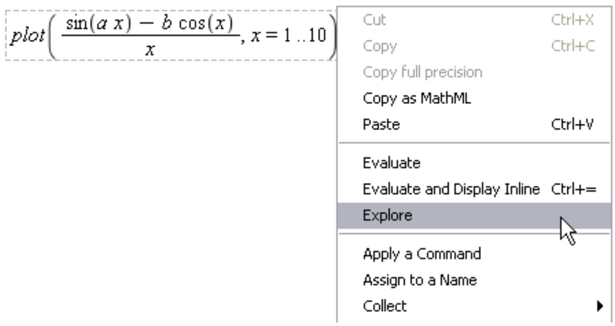
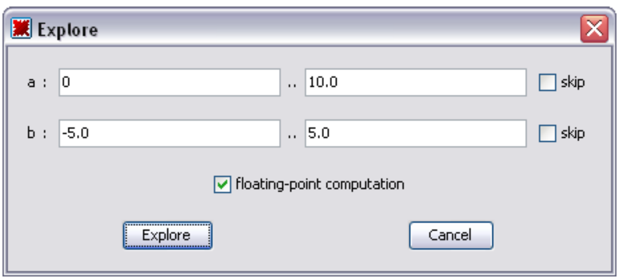
Select **skip** for any of the parameters to leave that parameter as a variable.

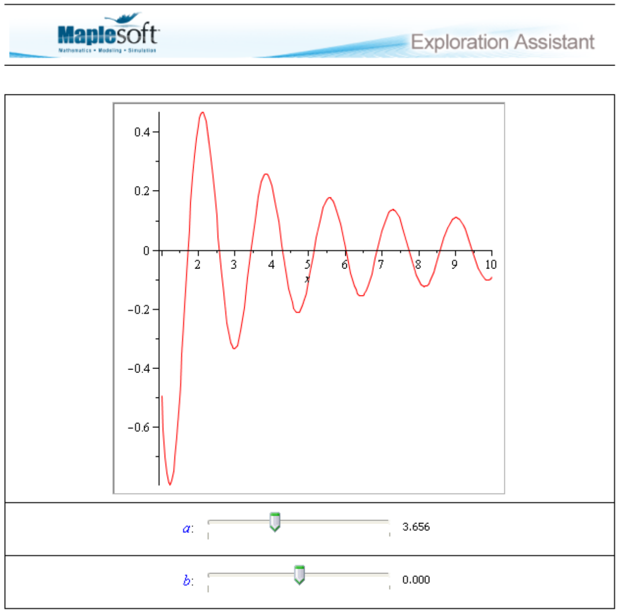
4. Click **Explore** to continue to the **Exploration Assistant**. The assistant opens in a new document. You can use the slider or sliders to vary the parameters and see your changes as the expression output is updated.

5. Once you are finished interacting with the assistant, you can copy and paste the results into your document, or save the interactive document for later use.

Example 8 - Use the Exploration Assistant to Explore a Plot

In this example, we will explore how the plot of $\frac{\sin(a\,x) - b\cos(x)}{x}$ changes as we vary the parameters a and b.

Action	**Result in Document**
1. Enter the plot command shown.	$plot\left(\frac{\sin(a\,x) - b\cos(x)}{x}, x = 1\,..10\right)$
2. Right-click (**Control**-click for Macintosh) the expression and select **Explore**.	
3. In the Explore parameter selection dialog, set the ranges **a = 0..10.0** and **b = -5.0..5.0**. Select **floating-point computation**.	

Action	Result in Document
4. Click **Explore**. The **Exploration Assistant** opens in a new document. Move the sliders to see the plot as the parameters change.	

1.4 Commands

Even though Maple comes with many features to solve problems and manipulate results without entering any commands, you may find that you prefer greater control and flexibility by using the set of commands and programming language that Maple offers.

The Maple Library

Commands are contained in the Maple library, which is divided into two groups: the *main library* and *packages*.

- The main library contains the most frequently used Maple commands.
- Packages contain related commands for performing tasks from disciplines such as Student Calculus, Statistics, or Differential Geometry. For example, the **Optimization** package contains commands for numerically solving optimization problems.

For details on top-level and package commands, see *Commands (page 82)*.

Entering Commands

If you want to interact with Maple using commands, simply enter the command using 2-D math. Notice that commands and variable names display in italics. Maple commands are constructed in a format similar to *command*(*arguments*), based on the command you are using.

For example, to factor an expression, enter:

$factor\left(x^2+2x+1\right)$

$$(x+1)^2$$

To differentiate an expression, enter:

$diff(\sin(x), x)$

$$\cos(x)$$

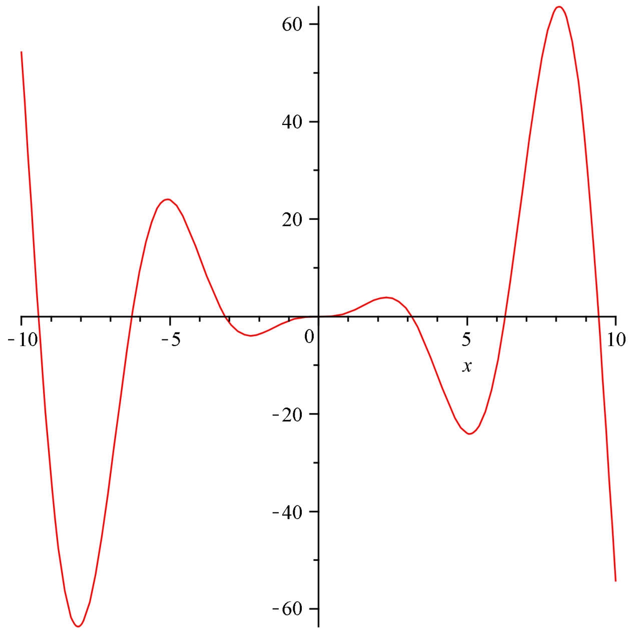
To integrate an expression on the interval $[0, 2\pi]$, enter:

$int\left(2x+\cos(x), x=0..2\pi\right)$

$$4\pi^2$$

To plot an expression, enter:

$plot\left(\sin(x)\,x^2, x=-10..10\right)$



For a list of the top commands in Maple, see *Top Commands (page 84)*.

Package Commands

There are two ways to access commands within a package, using the long form of the package command or the short form.

Long Form of Accessing Package Commands:

The long form specifies both the package and command names using the syntax *package***[***command***](***arguments***)**.

$LinearAlgebra[RandomMatrix](2)$

$$\begin{bmatrix} 44 & -31 \\ 92 & 67 \end{bmatrix}$$

Short Form of Accessing Package Commands:

The short form makes all of the commands in the package available using the **with** command, *with(package)*. If you are using a number of commands in a package, loading the entire package is recommended. When you execute the **with** command, a list of all commands in the package displays. To suppress the display of all command names, end the *with(package)* command with a colon. Alternatively, you can load packages through the **Tools** menu, by selecting **Load Package**, and then the package name.

$with(Optimization)$

$[ImportMPS, Interactive, LPSolve, LSSolve, Maximize, Minimize, NLPSolve, QPSolve]$

After loading a package, you can use the short-form names, that is, the command names, without the package name.

$LSSolve([x-2, x-6, x-9])$

$$[12.3333333333333322, [x = 5.66666666666666696]]$$

For a list of the top packages in Maple, see *Top Packages (page 86)*.

Command Completion

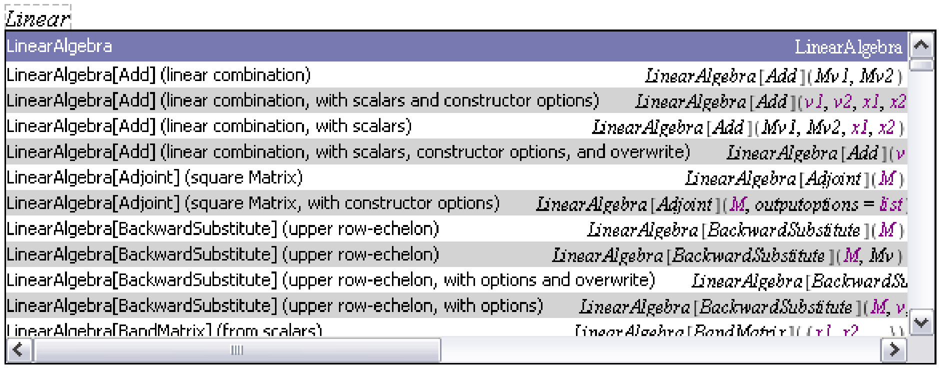
To help with syntax and reduce the amount of typing when entering Maple commands, you can use *command completion*. Command completion displays a list of all Maple packages, commands, and functions that match the entered text. If there are multiple ways to call a command, then the command completion list contains each one, with appropriate placeholders.

To use command completion:

1. Begin entering a command or package name.

2. Select **Tools → Complete Command** or use the shortcut key **Esc** (see *Shortcut Keys by Platform (page xiv)*). If there is a unique completion, it is inserted. Otherwise, a list of possible matches is displayed.

3. Select the correct completion from the list.



4. Some inserted commands have placeholders, denoted by purple text. The first placeholder is highlighted after you insert it into the document. Replace it with your parameter, then move to the next placeholder by pressing the **Tab** key.

Equation Labels

Equation labels help to save time entering expressions by referencing Maple output. See Figure 1.11.

By default, equation labels are displayed. If equation labels are not displayed,

1. From the **Tools** menu, select **Options**, and click the **Display** tab. Ensure that the **Show equation labels** check box is selected.

2. From the **Format** menu, select **Labels**. Ensure that both **Execution Group** and **Worksheet** are selected.

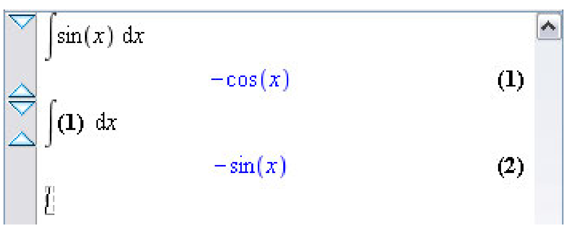


Figure 1.11: Equation Label

To apply equation labels:

1. Enter an expression and press **Enter**. Note that the equation label is displayed to the right of the answer in the document.

2. In a new execution group, enter another expression that will reference the output of the previous execution group.

3. From the **Insert** menu, select **Label**. Alternatively, press **Ctrl**+**L** (**Command**+**L**, for Macintosh) to open the **Insert Label** dialog. Enter the label number in the **Insert Label** dialog and click **OK**. The item is now a label. See Figure 1.12.

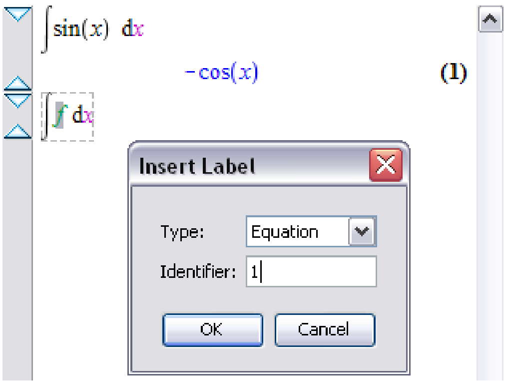


Figure 1.12: Inserting an Equation Label

4. Press **Enter** to obtain the result.

To change the format of equation labels:

- Select **Format** → **Labels** → **Label Display**. In the **Format Labels** dialog, select one of the numbering schemes.
- Optionally, enter an appropriate numbering prefix.

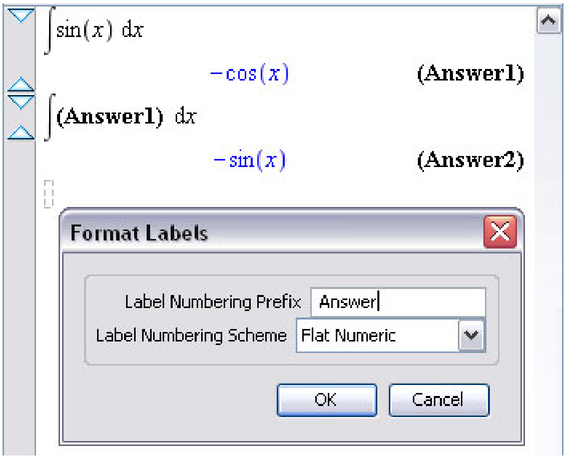


Figure 1.13: Format Labels Dialog: Adding a Prefix

The **Label Reference** menu item allows you to switch between the label name and its reference content. Place the cursor on the referenced equation label and select **Format** → **Labels** → **Label Reference**.

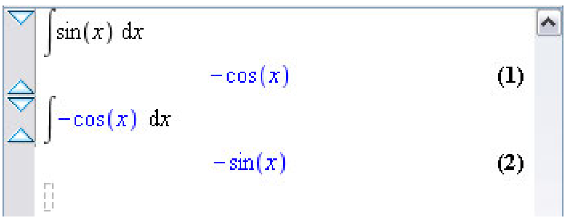


Figure 1.14: Label Reference

The label is associated with the last output within an execution group.

You cannot apply equation labels to the following:

- Error, warning, and information messages
- Tables, images, plots, sketches, or spreadsheets

Document Blocks

In Document mode, content is created as a series of document blocks. Document blocks allow you to hide the syntax used to perform calculations, which in turn lets you focus on the concept presented instead of the command used to manipulate or solve the problem. You can also create document blocks in Worksheet mode to perform the same function. Document blocks are typically collapsed to hide the Maple code, but these regions can also be expanded to reveal this code.

To create a document block:

From the **Format** menu, select **Create Document Block**. If text or math in one or more execution groups is selected, then a document block is created that contains those execution groups. If not, a new document block is created after the current execution group. For more information, see the next example.

Document block regions are identified using markers that are located in a vertical bar along the left pane of the document. See Figure 1.15. In addition to document block boundaries, these markers (icons) indicate the presence of hidden attributes in the document such as annotations, bookmarks, and numeric formatting.

To activate markers:

From the **View** menu, select **Markers**. See Figure 1.15.

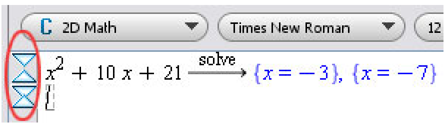


Figure 1.15: Document Block Markers

To view code in a document block:

1. Place the cursor in a document block to be expanded.

2. From the **View** menu, select **Expand Document Block**.

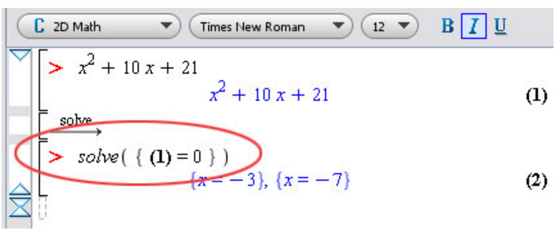


Figure 1.16: Expanded Document Block

With the Document Block expanded, you can see the Maple command that was used to perform this calculation. In Figure 1.16, the *solve* command was used.

Also notice a red prompt (>) before the original expression and the *solve* command. Entering commands outside of a document block region is done at this input region. To insert an input region, click the [> button in the toolbar menu.

In Figure 1.16, an equation label was used to refer to the expression. For more information, see *Equation Labels (page 49)*.

To collapse a Document Block:

- With your cursor inside the document block, select **View → Collapse Document Block**.

You can use this process of expanding document blocks to begin learning Maple commands.

Changing the Display:

You can specify which parts of the input and output are displayed when the document block is collapsed. For each execution group in the block, you can choose to display either the input or the output.

- Place the cursor in the execution group.
- From the **View** menu, select **Toggle Input/Output Display**.

Also, you can choose to display output either inline or centered on a new line.

- From the **View** menu, select **Inline Document Output**.

Example 9 - Creating a Document Block in Worksheet Mode

In Worksheet mode, you can create the content using commands, and then use a document block to choose how much information to display.

Enter the following sentence using text and 2-D Math input and output:

The answer to $\int \sin(x)\, dx$ is $-\cos(x)$.

1. At an input prompt, click the text icon, T, to enter plain text. Enter "The answer to ". **Note:** these instructions are for Worksheet mode.	[The answer to
2. Click the input prompt icon, [>, to enter Maple commands. Enter $\int \sin(x)\, dx$, and then press **Enter** to execute the command.	The answer to > $\int \sin(x)\, dx$ $-\cos(x)$ (1.3) >
3. Again, click the text icon to insert the rest of the text, "is", and then enter another input prompt icon. Make sure to put spaces around all of the text, so the sentence displays properly.	The answer to > $\int \sin(x)\, dx$ $-\cos(x)$ (1.3) is >

4. To display the same output again, use the **value** command and an equation label. This allows you to insert text between the input and output of a single command: there are really two commands. Enter and execute the command, as shown.	The answer to > $\int \sin(x)\,dx$ $-\cos(x)$ (1.3) is > *value*((1.3)) $-\cos(x)$ (1.4) >
5. To finish the sentence, click the text icon in the last execution group and enter a period.	The answer to > $\int \sin(x)\,dx$ $-\cos(x)$ (1.3) is > *value*((1.3)) $-\cos(x)$ (1.4) .
6. Select the entire sentence, then from the **Format** menu, select, **Create Document Block**. By default, only the text and output remains visible, and output is centered on a new line.	The answer to $-\cos(x)$ (1.3) is $-\cos(x)$ (1.4) .
7. To display the text and output on one line, place the cursor in the document block. From the **View** menu, select **Inline Document Output**.	The answer to $-\cos(x)$ is $-\cos(x)$.
8. To display input instead of output for the first expression, place the cursor in the first expression. From the **View** menu, select **Toggle Input/Output Display**. Only the first region displays input.	The answer to $\int \sin(x)\,dx$ is $-\cos(x)$.

1.5 The Maple Help System

The Maple program provides a custom help system consisting of almost 5000 reference pages. The help system is a convenient resource for determining the syntax of Maple commands and for learning about Maple features.

Accessing the Help System

There are several ways to access the Maple help system:

- From the **Help** menu, select **Maple Help**
- Click in the toolbar

To get help on a specific word:

- In a document, place the insertion point in a word for which you want to obtain help. From the **Help** menu, select **Help on ...**. Alternatively, press **F2** (**Control** + **?**, for Macintosh) to access context-sensitive help.

- In a document, execute the command **?topic**, for example, enter **?LinearAlgebra** and press **Enter**

The Maple help system opens in a separate window with two panes. The left pane contains the Help Navigator where you initiate searches and browse the table of contents, and the right pane displays the final search result, such as a specific help page.

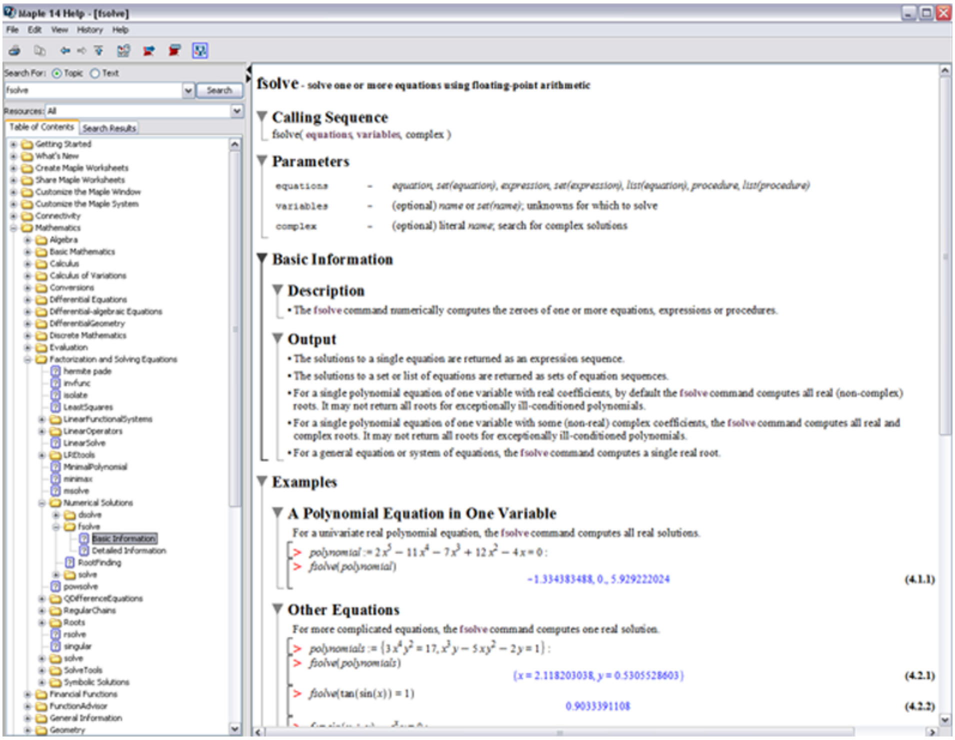


Figure 1.17: Example Help Page

Every help page in Maple lists the command's calling sequence, parameters, and a description, with examples of the command at the end of the page. Some help pages also contain hyperlinks to related help pages and hyperlinks to dictionary definitions. Hyperlinks to help pages display in green, while hyperlinks to dictionary definitions display in dark red.

Using the Help Navigator

The Help Navigator contains a field for topic or text-based searches. The **Table of Contents** tab provides a structured list of all topics in the help system.

To search the help system:

1. In the left pane, enter a string in the search field.

2. By default, a topic search is performed. To perform a text search, select the **Text** radio button.

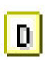
3. Enter the term and click **Search**.

- **Topic** searches reveal a list of matching topics sorted by the precision of the match.
- **Text** searches reveal a list of topics based on keyword frequency.
- You can search all of the help system or specific Resources such as Help Pages, Tasks, Tutorials, and Manuals by selecting the **Resources** drop-down menu.

Search results are displayed as a list in the **Search Results** tab of the left pane. Click the **Table of Contents** tab to view a structured list of all topics in the help system.

To display potential matches in the right pane, click a topic preceded by an icon. Table 1.9 describes the different icons.

Table 1.9: Help Page Icons

Icon	Description
	A folder icon in the **Table of Contents** tab indicates that a topic can be expanded into subtopics.
	Question mark icon indicates a help page and displays the associated help page in the right pane when selected.
	**WS** icon indicates an example worksheet. Example worksheets open in a new tab in the Maple document.
	**D** icon indicates a definition and displays the associated dictionary definition in the right pane when selected.
	**T** icon indicates a Task template and displays the associated Task Template in the right pane when selected.
	**M** icon indicates a manual. Manuals open in a new tab in the Maple document.

Viewing Help Pages as Documents

In the help system, examples are not executable.

The Maple help system allows you to open help pages as documents that you can execute.

To open a help page as a document or worksheet:

- With the help page displayed in the right pane of the help system, from the **View** menu, select **Open Page as Worksheet.** A new worksheet tab opens and displays the help page as an executable document.

	Alternatively, in the help system toolbar, click the *open current help page in a worksheet window* icon.

Viewing Examples in 2-D Math

You can choose to view the examples in most help pages in either 1-D Math (Maple input) or 2-D Math mode. The default is 1-D Math.

To change the math mode:

In the Maple help system:

- From the **View** menu, select or clear the **Display Examples with 2D math** check box.
- Click the **2-D Math** icon, .
- **Note**: Some input in help pages displays as 1-D Math, no matter which option you have chosen. This is for Maple procedures and other code that is best input in 1-D Math. For more information, see the **?helpnavigator** help page.

Copying Examples

Instead of opening the entire page as a document, you can copy the **Examples** section only.

To copy examples:

1. With the help page displayed in the right pane of the help system, from the **Edit** menu, select **Copy Examples.**

2. Close or minimize the Help Navigator and return to your document.

3. In your document, place the cursor at the location where you want to paste the examples.

4. From the **Edit** menu, select **Paste**. The **Examples** section of the help page is inserted as executable content in your document.

1.6 Available Resources

Your work with Maple is supported by numerous resources.

Resources Available through the Maple Help System

Help Pages

Use the help system to find information about a specific topic, command, package, or feature. For more information, see *The Maple Help System (page 54)*.

Dictionary

More than 5000 mathematical and engineering terms with over 300 figures and plots.

1. From the **Help** menu, select **Maple Help**.

2. Enter a search term. Dictionary entries that match your query are displayed in the left pane with a icon.

Tutorials and the Maple Portal

The **Maple Portal** includes material designed for all Maple users, from new users to users who want more advanced tutorials. The Maple Portal also includes specific sections for students, math educators, and engineers. The Maple Portal includes:

- How Do I... topics that give quick answers to essential questions
- Tutorials that provide an overview of topics from getting started to plotting, data manipulation, and interactive application development
- Navigation to portals with specialized information for students, math educators, and engineers

Access the portal from the **Help** menu (**Help → Manuals, Resources, and More → Maple Portal)**.

Applications and Example Worksheets

Applications

Sample applications demonstrate how Maple can be used to find and document a solution to a specific problem. Some applications allow for input or contain animations that you can run; however, their primary use is for demonstrations. Topics include DC Motor Control Design, Digital Filter Design, Frequency Domain System Identification, Harmonic Oscillator, Image Processing, and Radiator Design with CAD Systems.

Examples

Example worksheets are executable documents covering topics that demonstrate syntax or invoke a user interface to make complex problems easy to solve and visualize. You can copy and modify the examples as needed. Topics include Algebra, Calculus, Connectivity, Discrete Mathematics, General Numerics and Symbolics, and Integral Transforms.

- From the **Help** menu, select **Manuals, Resources, and more**, and then **Applications and Examples**.

Manuals

You can access all of Maple's manuals from within Maple, including the *Maple Programming Guide* and this manual. You can execute examples, copy content into other documents, and search the contents using the Maple Help System.

- From the **Help** menu, select **Manuals, Resources, and more** and then **Manuals**.

Task Templates

Set of commands with placeholders that you can use to quickly perform a task. For details, see *Task Templates (page 41)*.

- From the **Tools** menu, select **Tasks**, and then **Browse**.

Maple Tour and Quick Resources

Maple Tour

The **Maple Tour** consists of interactive sessions on several of the following topics: Ten Minute Tour, Numeric and Symbolic Computations, Matrix Computations, Differential Equations, Statistics, Programming and Code Generation, Units and Tolerances, and Education Assessment with Maple T.A.

- From the **Help** menu, select **Take a Tour of Maple**.

Quick Help and Quick References

The Quick Help dialog is a list of key commands and concepts.

- From the **Help** menu, select **Quick Help**. Alternatively, press **F1**. For additional information, click an item in the Quick Help.

The Quick Reference is a table of commands and information for new users that opens in a new window. It contains hyperlinks to help pages for more information.

- From the **Help** menu, select **Quick Reference**. Alternatively, press **Ctrl** + **F2** (**Command** + **F2**, for Macintosh).

Web Site Resources

Welcome Center

A Maple web site offering all of Maplesoft's key user resources in one central location. In the Welcome Center, you can view sample applications, participate in user forums, access exclusive premium content, and listen to podcasts. You can also access our support services, view training videos, download user manuals, and more.

(http://www.maplesoft.com/welcome)

Student Help Center

The Student Help Center offers a Maple student forum, online math Oracles, training videos, and a math homework resource guide.

(http://www.maplesoft.com/studentcenter)

Teacher Resource Center

The Teacher Resource Center is designed to ensure you get the most out of your Maple teaching experience. It provides sample applications, course material, training videos, white papers, e-books, podcasts, and tips.

(http://www.maplesoft.com/teachercenter)

Application Center

Maple web site resource for free applications related to mathematics, education, science, engineering, computer science, statistics and data analysis, finance, communications, and graphics. Many applications are available in translations (French, Spanish, and German).

You can also search for Education and Research PowerTools, which provide free course curricula and are available as add-on Maple packages and courses. PowerTools are developed by experts in their fields to help users configure Maple for research in specific application areas.

(http://www.maplesoft.com/applications)

Training

Maplesoft offers a comprehensive set of complementary training materials. From complete training videos to recorded training seminars to downloadable documentation, you have many options to get familiar with Maplesoft products. In addition, whether you are an expert or someone who is considering a new license purchase, a custom training session that is right for you and/or your organization can be created.

(**http://www.maplesoft.com/support/training)**

MaplePrimes

A web community dedicated to sharing experiences, techniques, and opinions about Maple and related products, as well as general interest topics in math and computing.

(http://www.mapleprimes.com)

Online Help

All of Maple's help pages are available online.

(**http://www.maplesoft.com/support/help**)

Technical Support

A Maple web site containing FAQs, downloads and service packs, links to discussion groups, and a form for requesting technical support.

(http://www.maplesoft.com/support)

For a complete list of resources, refer to the **?MapleResources** help page.

2 Document Mode

Using the Maple software, you can create powerful interactive documents. You can visualize and animate problems in two and three dimensions. You can solve complex problems with simple point-and-click interfaces or easy-to-modify interactive documents. You can also devise custom solutions using the Maple programming language. While you work, you can document your process, providing text descriptions.

2.1 In This Chapter

Section	**Topics**
Introduction (page 63)	• Comparison of Document and Worksheet Modes
Entering Expressions (page 64) - Overview of tools for creating complex mathematical expressions	• Palettes • Symbol Names • Mathematical Functions
Evaluating Expressions (page 67) - How to evaluate expressions	• Displaying the Value Inline • Displaying the Value on the Following Line
Editing Expressions and Updating Output (page 68) - How to update expressions and regenerate results	• Updating a Single Computation • Updating a Group of Computations • Updating All Computations in a Document
Performing Computations (page 69)- Overview of tools for performing computations and solving problems	• Computing with Palettes • Context Menus • Assistants and Tutors

2.2 Introduction

Maple has two modes: *Document* mode and *Worksheet* mode.

Document mode is designed for quickly performing calculations. You can enter a mathematical expression, and then evaluate, manipulate, solve, or plot it with a few keystrokes or mouse clicks. This chapter provides an overview of Document mode.

Document mode sample:

Find the value of the derivative of $\ln(x^2+1)$ at $x=4$.

$$\ln(x^2+1) \xrightarrow{\text{differentiate w.r.t. x}} \frac{2x}{x^2+1} \xrightarrow{\text{evaluate at point}} \frac{8}{17}$$

Integrate $\sin\left(\frac{1}{x}\right)$ over the interval $[0, \pi]$.

$$\int_0^{\pi} \sin\left(\frac{1}{x}\right) \mathrm{d}x = \sin\left(\frac{1}{\pi}\right)\pi - \mathrm{Ci}\left(\frac{1}{\pi}\right)$$

Worksheet mode is designed for interactive use through commands and programming using the Maple language. The Worksheet mode supports the features available in Document mode described in this chapter. For information on using Worksheet mode, see Chapter 3, *Worksheet Mode (page 79)*. **Note**: To enter a Maple input prompt while in Document mode, click [> in the Maple toolbar.

Important: In any Maple document, you can use Document mode and Worksheet mode.

Interactive document features include:

- Embedded graphical interface components, like buttons, sliders, and check boxes
- Automatic execution of marked regions when a file is opened
- Tables
- Character and paragraph formatting styles
- Hyperlinks

These features are described in Chapter 7, *Creating Mathematical Documents (page 281)*.

Note: This chapter and Chapter 1 were created using Document mode. All of the other chapters were created using Worksheet mode.

2.3 Entering Expressions

Chapter 1 provided an introduction to entering simple expressions in 2-D Math (see *Entering Expressions (page 19)*). It is also easy to enter mathematical expressions, such as:

- Piecewise-continuous functions: $|x| = \begin{cases} -x & x < 0 \\ 0 & x = 0 \\ x & 0 < x \end{cases}$

- Limits: $\delta(x) = \lim_{\epsilon \to 0} \epsilon |x|^{\epsilon - 1}$

- Continued fractions: $\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cdots}}}$

and more complex expressions.

Mathematical expressions can contain the following objects.

- Numbers: integers, rational numbers, complex numbers, floating-point values, finite field elements, i , ∞ , ...
- Operators: $+$, $-$, $!$, $/$, $\cdot$, $\int$, $\lim_{x \to a}$, $\frac{\partial}{\partial x}$, ...
- Constants: π, e, ...
- Mathematical functions: $\sin(x)$, $\cos\left(\frac{\pi}{3}\right)$, $\Gamma(2)$, ...
- Names (variables): x, y, z, α, β, ...
- Data structures: sets, lists, Arrays, Vectors, Matrices, ...

Maple contains over a thousand symbols. For some numbers, operators, and names, you can press the corresponding key, for example, **9**, **=**, **>**, or **x**. Most symbols are not available on the keyboard, but you can insert them easily using two methods, palettes and symbol names.

Example 1 - Enter a Partial Derivative

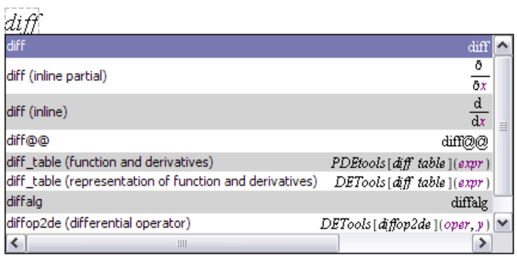
To insert a symbol, you can use palettes or symbol names.

Enter the partial derivative $\frac{\partial}{\partial t} \mathrm{e}^{-t^2}$ using palettes.

Action	Result in Document
1. In the **Expression** palette, click the partial differentiation item $\frac{\partial}{\partial x} f$. Maple inserts the partial derivative. The variable placeholder is selected.	$\frac{\partial}{\partial x} f$
2. Enter **t**, and then press **Tab**. The expression placeholder is selected.	$\frac{\partial}{\partial t} f$
3. Enter e^{-t^2}. **Note**: To enter the exponential e, use the expression palette or command completion.	$\frac{\partial}{\partial t} e^{-t^2}$

To evaluate the integral and display the result inline, press **Ctrl+=** (**Command**+=, for Macintosh) or **Enter**. For more information, see *Computing with Palettes (page 69)*.

You can enter any expression using symbol names and the symbol completion list.

Action	Result in Document
1. Begin typing the name of the symbol, **diff**, and press the symbol completion key (see *Shortcut Keys by Platform (page xiv)*).	
2. Select the partial differentiation item, diff (inline partial) $\frac{\partial}{\partial x}$	$\frac{\partial}{\partial x}$
3. Replace the placeholder with **t**. Use the right arrow to move out of the denominator. Enter e^{-t^2} as in the previous example.	$\frac{\partial}{\partial t} e^{-t^2}$

Example 2 - Define a Mathematical Function

Define the function *twice*, which doubles its input.

Action	Result in Document
1. In the **Expression** palette, click the single variable function definition item, $f := a \to y$.	$f := a \to y$
2. Replace the placeholder **f** with the function name, *twice*. Press **Tab** to move to the next placeholder.	$twice := a \to y$
3. Replace the parameter placeholder, **a**, with the independent variable x. Press **Tab**.	$twice := x \to y$
4. Replace the output placeholder, **y**, with the desired output, $2x$.	$twice := x \to 2x$ $x \to 2x$

$twice(1342) = 2684$

$twice(y - z) = 2y - 2z$

Note: To insert the right arrow symbol $\to$, you can also enter the characters **->** in Math mode. In this case, symbol completion is automatic.

Important: The expression $2x$ is different from the function $x \to 2x$.

For more information on functions, see *Functional Operators (page 343)*.

2.4 Evaluating Expressions

To evaluate a mathematical expression, place the cursor in the expression and press **Ctrl** + **=** (**Command** + =, for Macintosh). That is, *press and hold* the **Ctrl** (or **Command**) key, and then press the equal sign (=) key.

To the right of the expression, Maple inserts an equal sign and then the value of the expression.

$$\frac{2}{9} + \frac{7}{11} = \frac{85}{99}$$

You can replace the inserted equal sign with text or mathematical content.

To replace the equal sign:

1. Select the equal sign. Press **Delete**.

2. Enter the replacement text or mathematical content.

For example, you can replace the equal sign with the text "is equal to".

$\frac{2}{9} + \frac{7}{11}$ is equal to $\frac{85}{99}$

In mathematical content, pressing **Enter** evaluates the expression and displays it centered on the following line. The cursor moves to a new line below the output.

$\frac{2}{9} + \frac{7}{11}$

$$\frac{85}{99} \qquad (2.1)$$

By default, Maple labels output that is generated by pressing **Enter**. For information on equation labels, see *Equation Labels (page 97)*. In this manual, labels are generally not displayed.

In text, pressing **Enter** inserts a line break.

You can use the basic algebraic operators, such as $+$ and $-$, with most expressions, including polynomials—see *Polynomial Algebra (page 150)*—and matrices and vectors—see *Matrix Arithmetic (page 169)*.

$(2x^2 - x + 1) - (x^2 + 2x + 12) = x^2 - 3x - 11$

$3 \cdot \begin{bmatrix} -4 & 8 & 99 \\ 27 & 69 & 29 \end{bmatrix} = \begin{bmatrix} -12 & 24 & 297 \\ 81 & 207 & 87 \end{bmatrix}$

2.5 Editing Expressions and Updating Output

One important feature of Maple is that your documents are *live*. That is, you can edit expressions and quickly recalculate results.

To update one computation:

1. Edit the expression.

2. Press **Ctrl** + **=** (**Command** + **=**, for Macintosh) or **Enter**.

The result is updated.

To update a group of computations:

1. Edit the expressions.

2. Select all edited expressions and the results to recalculate.

3. Click the Execute toolbar icon **!**.

All selected results are updated.

To update all output in a Maple document:

- Click the Execute All toolbar icon **!!!**.

All results in the document are updated.

2.6 Performing Computations

Using the Document mode, you can access the power of the advanced Maple mathematical engine without learning Maple syntax. In addition to solving problems, you can also easily plot expressions.

The primary tools for syntax-free computation are:

- Palettes
- Context menus
- Assistants and tutors

Note: The Document mode is designed for quick calculations, but it also supports Maple commands. For information on commands, see *Commands (page 82)* in Chapter 3, *Worksheet Mode (page 79)*.

Important: In Document mode, you can execute a statement *only if* you enter it in Math mode. To use a Maple command, you must enter it in Math mode.

Computing with Palettes

As discussed in *Entering Expressions (page 64)*, some palettes contain mathematical operations.

To perform a computation using a palette mathematical operation:

1. In a palette that contains operators, such as the **Expression** palette, click an operator item.

2. In the inserted item, specify values in the placeholders.

3. To execute the operation and display the result, press **Ctrl**+**=** (**Command**+=, for Macintosh) or **Enter**.

For example, to evaluate $\frac{\partial}{\partial t} e^{-t^2}$ **inline:**

1. Using the **Expression** palette, enter the partial derivative. See *Example 1 - Enter a Partial Derivative (page 65)*.

2. Press **Ctrl**+**=** (**Command**+=, for Macintosh).

$$\frac{\partial}{\partial t} e^{-t^2} = -2\, t\, e^{-t^2}$$

Context Menus

A *context menu* is a pop-up menu that lists the operations and actions you can perform on a particular expression. See Figure 2.1.

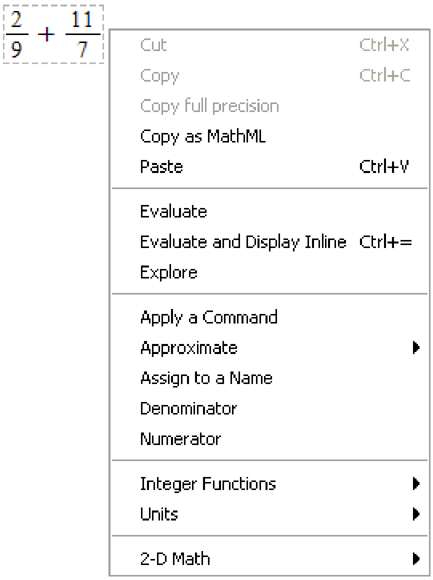


Figure 2.1: Context Menu

To display the context menu for an expression:

- Right-click (**Control**-click, for Macintosh) the expression.

The context menu is displayed beside the mouse pointer.

You can evaluate expressions using context menus.

- The **Evaluate and Display Inline** operation (see Figure 2.1) is equivalent to pressing **Ctrl**+**=** (**Command**+**=**, for Macintosh). That is, it inserts an equal sign (=) and then the value of the expression.

- The **Evaluate** operation (see Figure 2.1) is equivalent to pressing **Enter**. That is, it evaluates the expression and displays the result centered on the following line.

For more information on evaluation, see *Evaluating Expressions (page 67)*.

From the context menu, you can also select operations different from evaluation. To the right of the expression, Maple inserts a right arrow symbol (→) and then the result.

For example, use the **Approximate** operation to approximate a fraction:

$$\frac{2}{3} \xrightarrow{\text{at 10 digits}} 0.6666666667$$

You can perform a sequence of operations by repeatedly using context menus. For example, to compute the derivative of $\cos(x^2)$, use the **Differentiate** operation on the expression, and then to evaluate the result at a point, use the **Evaluate at a Point** operation on the output and enter 10:

$$\cos(x^2) \xrightarrow{\text{differentiate w.r.t. x}} -2\sin(x^2)\,x \xrightarrow{\text{evaluate at point}} -20\sin(100)$$

The following subsections provide detailed instructions on performing a few of the numerous operations available using context menus. Figures in the subsections show related context menus or palettes.

Approximating the Value of an Expression

To approximate a fraction numerically:

1. Enter a fraction.

2. Display the context menu. See Figure 2.2.

3. From the context menu, select **Approximate**, and then the number of significant digits to use: **5**, **10**, **20**, **50**, or **100**.

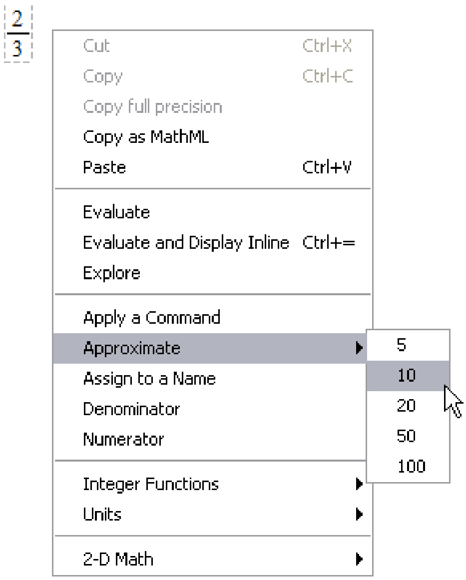


Figure 2.2: Approximating the Value of a Fraction

$$\frac{2}{3} \xrightarrow{\text{at 10 digits}} 0.6666666667$$

You can replace the inserted right arrow with text or mathematical content.

To replace the right arrow ($\rightarrow$):

1. Select the arrow and text. Press **Delete**.

2. Enter the replacement text or mathematical content.

Note: To replace the right arrow with text, you must first press **F5** to switch to Text mode.

For example, you can replace the arrow with the text "is approximately equal to" or the symbol ≈.

$$\frac{2}{3} \text{ is approximately equal to } 0.6666666667$$

$$\frac{2}{3} \approx 0.6666666667$$

Solving an Equation

You can find an exact (*symbolic*) solution or an approximate (*numeric*) solution of an equation. For more information on symbolic and numeric computations, see *Symbolic and Numeric Computation (page 102)*.

To solve an equation:

1. Enter an equation.
2. Display the context menu. See Figure 2.3.
3. From the context menu, select **Solve** or **Numerically Solve** in the **Solve** menu item.

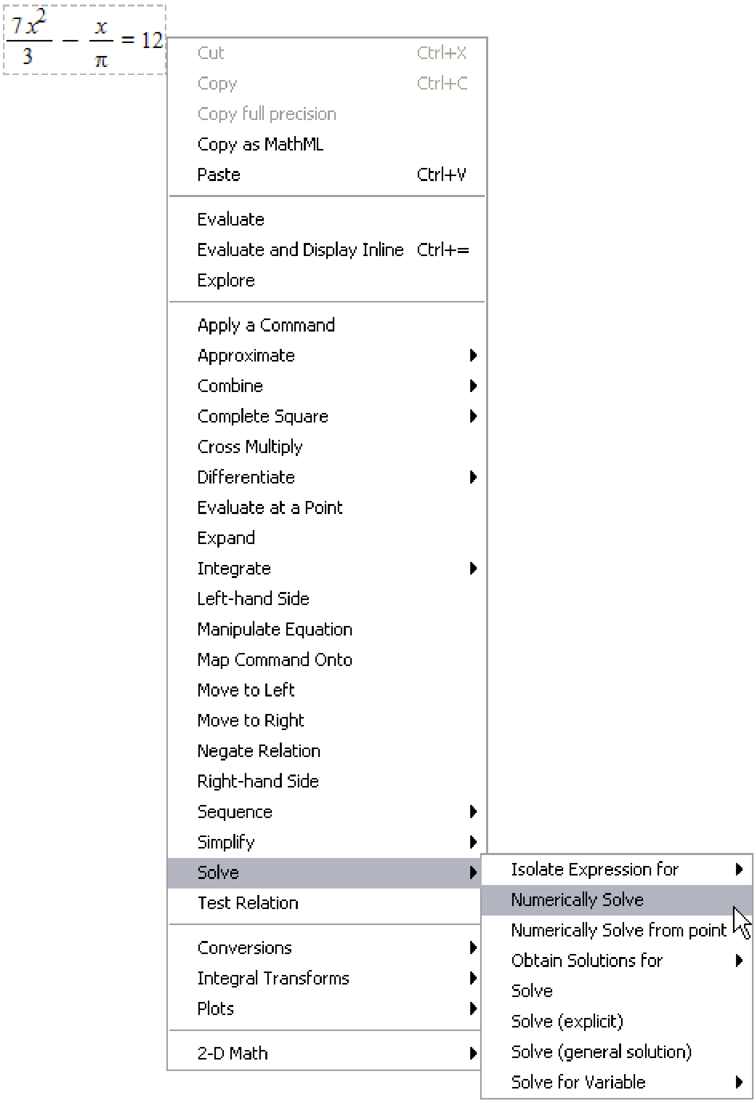


Figure 2.3: Finding the Approximate Solution to an Equation

$$\frac{7x^2}{3} - \frac{x}{\pi} = 12 \xrightarrow{\text{solve}} \left\{x = \frac{3}{14} \frac{1 + \sqrt{1 + 112\pi^2}}{\pi}\right\}, \left\{x = -\frac{3}{14} \frac{-1 + \sqrt{1 + 112\pi^2}}{\pi}\right\}$$

$$\frac{7x^2}{3} - \frac{x}{\pi} = 12 \xrightarrow{\text{solve}} -2.200603126, 2.337021648$$

For more information on solving equations, including solving inequations, differential equations, and other types of equations, see *Solving Equations (page 111)*.

Using Units

You can create expressions with units. To specify a unit for an expression, use the **Units** palettes. The **Units (FPS)** palette (Figure 2.4) contains important units from the foot-pound-second (FPS) system of units used in the United States. The **Units (SI)** palette (Figure 2.5) contains important units from the international system (SI) of units.

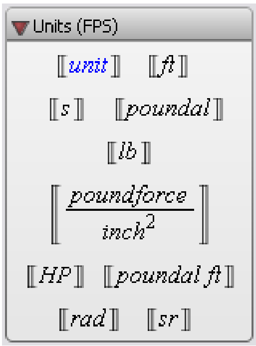


Figure 2.4: FPS Units Palette

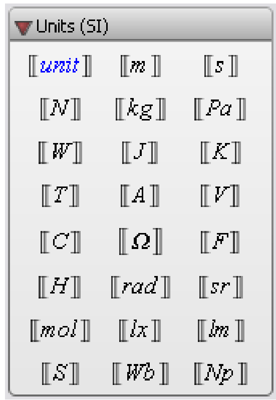


Figure 2.5: SI Units Palette

To insert an expression with a unit:

1. Enter the expression.

2. In a unit palette, click a unit symbol.

Note: To include a reciprocal unit, divide by the unit.

To evaluate an expression that contains units:

1. Enter the expression using the units palettes to insert units.

2. Right-click (**Control**-click, for Macintosh) the expression.

3. From the context menu, select **Units** and then **Simplify**.

For example, compute the electric current passing through a wire that conducts 590 coulombs in 2.9 seconds.

$$\frac{590[\![C]\!]}{2.9[\![s]\!]} \xrightarrow{\text{simplify units}} 203.4482759\,[\![A]\!]$$

For more information on using units, see *Units (page 128)*.

Assistants and Tutors

Assistants and tutors provide point-and-click interfaces with buttons, text input regions, and sliders. For details on assistants and tutors, see *Point-and-Click Interaction (page 32)*.

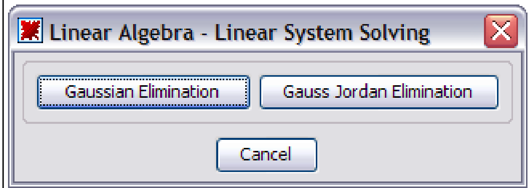
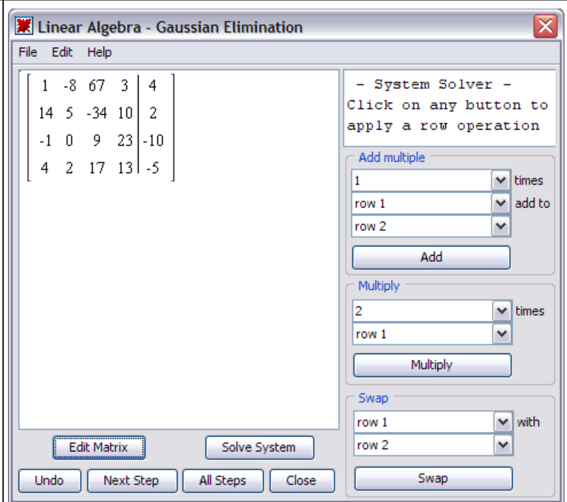
Assistants and tutors can be launched from the **Tools** menu or the context menu for an expression. For example, you can use the **Linear System Solving** tutor to solve a linear system, specified by a matrix or a set of equations.

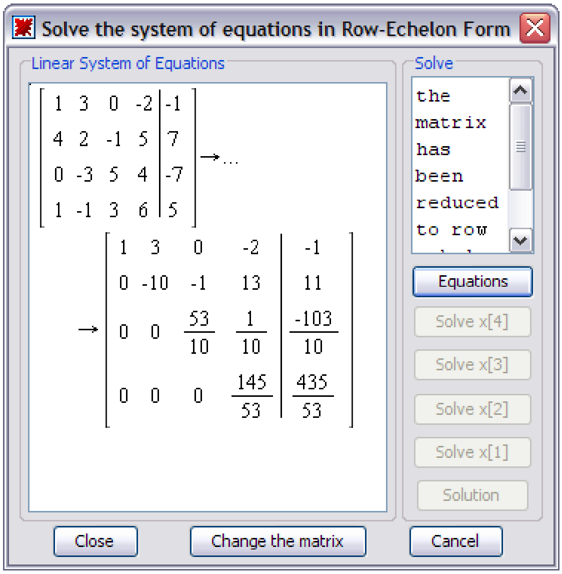
Example 3 - Using a Context Menu to Open the Linear System Solving Tutor

Use the **Linear System Solving** tutor to solve the following system of linear equations, written in matrix form:

$$\begin{bmatrix} 1 & 3 & 0 & -2 & -1 \\ 4 & 2 & -1 & 5 & 7 \\ 0 & -3 & 5 & 4 & -7 \\ 1 & -1 & 3 & 6 & 5 \end{bmatrix}$$

Action	Result in Document
1. In a new document block, create the matrix or set of linear equations to be solved.	$\begin{bmatrix} 1 & 3 & 0 & -2 & -1 \\ 4 & 2 & -1 & 5 & 7 \\ 0 & -3 & 5 & 4 & -7 \\ 1 & -1 & 3 & 6 & 5 \end{bmatrix}$
2. Load the **Student[LinearAlgebra]** package. From the **Tools** menu, select **Load Package → Student Linear Algebra**. This makes the tutors in that package available. For details, see *Package Commands (page 47)*.	Loading Student:-LinearAlgebra

Action	Result in Document
3. Right-click the matrix and select **Tutors → Linear Algebra → Linear System Solving...**. The **Linear System Solving** dialog appears, where you can choose the solving method. **Gaussian Elimination** reduces the matrix to row-echelon form, then performs back-substitution to solve the system. **Gauss Jordan Elimination** reduces the matrix to reduced row-echelon form, where the equations are already solved. For this example, choose **Gaussian Elimination**.	
4. The **Gaussian Elimination** dialog opens. You can specify the Gaussian elimination step-by-step, or you can use the **Next Step** or **All Steps** buttons to have Maple perform the steps for you. 5. Once the matrix is in row-echelon (upper-triangular) form, click the **Solve System** button to move to the next step.	

Action	Result in Document
6. The **Solve the system of equations in Row-Echelon Form** dialog appears. Click the buttons on the right to calculate the solution: first find the **Equations**, then solve for each variable. Click the **Solution** button to display the solution in the tutor.	
7. Click the **Close** button to return the solution to your document.	$\begin{bmatrix} 1 & 3 & 0 & -2 & -1 \\ 4 & 2 & -1 & 5 & 7 \\ 0 & -3 & 5 & 4 & -7 \\ 1 & -1 & 3 & 6 & 5 \end{bmatrix} \xrightarrow{\text{linear solve tutor}} \begin{bmatrix} -4 \\ 3 \\ -2 \\ 3 \end{bmatrix}$

For more information on linear systems and matrices, see *Linear Algebra (page 159)*.

3 Worksheet Mode

The *Worksheet* mode of the Standard Worksheet interface is designed for:

- Interactive use through Maple commands, which offers advanced functionality and customized control not available using context menus or other syntax-free methods
- Programming using the powerful Maple language

Using Worksheet mode, you have access to all of the Maple features described in Chapter 1, and most of those described in Chapter 2, including:

- Math and Text modes
- Palettes
- Context menus
- Assistants and tutors

For information on these features, see Chapter 1, *Getting Started (page 1)* and Chapter 2, *Document Mode (page 63)*.

Note: Using a document block, you can use all Document mode features in Worksheet mode. For information on document blocks, see *Document Blocks (page 51)*.

Note: This chapter and the following chapters except Chapter 7 were created using Worksheet mode.

3.1 In This Chapter

Section	**Topics**
Input Prompt (page 80) - Where you enter input	• The Input Prompt (>) • Suppressing Output • 2-D and 1-D Math Input • Input Separators
Commands (page 82) - Thousands of routines for performing computations and other operations	• The Maple Library • Top-Level Commands • Package Commands • Lists of Common Commands and Packages
Palettes (page 87) - Items that you can insert by clicking or dragging	• Using Palettes
Context Menus (page 89)- Pop-up menus of common operations	• Using Context Menus

Section	Topics
Assistants and Tutors (page 91)- Graphical interfaces with buttons and sliders	• Launching Assistants and Tutors
Task Templates (page 91) - Sets of commands with placeholders that you can insert and use to perform a task	• Viewing Task Templates • Inserting a Task Template • Performing the Task
Text Regions (page 93) - Areas in the document in which you can enter text	• Inserting a Text Region • Formatting Text
Names (page 94) - References to the expressions you assign to them	• Assigning to Names • Unassigning Names • Valid Names
Equation Labels (page 97) - Automatically generated labels that you can use to refer to expressions	• Displaying Equation Labels • Referring to a Previous Result • Execution Groups with Multiple Outputs • Label Numbering Schemes • Features of Equation Labels

3.2 Input Prompt

In Worksheet mode, you enter input at the Maple *input prompt* (>). The default mode for input is Math mode (*2-D Math*).

To evaluate input:

- Press **Enter**.

Maple displays the result (output) below the input.

For example, to find the value of $\sin^3\left(\frac{\pi}{3}\right)$, enter the expression, and then press **Enter**.

> $\sin^3\left(\frac{\pi}{3}\right)$

$$\frac{3}{8}\sqrt{3} \tag{3.1}$$

For example, compute the sum of two fractions.

$$> \frac{2}{9} + \frac{7}{11}$$

$$\frac{85}{99} \tag{3.2}$$

Suppressing Output

To suppress the output, enter a colon (**:**) at the end of the input.

$$> \frac{2}{9} + \frac{7}{11} :$$

A set of Maple input and its output are referred to as an *execution group*.

1-D Math Input

You can also insert input using Text mode (*1-D Math*). The input is entered as a one-dimensional sequence of characters. 1-D Math input is red.

To enter input using 1-D Math:

- At the input prompt, press **F5** or click the **Text** button in the toolbar, 

, to switch from 2-D Math to 1-D Math.

```
> 123^2 - 29857/120;
```

$$\frac{1785623}{120}$$

Important: 1-D Math input must end with a semicolon or colon. If you use a semicolon, Maple displays the output; if you use a colon, Maple suppresses the output.

```
> 123^2 - 29857/120:
```

To set the default input mode to 1-D Math:

1. From the **Tools** menu, select **Options**. The **Options** dialog is displayed.

2. On the **Display** tab, in the **Input display** drop-down list, select **Maple Notation**.

3. Click **Apply to Session** (to set for only the current session) or **Apply Globally** (to set for all Maple sessions).

To convert 2-D Math input to 1-D Math input:

1. Select the 2-D Math input.

2. From the **Format** menu, select **Convert To**, and then **1-D Math Input**.

Important: In Document mode, you can execute a statement *only if* you enter it in Math mode.

Input Separators

In 1-D and 2-D Math input, you can use a semicolon or colon to separate multiple inputs in the same input line.

> $\sqrt{4.4}$; $\tan(3.2)$

$$2.097617696$$

$$0.05847385446$$

If you do not specify a semicolon or colon, Maple interprets it as a single input. This can either give unexpected results, as below, or an error.

> $\sqrt{4.4}\ \tan(3.2)$

$$0.1226557919$$

3.3 Commands

Maple contains a large set of commands and a powerful programming language. Most Maple commands are written using the Maple programming language.

You can enter commands using 1-D or 2-D Math. You must use 1-D Math input when programming in Maple. *Basic Programming (page 369)* provides an introduction to Maple programming.

To learn how to use Maple commands, see the appropriate help page, or use task templates. For more information, see *The Maple Help System (page 54)* and *Task Templates (page 91)*.

The Maple Library

Maple's commands are contained in the Maple library. There are two types of commands: *top-level commands* and *package commands*.

- The top-level commands are the most frequently used Maple commands.
- Packages contain related specialized commands in areas such as calculus, linear algebra, vector calculus, and code generation.

For a complete list of packages and commands, refer to the **index** help pages. To access the index overview help page, enter **?index**, and then press **Enter**. For information on the Maple Help System, see *The Maple Help System (page 54)*.

Top-Level Commands

To use a top-level command, enter its name followed by parentheses (**( )**) containing any parameters. This is referred to as a *calling sequence* for the command.

command(*arguments*)

Note: In 1-D Math input, include a semicolon or colon at the end of the calling sequence.

For example, to differentiate an expression, use the **diff** command. The required parameters are the expression to differentiate, which must be specified first, and the independent variable.

> $diff(\tan(x)\sin(x), x)$

$$\left(1+\tan(x)^2\right)\sin(x)+\tan(x)\cos(x)$$

For a complete list of functions (commands that implement mathematical functions), such as **BesselI** and **AiryAi**, available in the library, refer to the **?initialfunctions** help page.

> $\frac{\mathrm{BesselI}(0.1, 1)}{\mathrm{AiryAi}(2.2)}$

$$47.53037086$$

For detailed information on the properties of a function, use the **FunctionAdvisor** command.

> $FunctionAdvisor('definition', \mathrm{BesselI})$

$$\left[\mathrm{BesselI}(a, z) = \frac{z^a\,\mathrm{hypergeom}\left([\ ], [1+a], \frac{1}{4}z^2\right)}{\Gamma(1+a)\,2^a}, \textit{with no restrictions on } (a, z)\right]$$

For detailed information on how to use a function in Maple, refer to its help page.

For example:

> ?*Bessel*

Note: In 1-D and 2-D Math input, when accessing a help page using **?**, you do not need to include a trailing semicolon or colon.

Top Commands

Here are a few of the most frequently used Maple commands. A complete list of top-level commands is available at **Help → Manuals, Resources, and more → List of Commands**.

Table 3.1: Top Commands

Command Name	Description
plot and **plot3d**	Create a two-dimensional and three-dimensional plot of functions.
solve	Solve one or more equations or inequalities for their unknowns.
fsolve	Solve one or more equations using floating-point arithmetic.
eval	Evaluate an expression at a given point.
evalf	Numerically evaluate expressions.
dsolve	Solve ordinary differential equations (ODEs).
int	Compute an indefinite or definite integral.
diff	Compute an ordinary or partial derivative, as the context dictates.
limit	Calculate the limiting value of a function.
sum	For symbolic summation. It is used to compute a closed form for an indefinite or definite sum.
assume/is	Set variable properties and relationships between variables. Similar functionality is provided by the **assuming** command.
assuming	Compute the value of an expression under assumptions.
simplify	Apply simplification rules to an expression.
expand	Distribute products over sums.
normal	Normalize a rational expression.
convert	Convert an expression to a different type or form.
type	Type-checking command. In many contexts, it is not necessary to know the exact value of an expression; it suffices to know that an expression belongs to a broad class, or group, of expressions that share some common properties. These classes or groups are known as *types*.
series	Generalized series expansion.
map	Apply a procedure to each operand of an expression.

Package Commands

To use a package command, the calling sequence must include the package name, and the command name enclosed in square brackets (**[ ]**).

package[*command*](*arguments*)

If you are frequently using the commands in a package, load the package.

To load a package:

- Use the **with** command, specifying the package as an argument.

The **with** command displays a list of the package commands loaded (unless you suppress the output by entering a colon at the end of the calling sequence).

After loading a package, you can use the short form names of its commands. That is, you can enter the commands without specifying the package name.

For example, use the **NLPSolve** command from the **Optimization** package to find a local minimum of an expression and the value of the independent variable at which the minimum occurs.

> $Optimization[NLPSolve]\left(\frac{\sin(x)}{x}, x=1..15\right)$

$$[-0.0913252028230576718, [x=10.9041216700744900]]$$

> $with(Optimization);$

$$[ImportMPS, Interactive, LPSolve, LSSolve, Maximize, Minimize, NLPSolve, QPSolve]$$

> $NLPSolve\left(\frac{\sin(x)}{x}, x=1..15\right)$

$$[-0.0913252028230576718, [x=10.9041216700744900]]$$

For more information on optimization, see *Optimization (page 188)*.

To unload a package:

- Use the **unwith** command, specifying the package as an argument.

> $unwith(Optimization)$

Alternatively, use the **restart** command. The restart command clears Maple's internal memory. The effects include unassigning all names and unloading all packages. For more information, refer to the **?restart** help page.

Note: To execute the examples in this manual, you may be required to use the **unassign** or **restart** command between examples.

Some packages contain commands that have the same name as a top-level command. For example, the **plots** package contains a **changecoords** command. Maple also contains a top-level **changecoords** command.

> $with(plots):$

After the plots package is loaded, the name **changecoords** refers to the **plots[changecoords]** command. To use the top-level **changecoords** command, unload the package or use the restart command. (For alternative methods of accessing the top-level command, see the **?rebound** help page.)

Top Packages

Here are a few of the most frequently used Maple packages. A complete list of packages is available in the Maple help system at **Help → Manuals, Resources, and more → List of Packages**.

Table 3.2: Top Packages

Package Name	Description
CodeGeneration	The **Code Generation** package is a collection of commands and subpackages that enable the translation of Maple code to other programming languages, such as C, Fortran, MATLAB®, Visual Basic®, and Java™.
LinearAlgebra	The **Linear Algebra** package contains commands to construct and manipulate Matrices and Vectors, and solve linear algebra problems. **LinearAlgebra** routines operate on three principal data structures: Matrices, Vectors, and scalars.
Optimization	The **Optimization** package is a collection of commands for numerically solving optimization problems, which involve finding the minimum or maximum of an objective function possibly subject to constraints.
Physics	The **Physics** package implements computational representations and related operations for most of the objects used in mathematical physics computations.
RealDomain	The **Real Domain** package provides an environment in which Maple assumes that the basic underlying number system is the field of real numbers instead of the complex number field.
ScientificConstants	The **Scientific Constants** package provides access to the values of various physical constants, for example, the velocity of light and the atomic weight of sodium. This package provides the units for each of the constant values, allowing for greater understanding of an equation. The package also provides units-matching for error checking of the solution.
ScientificErrorAnalysis	The **Scientific Error Analysis** package provides representation and construction of numerical quantities that have a central value and an associated uncertainty (or error), which is a measure of the degree of precision to which the quantity's value is known. Various first-order calculations of error analysis can be performed with these quantities.
Statistics	The **Statistics** package is a collection of tools for mathematical statistics and data analysis. The package supports a wide range of common statistical tasks such as quantitative and graphical data analysis, simulation, and curve fitting.

Package Name	Description
Student	The **Student** package is a collection of subpackages designed to assist with teaching and learning standard undergraduate mathematics. The many commands display functions, computations, and theorems in various ways, including stepping through important computations. The **Student** package contains the following subpackages: • **Calculus1** - single-variable calculus • **LinearAlgebra** - linear algebra • **MultivariateCalculus** - multivariate calculus • **NumericalAnalysis** - numerical analysis • **Precalculus** - precalculus • **VectorCalculus** - multivariate vector calculus
Units	The **Units** package contains commands for unit conversion and provides environments for performing calculations with units. It accepts approximately 300 distinct unit names (for example, meters and grams) and over 550 units with various contexts (for example, standard miles and U.S. survey miles). Maple also contains two **Units** palettes that allow you to enter the unit for an expression quickly.
VectorCalculus	The **Vector Calculus** package is a collection of commands that perform multivariate and vector calculus operations. A large set of predefined orthogonal coordinate systems is available. All computations in the package can be performed in any of these coordinate systems. It contains a facility for adding a custom but orthogonal coordinate system and using that new coordinate system for your computations.

3.4 Palettes

Palettes are collections of related items that you can insert by clicking or dragging. For example, see Figure 3.1.

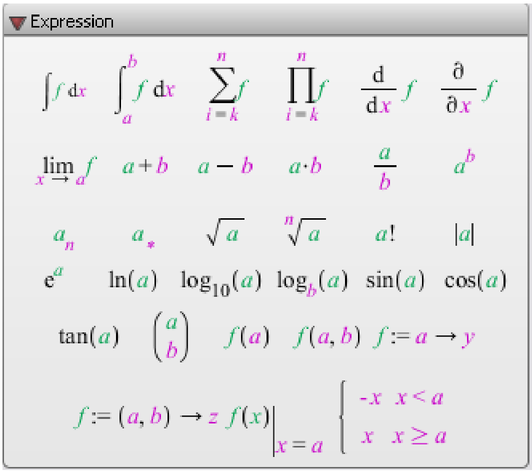
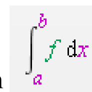


Figure 3.1: Expression Palette

You can use palettes to enter input.

For example, evaluate a definite integral using the definite integration item  in the **Expression** palette.

In 2-D Math, clicking the definite integration item inserts:

$$> \int_a^b f \, dx$$

1. Enter values in the placeholders. To move to the next placeholder, press **Tab**. **Note:** If pressing the **Tab** key inserts a tab, click the Tab icon in the toolbar.

2. To evaluate the integral, press **Enter**.

$$> \int_0^1 \tanh(x) \, dx$$

$$-\ln(2) + \ln\left(e^{-1} + e\right)$$

In 1-D Math, clicking the definite integration item inserts the corresponding command calling sequence.

```
> int(f,x=a..b);
```

Specify the problem values (using the **Tab** to move to the next placeholder), and then press **Enter**.

```
> int(tanh(x), x = 0..1);
```

$$-\ln(2) + \ln\left(e^{-1} + e\right)$$

Note: Some palette items cannot be inserted into 1-D Math because they are not defined in the Maple language. When the cursor is in 1-D Math input, unavailable palette items are dimmed.

For more information on viewing and using palettes, see *Palettes (page 22)* in Chapter 1.

3.5 Context Menus

A *context menu* is a pop-up menu that lists the operations and actions you can perform on a particular expression. See Figure 3.2.

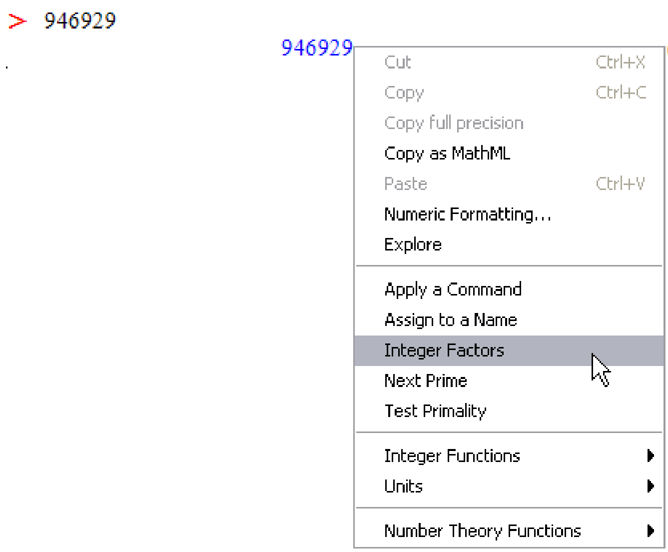


Figure 3.2: Integer Context Menu

In Worksheet mode, you can use context menus to perform operations on 2-D Math and output.

To use a context menu:

1. Right-click (**Control**-click, for Macintosh) the expression. The context menu is displayed.

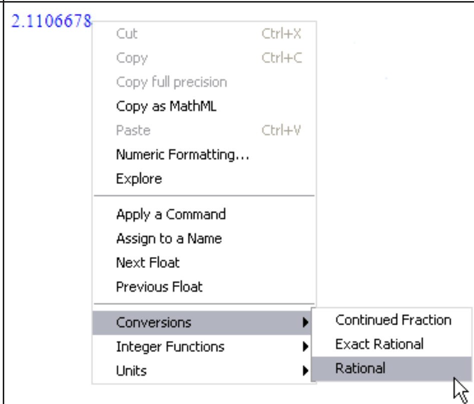
2. From the context menu, select an operation.

Maple inserts a new execution group containing:

- The calling sequence that performs the operation
- The result of the operation

Example 1 - Using Context Menus

Determine the rational expression (fraction) that approximates the floating-point number $0.3463678 + 1.7643$.

Action	Result in Document
1. Enter and execute the expression.	> $0.3463678 + 1.7643$ 2.1106678 (3.3)
2. Right-click (**Control-click**, for Macintosh) the output floating-point number.	
3. From the context menu, select **Conversions** → **Rational**. The inserted calling sequence includes an equation label reference to the number you are converting.	> $convert(\ \mathbf{(3.3)}, 'rational'\ )$ $\frac{32270}{15289}$

Notice that an equation label reference has been used. For information on equation labels and equation label references, see *Equation Labels (page 97)*.

For more information on context menus, see *Context Menus (page 70)* in Chapter 2.

3.6 Assistants and Tutors

Assistants and tutors provide point-and-click interfaces with buttons, text input regions, and sliders. See Figure 3.3.

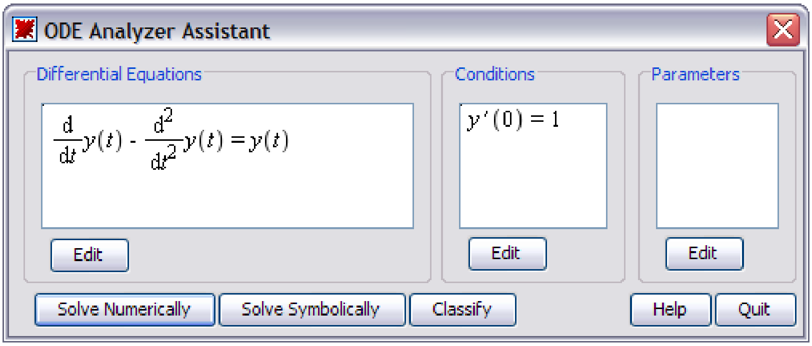


Figure 3.3: ODE Analyzer Assistant

Launching an Assistant or Tutor

To launch an assistant or tutor:

1. Open the **Tools** menu.

2. Select **Assistants** or **Tutors**.

3. Navigate to and select one of the assistants or tutors.

For more information on assistants and tutors, see *Assistants (page 32)* in Chapter 1.

3.7 Task Templates

Maple can solve a diverse set of problems. The task template facility helps you quickly find and use the commands required to perform common tasks.

After inserting a task template, specify the parameters of your problem in the placeholders, and then execute the commands, or click a button.

The **Task Browser** (Figure 3.4) organizes task templates by subject.

To launch the Task Browser:

- From the **Tools** menu, select **Tasks**, and then **Browse**.

You can also browse the task templates in the Table of Contents of the Maple Help System.

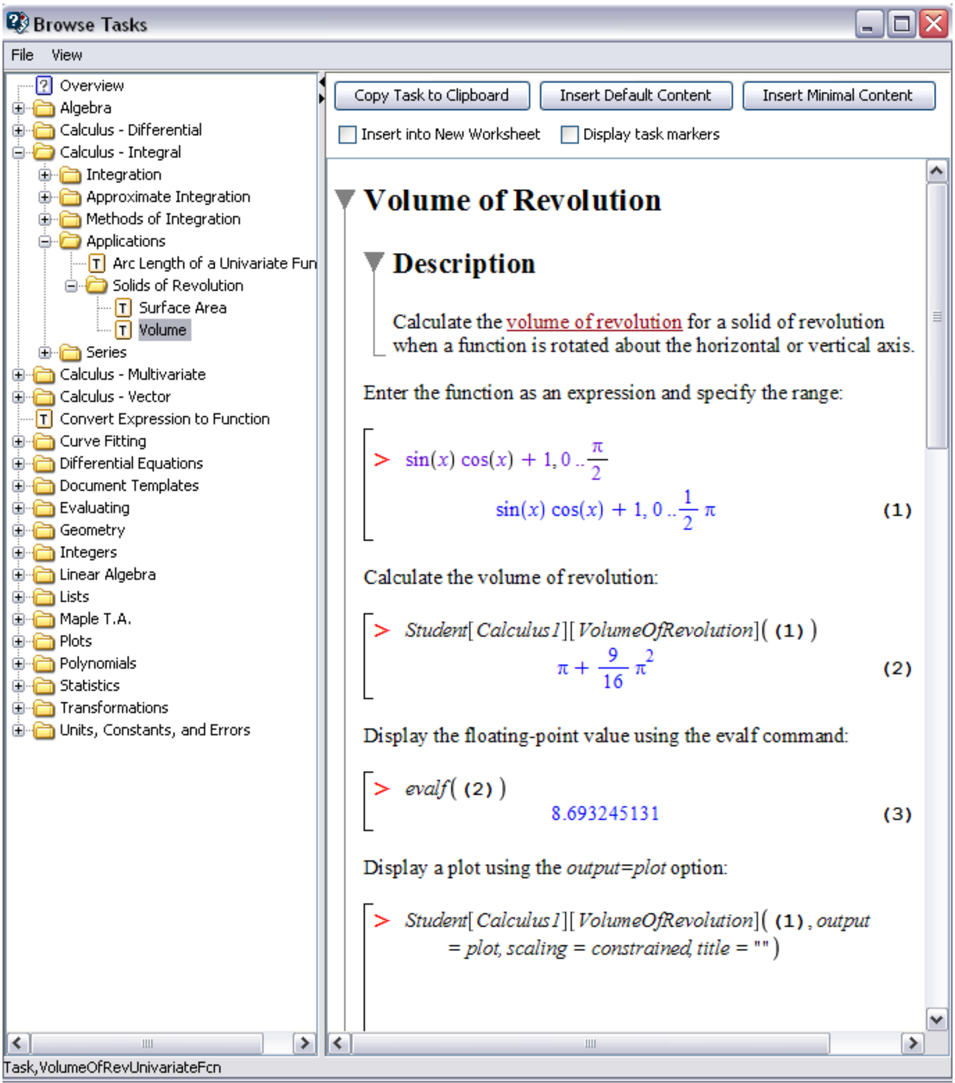


Figure 3.4: Task Browser

For details on inserting and using task templates, see *Task Templates (page 41)*. You can also create your own task templates for performing common tasks. For details, refer to the **?creatingtasks** help page.

3.8 Text Regions

To add descriptive text in Worksheet mode, use a *text region*.

To insert a text region:

- In the toolbar, click the Text region icon T.

The default mode in a text region is Text mode.

In a text region, you can:

- Enter text with inline mathematical content by switching between Text and Math modes. To toggle between Text mode and Math mode, press **F5** or click the Math and Text toolbar icons, Text Math. **Note:** The mathematical content in a text region is not evaluated. To enter mathematical content that is evaluated, enter it at an *Input Prompt (page 80)*.
- Insert any palette item. Palette items are inserted in Math mode (2-D Math). **Note**: After you insert a palette item, you must press **F5** or click the toolbar icon to return to Text mode.

You can format text in a text region. Features include:

- Character styles
- Paragraph styles
- Sections and subsections
- Tables

For more information on formatting documents, see *Creating Mathematical Documents (page 281)*.

3.9 Names

Instead of re-entering an expression every time you need it, you can assign it to a *name* or add an *equation label* to it. Then you can quickly refer to the expression using the name or an equation label reference. For information on labels, see the following section, *Equation Labels (page 97)*.

Assigning to Names

You can assign any Maple expression to a name: numeric values, data structures, procedures (a type of Maple program), and other Maple objects.

Initially, the value of a name is itself.

> a

$$a$$

The assignment operator (**:=**) associates an expression with a name.

> $a := \pi$

$$a := \pi$$

Recall that you can enter π using the following two methods.

- Use the **Common Symbols** palette
- In 2-D Math enter *pi*, and then press the symbol completion shortcut key. See *Shortcuts for Entering Mathematical Expressions (page 7)*.

When Maple evaluates an expression that contains a name, it replaces the name with its value. For example:

> $\cos(a)$

$$-1$$

For information on Maple evaluation rules, see *Evaluating Expressions (page 359)*.

Mathematical Functions

To define a function, assign it to a name.

For example, define a function that computes the cube of its argument.

> $cube := x \rightarrow x^3$:

For information on creating functions, see *Define a Mathematical Function (page 66)*.

> $cube(3); cube(1.666)$

$$27$$

$$4.624076296$$

Note: To insert the right arrow, enter the characters **->**. In 2-D Math, Maple replaces -> with the right arrow symbol $\rightarrow$. In 1-D Math, the characters are not replaced.

For example, define a function that squares its argument.

```
> square := x -> x^2:
```

```
> square(32);
```

$$1024$$

For more information on functions, see *Functional Operators (page 343)*.

Protected Names

Protected names are valid names that are predefined or reserved.

If you attempt to assign to a protected name, Maple returns an error.

> $\sin := 2$

```
Error, attempting to assign to `sin` which is protected
```

For more information, refer to the **?type/protected** and **?protect** help pages.

Unassigning Names

The **unassign** command resets the value of a name to itself. **Note:** You must enclose the name in right single quotes (**' '**).

> $unassign('a')$

> a

$$a$$

Right single quotes (*unevaluation quotes*) prevent Maple from evaluating the name. For more information on unevaluation quotes, see *Delaying Evaluation (page 366)* or refer to the **?uneval** help page.

See also *Unassigning a Name Using Unevaluation Quotes (page 368)*.

Unassigning all names:

The **restart** command clears Maple's internal memory. The effects include unassigning all names. For more information, refer to the **?restart** help page.

Note: To execute the examples in this manual, you may be required to use the **unassign** or **restart** command between examples.

Valid Names

A Maple name must be one of the following.

- A sequence of alphanumeric and underscore (_) characters that begins with an alphabetical character. **Note:** To enter an underscore character in 2-D Math, enter a backslash character followed by an underscore character, that is, _.
- A sequence of characters enclosed in left single quotes (\` \`).

Important: Do not begin a name with an underscore character. Maple reserves names that begin with an underscore for use by the Maple library.

Examples of valid names:

- **a**
- **a1**
- **polynomial**
- **polynomial1_divided_by_polynomial2**
- **\`2a\`**
- **\`x y\`**

3.10 Equation Labels

Maple marks the output of each execution group with a unique equation label.

Note: The equation label is displayed to the right of the output.

> $\int \sin(x)\, dx$

$$-\cos(x) \tag{3.4}$$

Using equation labels, you can refer to the result in other computations.

> $\int \mathbf{(3.4)}\, dx$

$$-\sin(x) \tag{3.5}$$

Displaying Equation Labels

Important: By default, equation labels are displayed. If equation label display is turned off, complete **both** of the following operations.

- From the **Format** menu, select **Equation Labels**, and then ensure that **Worksheet** is selected.
- In the **Options** dialog (**Tools→Options**), on the **Display** tab, ensure that **Show equation labels** is selected.

Referring to a Previous Result

Instead of re-entering previous results in computations, you can use equation label references. Each time you need to refer to a previous result, insert an equation label reference.

To insert an equation label reference:

- From the **Insert** menu, select **Label**. (Alternatively, press **Ctrl+L**; **Command+L**, Macintosh.)
- In the **Insert Label** dialog (see Figure 3.5), enter the label value, and then click **OK**.

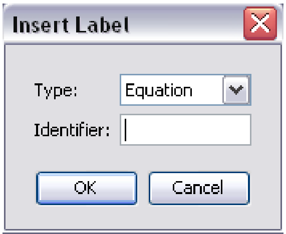
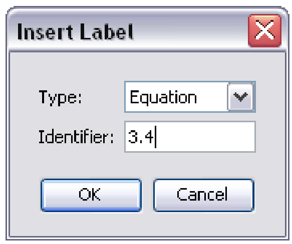


Figure 3.5: Insert Label Dialog

Maple inserts the reference.

For example:

To integrate the product of (3.4) **and** (3.5)**:**

Action	**Result in Document**
1. In the **Expression** palette, click the indefinite integration item $\int f\,dx$. The item is inserted and the integrand placeholder is highlighted.	> $\int f\,dx$
2. Press **Ctrl+L** (**Command+L**, for Macintosh). 3. In the **Insert Label** dialog, enter **3.4**. Click **OK**.	> $\int f\,dx$ 
4. Press *. 5. Press **Ctrl+L** (**Command+L**, for Macintosh). 6. In the **Insert Label** dialog, enter **3.4**. Click **OK**.	> $\int (3.4)\cdot(3.5)\,dx$
7. To move to the variable of integration placeholder, press **Tab**. 8. Enter **x**. 9. To evaluate the integral, press **Enter**.	> $\int (3.4)\cdot(3.5)\,dx$ $-\frac{1}{2}\cos(x)^2$ (3.6)

Execution Groups with Multiple Outputs

An equation label is associated with the *last output* within an execution group.

> $\left(\frac{2}{3.5}\right)^2$; $\cos\left(\frac{\pi}{6}\right)$

$$0.3265306122$$

$$\frac{1}{2}\sqrt{3} \tag{3.7}$$

> **(3.7)**^{2}

$$\frac{3}{4} \tag{3.8}$$

Label Numbering Schemes

You can number equation labels in two ways:

- **Flat** - Each label is a single number, for example, 1, 2, or 3.
- **Sections** - Each label is numbered according to the section in which it occurs. For example, 2.1 is the first equation in the second section, and 1.3.2 is the second equation in the third subsection of the first section.

To change the equation label numbering scheme:

- From the **Format** menu, select **Equation Labels → Label Display**. In the **Format Labels** dialog (Figure 3.6), select one of the formats.
- Optionally, enter a prefix.

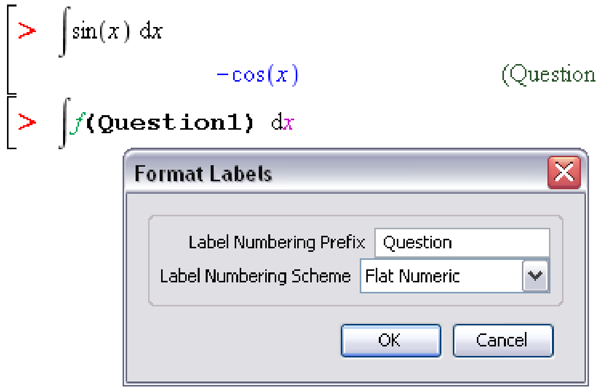


Figure 3.6: Format Labels Dialog: Adding a Prefix

Features of Equation Labels

Although equation labels are not descriptive names, labels offer other important features.

- Each label is unique, whereas a name may be inadvertently assigned more than once for different purposes.
- Maple labels the output values sequentially. If you remove or insert an output, Maple automatically re-numbers all equation labels and updates the label references.
- If you change the equation label format (see *Label Numbering Schemes (page 99)*), Maple automatically updates all equation labels and label references.

For information on assigning to, using, and unassigning names, see *Names (page 94)*.

For more information on equation labels, refer to the **?equationlabel** help page.

The following chapters describe how to use Maple to perform tasks such as solving equations, producing plots and animations, and creating mathematical documents. The chapters were created using Worksheet mode. Except where noted, all features are available in both Worksheet mode and Document mode.

4 Basic Computations

This chapter discusses key concepts related to performing basic computations with Maple. It discusses important features that are relevant to all Maple users. After learning about these concepts, you will learn how to use Maple to solve problems in specific mathematical disciplines in the following chapter.

4.1 In This Chapter

Section	Topics
Symbolic and Numeric Computation (page 102)- An overview of exact and floating-point computation	• Exact Computations • Floating-Point Computations • Converting Exact Quantities to Floating-Point Values • Sources of Error
Integer Operations (page 106) - How to perform integer computations	• Important Integer Commands • Non-Base 10 Numbers • Finite Rings and Fields • Gaussian Integers
Solving Equations (page 111) - How to solve standard mathematical equations	• Equations and Inequations • Ordinary Differential Equations • Partial Differential Equations • Integer Equations • Integer Equations in a Finite Field • Linear Systems • Recurrence Relations

<table>
<tr><th>Section</th><th>Topics</th></tr>
<tr><td>Units, Scientific Constants, and Uncertainty (page 128) - How to construct and compute with expressions that have units, scientific constants, or uncertainty</td><td>Units
Conversions
Applying Units to an Expression
Performing Computations with Units
Changing the Current System of Units
Extensibility
Scientific Constants
Scientific Constants
Element and Isotope Properties
Value, Units, and Uncertainty
Performing Computations
Modification and Extensibility
Uncertainty Propagation
Quantities with Uncertainty
Performing Computations with Quantities with Uncertainty</td></tr>
<tr><td>Restricting the Domain (page 143) - How to restrict the domain for computations</td><td>Real Number Domain
Assumptions on Variables</td></tr>
</table>

4.2 Symbolic and Numeric Computation

Symbolic computation is the mathematical manipulation of expressions involving symbolic or abstract quantities, such as variables, functions, and operators; and exact numbers, such as integers, rationals, π, and e^2. The goal of such manipulations may be to transform an expression to a simpler form or to relate the expression to other, better understood formulas.

Numeric computation is the manipulation of expressions in the context of finite-precision arithmetic. Expressions involving exact numbers, for example, $\sqrt{2}$, are replaced by close approximations using floating-point numbers, for example 1.41421. These computations generally involve some error. Understanding and controlling this error is often of as much importance as the computed result.

In Maple, numeric computation is normally performed if you use floating-point numbers (numbers containing a decimal point) or the **evalf** command. The **plot** command (see *Plots and Animations (page 237)*) uses numeric computation, while commands such as **int**, **limit**, and **gcd** (see *Integer Operations (page 106)* and *Mathematical Problem Solving (page 149)*) generally use only symbolic computation to achieve their results.

Exact Computations

In Maple, integers, rational numbers, mathematical constants such as π and ∞, and mathematical structures such as matrices with these as entries are treated as exact quantities. Names, such as x, y, my_variable, and mathematical functions, such as $\sin(x)$ and $\mathrm{LambertW}(k, z)$, are *symbolic* objects. Names can be assigned exact quantities as their values, and functions can be evaluated at symbolic or exact arguments.

> $\frac{3}{2} + \frac{1}{3}, 1 + \frac{\pi}{2}$

$$\frac{11}{6}, 1 + \frac{1}{2}\pi$$

Important: Unless requested to do otherwise (see the following section), Maple evaluates expressions containing exact quantities to exact results, as you would do if you were performing the calculation by hand, and not to numeric approximations, as you normally obtain from a standard hand-held calculator.

> $\sin(1), \sin(\pi), \sin(x)$

$$\sin(1), 0, \sin(x)$$

> $\int \tan(t)\, \mathrm{d}t$

$$-\ln(\cos(t))$$

> $\sqrt{32}$

$$4\sqrt{2}$$

Floating-Point Computations

In some situations, a numeric approximation of an exact quantity is required. For example, the **plot** command requires the expression it is plotting to evaluate to numeric values that can be rendered on the screen: π cannot be so rendered, but 3.14159 can be. Maple distinguishes *approximate* from *exact* quantities by the presence or absence of a decimal point: 1.9 is approximate, while $\frac{19}{10}$ is exact.

Note: An alternative representation of floating-point numbers, called *e-notation*, may not include an explicit decimal point: *1e5* $= 100000.$, *3e-2* $= .03$.

In the presence of a floating-point (approximate) quantity in an expression, Maple generally computes using numeric approximations. Arithmetic involving mixed exact and floating-point quantities results in a floating-point result.

> $1.5 + \frac{2}{3}$

$$2.166666667$$

If a mathematical function is passed a floating-point argument, it normally attempts to produce a floating-point approximation of the result.

> $\sin(1.5), \int_{0.0}^{1.0} e^{x}\, dx$

$$0.9974949866, 1.718281828$$

Converting Exact Quantities to Floating-Point Values

To convert an exact quantity to a numeric approximation of that quantity, use the **evalf** command or the **Approximate** context menu operation (see *Approximating the Value of an Expression (page 71)*).

> $evalf(\pi), evalf(\sin(3)), evalf\left(\frac{3}{2} + \frac{1}{3}\right)$

$$3.141592654, 0.1411200081, 1.833333333$$

By default, Maple computes such approximations using 10 digit arithmetic. You can modify this in one of two ways:

- *Locally*, you can pass the precision as an index to the **evalf** call.

> $evalf[20](\exp(2)), evalf\left(\Gamma\left(\frac{2}{3}\right)\right)$

$$7.3890560989306502272, 1.354117939$$

- *Globally*, you can set the value of the **Digits** environment variable.

> $Digits := 25 :$

> $evalf\left(\tan\left(\frac{\pi}{3}\right)\right)$

$$1.732050807568877293527446$$

For more information, see the **?evalf** and **?Digits** help pages.

Note: When appropriate, Maple performs floating-point computations directly using your computer's underlying hardware.

Sources of Error

By its nature, floating-point computation normally involves some error. Controlling the effect of this error is the subject of active research in *Numerical Analysis*. Some sources of error are:

- An exact quantity may not be exactly representable in decimal form: $\frac{1}{3}$ and π are examples.
- Small errors can accumulate after many arithmetic operations.
- Subtraction of nearly equal quantities can result in essentially no useful information. For example, consider the computation $x - \sin(x)$ for $x \approx 0.$

> $(x-\sin(x))\Big|_{x=.00001}$

$$0.$$

No correct digits remain. If, however, you use Maple to analyze this expression, and replace this form with a representation that is more accurate for small values of $x,$ a fully accurate 10-digit result can be obtained.

> $t := taylor(x - \sin(x), x)$

$$t := \frac{1}{6}x^3 - \frac{1}{120}x^5 + \mathrm{O}(x^6)$$

> $t\Big|_{x = 0.00001}$

$$1.666666667\ 10^{-16}$$

For information on evaluating an expression at a point, see *Substituting a Value for a Subexpression (page 359)*. For information on creating a series approximation, see *Series (page 182)*. For more information on floating-point numbers, refer to the **?float** and **?type/float** help pages.

4.3 Integer Operations

In addition to the basic arithmetic operators, Maple has many specialized commands for performing more complicated integer computations, such as factoring an integer, testing whether an integer is a prime number, and determining the greatest common divisor (GCD) of a pair of integers.

Note: Many integer operations are available as task templates (**Tools→Tasks→Browse**, under **Integer Operations**).

You can quickly perform many integer operations using context menus. Selecting an integer, and then right-clicking (for Macintosh, **Control**-clicking) displays a context menu with integer commands. For example, the context menu item **Integer Factors** applies the **ifactor** command to compute the prime factorization of the given integer. See Figure 4.1.

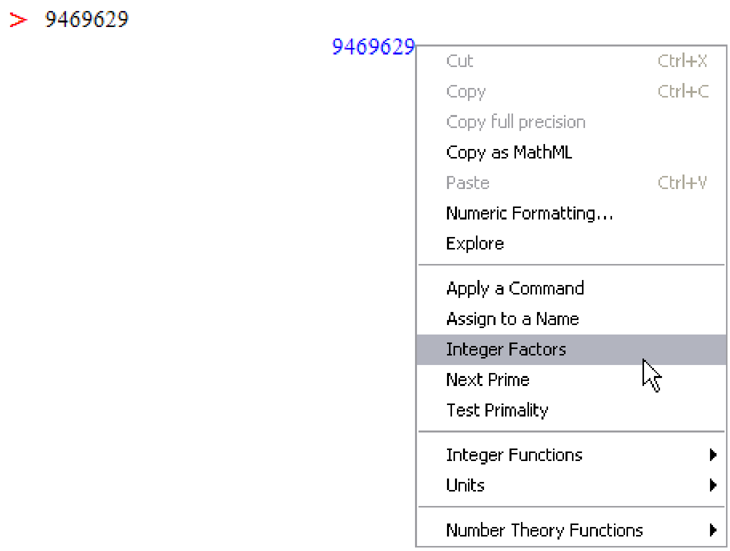


Figure 4.1: Context Menu for an Integer

The result of applying **Integer Factors** is shown:

> 9469629

$$9469629 \quad (4.1)$$

> *ifactor*(**(4.1)**)

$$(3)^4 (13) (17) (23)^2 \quad (4.2)$$

Maple inserts the command **ifactor**, using an equation label reference to the integer 946929. For more information on equation labels, see *Equation Labels (page 97)*.

For more information on using context menus in Worksheet mode, see *Context Menus (page 89)*. For information on using context menus in Document mode, see *Context Menus (page 70)*.

Maple has many other integer commands, including those listed in Table 4.1.

Table 4.1: Select Integer Commands

Command	Description
abs	absolute value (displays in 2-D math as $\|a\|$)

Command	Description
factorial	factorial (displays in 2-D math as $a!$)
ifactor	prime factorization
igcd	greatest common divisor
iquo	quotient of integer division
irem	remainder of integer division
iroot	integer approximation of nth root
isprime	test primality
isqrt	integer approximation of square root
max, min	maximum and minimum of a set
mod	modular arithmetic (See *Finite Rings and Fields (page 109)*.)
numtheory[divisors]	set of positive divisors

> $iquo(209, 17)$

$$12$$

> $irem(209, 17)$

$$5$$

> $igcd(2024, 4862)$

$$22$$

> $iroot(982523, 4)$

$$31$$

For information on finding integer solutions to equations, see *Integer Equations (page 126)*.

Non-Base 10 Numbers and Other Number Systems

Maple supports:

- Non-base 10 numbers
- Finite ring and field arithmetic
- Gaussian integers

Non-Base 10 Numbers

To represent an expression in another base, use the **convert** command.

> *convert*(6000, '*binary*')

$$1011101110000$$

> *convert*(34271, '*hex*')

$$85DF$$

For information on enclosing keywords in right single quotes ('), see *Delaying Evaluation (page 366)*.

You can also use the **convert/base** command.

> *convert*(34271, '*base*', 16)

$$[15, 13, 5, 8]$$

Note: The **convert/base** command returns a list of digit values in order of *increasing significance*.

Finite Rings and Fields

Maple supports computations over the integers modulo m.

The **mod** operator evaluates an expression over the integers modulo m.

> 27 **mod** 4

$$3$$

By default, the **mod** operator uses positive representation (**modp** command). Symmetric representation is available using the **mods** command.

> *modp*(27, 4)

$$3$$

> *mods*(27, 4)

$$-1$$

For information on setting symmetric representation as the default, refer to the **?mod** help page.

The modular arithmetic operators are listed in Table 4.2.

Table 4.2: Modular Arithmetic Operators

Operation	Operator	Example
Addition	+	> $7 + 6 \bmod 5$ 3
Subtraction	-	> $mods(3 - 16, 11)$ -2
Multiplication (displays in 2-D Math as $\cdot$)	*	> $13 \cdot 5 \bmod 3$ 2
Multiplicative inverse (displays in 2-D Math as a superscript)	^(-1)	> $3^{(-1)} \bmod 5$ 2
Division (displays in 2-D Math as $\frac{a}{b}$)	/	> $\frac{2}{3} \bmod 5$ 4
Exponentiation[1]	&^	> $(100\&\hat{}100) \bmod 7$ 2

[1]To enter a caret (^) in 2-D Math, enter a backslash character followed by a caret, that is, \^.

For information on solving an equation modulo an integer, see *Integer Equations in a Finite Field (page 126)*.

The **mod** operator also supports polynomial and matrix arithmetic over finite rings and fields. For more information, refer to the **?mod** help page.

Gaussian Integers

Gaussian integers are complex numbers in which the real and imaginary parts are integers.

The **GaussInt** package contains commands that perform Gaussian integer operations.

The **GIfactor** command returns the Gaussian integer factorization.

> *GaussInt*[*GIfactor*](173 + 16 *I*)

$$(1 + 2\,\mathrm{I})\,(41 - 66\,\mathrm{I})$$

In Maple, complex numbers are represented as **a+b*I**, where the uppercase I represents the imaginary unit $\sqrt{-1}$.

You can also enter the imaginary unit using the following two methods.

- In the **Common Symbols** palette, click the I , i or j item. See *Palettes (page 22)*.
- Enter *i* or *j*, and then press the symbol completion key. See *Symbol Names (page 29)*.

Note that the output will still be displayed with I, no matter what symbol was used for input. You can customize Maple's settings to use a different symbol for $\sqrt{-1}$. For more information on entering complex numbers, including how to customize this setting, refer to the ?HowDoI help page.

The **GIsqrt** command approximates the square root in the Gaussian integers.

> *GaussInt*[*GIsqrt*](9 − 5 j)

$$3 - \mathrm{I}$$

For more information on Gaussian integers including a list of **GaussInt** package commands, refer to the **?GaussInt** help page.

4.4 Solving Equations

You can solve a variety of equation types, including those described in Table 4.3.

Table 4.3: Overview of Solution Methods for Important Equation Types

Equation Type	Solution Method
Equations and inequations	**solve** and **fsolve** commands
Ordinary differential equations	**ODE Analyzer Assistant** (and **dsolve** command)
Partial differential equations	**pdsolve** command
Integer equations	**isolve** command
Integer equations in a finite field	**msolve** command
Linear integral equations	intsolve command
Linear systems	**LinearAlgebra[LinearSolve]** command
Recurrence relations	**rsolve** command

Note: Many solve operations are available in context menus and as task templates (**Tools→Tasks→Browse**). Most of this section focuses on other methods.

Solving Equations and Inequations

Using Maple, you can symbolically solve equations and inequations. You can also solve equations numerically.

To solve an equation or set of equations using context menus:

1. Right-click (for Macintosh, **Control**-click) the equations.

2. From the context menu, select **Solve** (or **Solve Numerically**). See Figure 4.2.

$$> \frac{7x^2}{3} - x = 12$$

$$\frac{7}{3}x^2 - x = 12$$

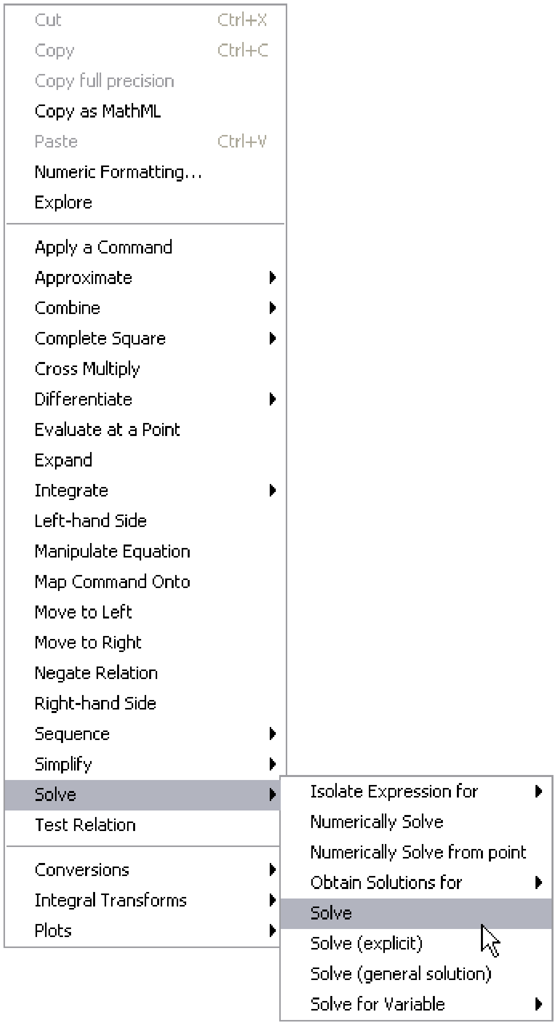


Figure 4.2: Context Menu for an Equation

In Worksheet mode, Maple inserts a calling sequence that solves the equation followed by the solutions.

If you select **Solve**, Maple computes exact solutions.

> $\frac{7x^2}{3} - x = 12$

$$\frac{7}{3}x^2 - x = 12 \quad (4.3)$$

> *solve*({**(4.3)**})

$$\left\{x = \frac{3}{14} + \frac{3}{14}\sqrt{113}\right\}, \left\{x = \frac{3}{14} - \frac{3}{14}\sqrt{113}\right\} \quad (4.4)$$

If you select **Solve Numerically**, Maple computes floating-point solutions.

> $\frac{7x^2}{3} - x = 12$

$$\frac{7}{3}x^2 - x = 12 \quad (4.5)$$

> *fsolve*({**(4.5)**})

$$\{x = -2.063602674\}, \{x = 2.492174103\} \quad (4.6)$$

For information on solving equations and inequations symbolically using the **solve** command, see the following section. For information on solving equations numerically using the **fsolve** command, see *Numerically Solving Equations (page 117)*.

Symbolically Solving Equations and Inequations

The **solve** command is a general solver that determines exact symbolic solutions to equations or inequations. The solutions to a single equation or inequation are returned as an expression sequence. For details, see *Creating and Using Data Structures (page 337)*. If Maple does not find any solutions, the **solve** command returns the empty expression sequence.

> *solve*$\left(x^2 + 3x + 14 = 0\right)$

$$-\frac{3}{2} + \frac{1}{2}\mathrm{I}\sqrt{47}, -\frac{3}{2} - \frac{1}{2}\mathrm{I}\sqrt{47}$$

In general, **solve** computes solutions in the field of complex numbers. To restrict the problem to only real solutions, see *Restricting the Domain (page 143)*.

It is recommended that you verify the solutions returned by the **solve** command. For details, see *Working with Solutions (page 119)*.

To return the solutions as a list, enclose the calling sequence in brackets (**[ ]**).

> $[solve(x^2 + x = 256\,y, x)]$

$$\left[-\frac{1}{2} + \frac{1}{2}\sqrt{1 + 1024\,y}, -\frac{1}{2} - \frac{1}{2}\sqrt{1 + 1024\,y}\right]$$

Expressions: You can specify expressions instead of equations. The **solve** command automatically equates them to zero.

> $solve(e^z + z)$

$$-\mathrm{LambertW}(1)$$

Multiple Equations: To solve multiple equations or inequations, specify them as a *Creating and Using Data Structures (page 337)*.

> $solve([x\,y^2 - y = 5, x > 0])$

$$\left\{x = \frac{y+5}{y^2}, \frac{y+5}{y^2} = \frac{y+5}{y^2}, 0 < y\right\}, \left\{x = \frac{y+5}{y^2}, \frac{y+5}{y^2} = \frac{y+5}{y^2}, -5 < y, y < 0\right\}$$

> $solve(\{x\,y^2 - y = 5, x < 0\})$

$$\left\{x = \frac{y+5}{y^2}, \frac{y+5}{y^2} = \frac{y+5}{y^2}, y < -5\right\}$$

Solving for Specific Unknowns: By default, the **solve** command returns solutions for all unknowns. You can specify the unknowns for which to solve.

> $solve\left(q^2 - r\,s + \frac{q}{r} = 5, q\right)$

$$\frac{1}{2}\,\frac{-1 + \sqrt{1 + 4\,r^3\,s + 20\,r^2}}{r}, -\frac{1}{2}\,\frac{1 + \sqrt{1 + 4\,r^3\,s + 20\,r^2}}{r}$$

To solve for multiple unknowns, specify them as a **list**.

$> solve\left(\left\{\frac{q}{s} - \frac{r}{s+1} + \frac{q}{r} = 5, r\,s = 1\right\}, [q, r]\right)$

$$\left[\left[q = \frac{5\,s^2 + 1 + 5\,s}{s + 1 + s^3 + s^2}, r = \frac{1}{s}\right]\right]$$

Transcendental Equations: In general, the **solve** command returns one solution to transcendental equations.

$> equation1 := \sin(x) = \cos(x) :$

$> solve(equation1)$

$$\frac{1}{4}\,\pi$$

To produce all solutions, use the **allsolutions** option.

$> solve(equation1, allsolutions = true)$

$$\frac{1}{4}\,\pi + \pi\_Z1\sim$$

Maple uses variables of the form **_ZN~**, where **N** is a positive integer, to represent arbitrary integers. The tilde (~) indicates that it is a quantity with an assumption. For information about names with assumptions, see *Assumptions on Variables (page 144)*.

RootOf Structure: The **solve** command may return solutions, for example, to higher order polynomial equations, in an implicit form using **RootOf** structures.

$> [solve(x^5 - 2\,x^4 + 3\,x^3 - 2)]$

$$[1, RootOf(\_Z^4 - \_Z^3 + 2\,\_Z^2 + 2\,\_Z + 2, index = 1), RootOf(\_Z^4 - \_Z^3 + 2\,\_Z^2 + 2\,\_Z + 2, index = 2), RootOf(\_Z^4 - \_Z^3 + 2\,\_Z^2 + 2\,\_Z + 2, index = 3), RootOf(\_Z^4 - \_Z^3 + 2\,\_Z^2 + 2\,\_Z + 2, index = 4)] \quad (4.7)$$

These **RootOf** structures are placeholders for the roots of the equation $z^4 - z^3 + 2\,z^2 + 2\,z + 2$. The **index** parameter numbers and orders the four solutions.

Like any symbolic expression, you can convert **RootOf** structures to a floating-point value using the **evalf** command.

> $evalf((4.7))$

$$[1., 0.984001051867989 + 1.52659083388421\,\mathrm{I}, -0.484001051867989 + 0.609947140486231\,\mathrm{I}, -0.484001051867989 - 0.609947140486231\,\mathrm{I}, 0.984001051867989 - 1.52659083388421\,\mathrm{I}]$$

Some equations are difficult to solve symbolically. For example, polynomial equations of order five and greater do not in general have a solution in terms of radicals. If the **solve** command does not find any solutions, it is recommended that you use the Maple numerical solver, **fsolve**. For information, see the following section, *Numerically Solving Equations*.

For more information on the **solve** command, including how to solve equations defined as procedures and how to find parametric solutions, refer to the **?solve/details** help page.

For information on verifying and using solutions returned by the **solve** command, see *Working with Solutions (page 119)*.

Numerically Solving Equations

The **fsolve** command solves equations numerically. The behavior of the **fsolve** command is similar to that of the **solve** command.

> $equation2 := z\cos(z) = 2:$

> $fsolve(equation2, z)$

$$23.64662473 \quad (4.8)$$

Note: You can also numerically solve equations using the context menus. See *Solving Equations and Inequations (page 112)*.

It is recommended that you verify the solutions returned by the **fsolve** command. For details, see *Working with Solutions (page 119)*.

Multiple Equations: To solve multiple equations, specify them as a set. For more information, see *Creating and Using Data Structures (page 337)*. The **fsolve** command solves for all unknowns.

> $fsolve(\{ln(x) = y^2 + 1, x\,y = e^y\})$

$$\{x = 3.396618823, y = 0.4719962637\}$$

Univariate Polynomial Equations: In general, the **fsolve** command finds one real solution. However, for a univariate polynomial equation, the **fsolve** command returns all *real* roots.

> $equation3 := y^4 - 3\,y^2 - 2\,y + 1:$

> *fsolve*(*equation3*, *y*)

0.3365322739, 1.940392664

Controlling the Number of Solutions: To limit the number of roots returned, specify the **maxsols** option.

> *fsolve*(*equation3*, *y*,'*maxsols*'= 1)

0.3365322739

To find additional solutions to a general equation, use the **avoid** option to ignore known solutions.

> *fsolve*(*equation2*, *z*, '*avoid*' = {*z* = `(4.8)`})

−2.498755763

Complex Solutions: To search for a complex solution or find all complex and real roots for a univariate polynomial, specify the **complex** option for the **fsolve** command.

> *fsolve*(*equation3*, *y*, '*complex*')

$-1.13846246879373 - 0.485062494059435\,\mathrm{I}, -1.13846246879373 + 0.485062494059435\,\mathrm{I}, 0.336532273926790, 1.94039266366067$

If the **fsolve** command does not find any solutions, it is recommended that you specify a range in which to search for solutions, or specify an initial value.

Range: To search for a solution in a range, specify the range in the calling sequence. The range can be real or complex.

> *fsolve*(*equation2*, *z*, {*z* = 100 ..200})

149.2390528

The syntax for specifying a region in the complex plane is **lower-left point..upper-right point**.

> *fsolve*(*equation3*, *y*, {*y* = −2 − *I* ..0}, '*complex*');

$-1.13846246879373 - 0.485062494059435\,\mathrm{I}$

Initial Values: You can specify a value for each unknown. The **fsolve** command uses these as initial values for the unknowns in the numerical method.

> $fsolve(equation2, \{z = 100\})$

$$\{z = 98.98037599\} \tag{4.9}$$

For more information and examples, refer to the **?fsolve/details** help page.

For information on verifying and using solutions returned by the **fsolve** command, see the following section, *Working with Solutions*.

Working with Solutions

Verifying: It is recommended that you always verify solutions (that the **solve** and **fsolve** commands return) using the **eval** command.

> $equation4 := sin(x) = -cos(x):$

> $solve(equation4)$

$$-\frac{1}{4}\pi \tag{4.10}$$

> $eval(equation4, x = \mathbf{(4.10)})$

$$-\frac{1}{2}\sqrt{2} = -\frac{1}{2}\sqrt{2} \tag{4.11}$$

> $equation5 := cos(z) = \frac{2}{z}:$

> $fsolve(equation5)$

$$-2.498755763 \tag{4.12}$$

> $eval(equation5, \{z = \mathbf{(4.12)}\})$

$$-0.8003983544 = -0.8003983540 \tag{4.13}$$

For more information, see *Substituting a Value for a Subexpression (page 359)*.

Assigning the Value of a Solution to a Variable: To assign the value of a solution to the corresponding variable as an *expression*, use the **assign** command.

For example, consider the numeric solution in (4.9), $\{z = 98.98037599\}$, found using the starting value $z = 100$.

> *assign*(**(4.9)**)

> *z*

$$98.98037599$$

Creating a Function from a Solution: The **assign** command assigns a value as an expression to a name. It does **not** define a function. To convert a solution to a function, use the **unapply** command.

Consider one of the solutions for **q** to the equation $q^2 - r\,s + \frac{q}{r} = 5$.

> $solutions := \left[solve\left(q^2 - r\,s + \frac{q}{r} = 5, q \right) \right]$

$$solutions := \left[\frac{1}{2} \frac{-1 + \sqrt{1 + 4\,r^3\,s + 20\,r^2}}{r}, -\frac{1}{2} \frac{1 + \sqrt{1 + 4\,r^3\,s + 20\,r^2}}{r} \right]$$

> $f := unapply(solutions[1], r, s)$

$$f := (r, s) \rightarrow \frac{1}{2} \frac{-1 + \sqrt{1 + 4\,r^3\,s + 20\,r^2}}{r}$$

Here, **solutions[1]** selects the first element of the list of solutions. For more information on selecting elements, see *Accessing Elements (page 338)*.

You can evaluate this function at symbolic or numeric values.

> $f(x, y)$

$$\frac{1}{2}\,\frac{-1+\sqrt{1+4\,x^3\,y+20\,x^2}}{x}$$

> $f\left(\frac{1}{\sqrt{2}}, 1\right)$

$$\frac{1}{2}\,\sqrt{2}\,\left(-1+\sqrt{11+\sqrt{2}}\right)$$

> $f(5.7, 2.1)$

$$4.032680522$$

For more information on defining and using functions, see *Functional Operators (page 343)*.

Other Specialized Solvers

In addition to equations and inequations, Maple can solve other equations including:

- Ordinary differential equations (ODEs)
- Partial differential equations (PDEs)
- Integer equations
- Integer equations in a finite field
- Linear systems
- Recurrence relations

Ordinary Differential Equations (ODEs)

Maple can solve ODEs and ODE systems, including initial value and boundary value problems, symbolically and numerically.

ODE Analyzer Assistant The **ODE Analyzer Assistant** is a point-and-click interface to the Maple ODE solving routines.

To open the ODE Analyzer:

- From the **Tools** menu, select **Assistants**, and then **ODE Analyzer**.

Maple inserts the *dsolve*[*interactive*]() calling sequence in the document. The **ODE Analyzer Assistant** (Figure 4.3) is displayed.

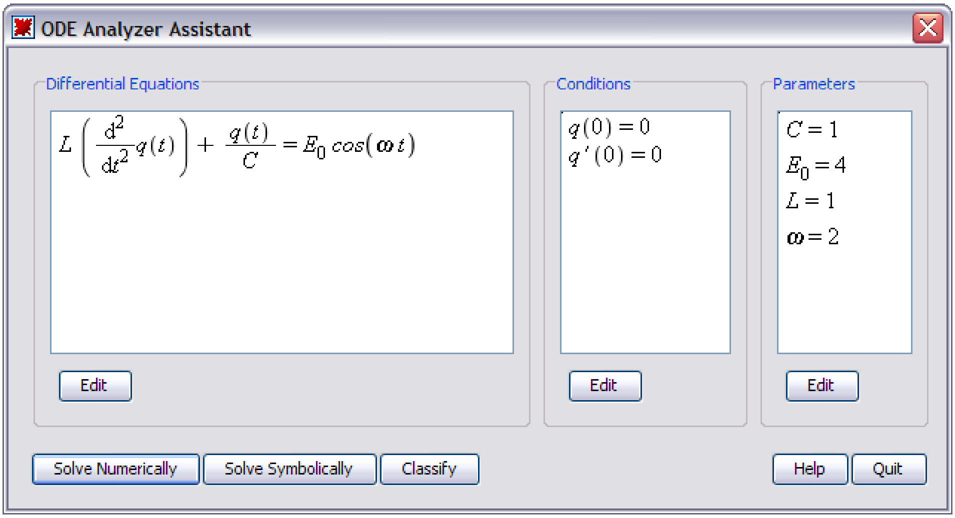


Figure 4.3: ODE Analyzer Assistant

In the main **ODE Analyzer Assistant** window, you can define ODEs, initial or boundary value conditions, and parameters. To define derivatives, use the **diff** command. For example, **diff(x(t), t)** corresponds to $\frac{dx(t)}{dt}$, and **diff(x(t), t, t)** corresponds to $\frac{d^2x(t)}{dt^2}$. For more information on the **diff** command, see *The diff Command (page 178)*.

After defining an ODE, you can solve it numerically or symbolically.

To solve a system numerically using the ODE Analyzer Assistant:

1. Ensure that the conditions guarantee uniqueness of the solution.

2. Ensure that all parameters have fixed values.

3. Click the **Solve Numerically** button.

4. In the **Solve Numerically** window (Figure 4.4), you can specify the numeric method and relevant parameters and error tolerances to use for solving the problem.

5. To compute solution values at a point, click the **Solve** button.

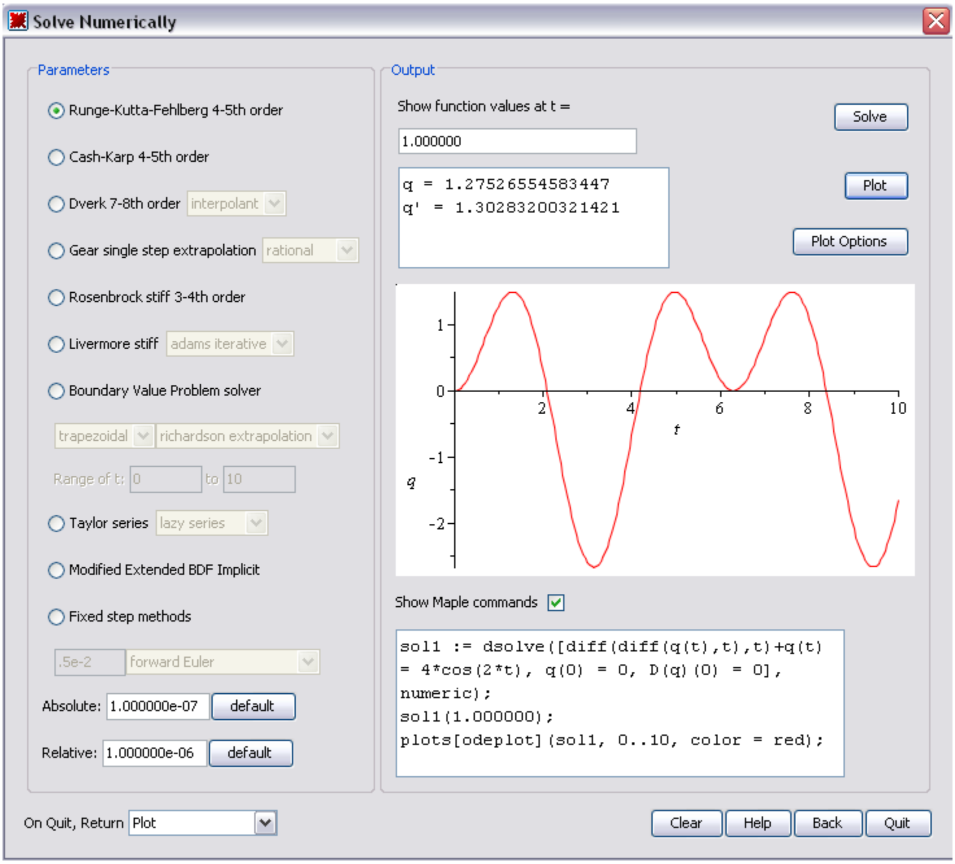


Figure 4.4: ODE Analyzer Assistant: Solve Numerically Dialog

To solve a system symbolically using the ODE Analyzer Assistant:

1. Click the **Solve Symbolically** button.

2. In the **Solve Symbolically** window (Figure 4.5), you can specify the method and relevant method-specific options to use for solving the problem.

3. To compute the solution, click the **Solve** button.

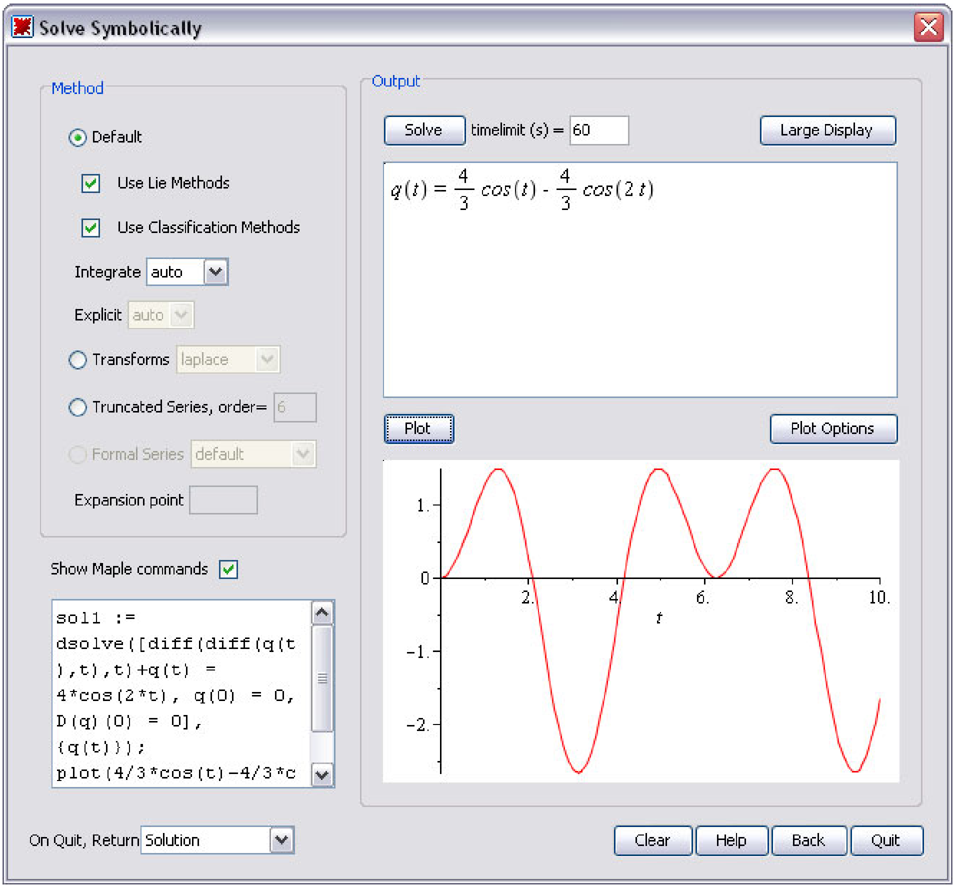


Figure 4.5: ODE Analyzer Assistant: Solve Symbolically Dialog

When solving numerically or symbolically, you can view a plot of the solution by clicking the **Plot** button.

- To plot the solution to a symbolic problem, all conditions and parameters must be set.
- To customize the plot, click the **Plot Options** button to open the **Plot Options** window.

To view the corresponding Maple commands as you solve the problem or plot the solution, select the **Show Maple** commands check box.

You can control the return value of the ODE Analyzer using the **On Quit, Return** drop-down list. You can select to return nothing, the displayed plot, the computed numeric procedure (for numeric solutions), the solution (for symbolic solutions), or the Maple commands needed to produce the solution values and the displayed plot.

For more information, refer to the **?ODEAnalyzer** help page.

The dsolve Command

The ODE Analyzer provides a point-and-click interface to the Maple **dsolve** command.

For ODEs or systems of ODEs, the **dsolve** command can find:

- Closed form solutions
- Numerical solutions
- Series solutions

In addition, the **dsolve** command can find:

- Formal power series solutions to linear ODEs with polynomial coefficients
- Formal solutions to linear ODEs with polynomial coefficients

To access all available functionality, use the **dsolve** command directly. For more information, refer to the **?dsolve** help page.

Partial Differential Equations (PDEs)

To solve a PDE or PDE system symbolically or numerically, use the **pdsolve** command. PDE systems can contain ODEs, algebraic equations, and inequations.

For example, solve the following PDE symbolically. For help entering a partial derivative, see *Example 1 - Enter a Partial Derivative (page 65)*.

$$> \; x\left(\frac{\partial}{\partial y} f(x,y)\right) - y\left(\frac{\partial}{\partial x} f(x,y)\right) = 0$$

$$x\left(\frac{\partial}{\partial y} f(x,y)\right) - y\left(\frac{\partial}{\partial x} f(x,y)\right) = 0 \tag{4.14}$$

> *pdsolve*(**(4.14)**)

$$f(x,y) = \_F1\left(x^2 + y^2\right)$$

The solution is an arbitrary univariate function applied to $x^2 + y^2$.

Maple generally prints only the return value, errors, and warnings during a computation. To print information about the techniques Maple uses, increase the **infolevel** setting for the command.

To return all information, set **infolevel** to 5.

> $infolevel[pdsolve] := 5:$

> $pdsolve(\textbf{(4.14)})$

```
Checking arguments ...
First set of solution methods (general or quase general solution)
Second set of solution methods (complete solutions)
Trying methods for first order PDEs
Second set of solution methods successful
```

$$f(x, y) = \_F1\left(x^2 + y^2\right)$$

For more information on solving PDEs, including numeric solutions and solving PDE systems, refer to the **?pdsolve** help page.

Integer Equations

To find only integer solutions to an equation, use the **isolve** command. The **isolve** command finds solutions for all variables. For more information, refer to the **?isolve** help page.

> $isolve\left(\{x^2 + y = 13\}\right)$

$$\left\{x = \_Z1, y = -\_Z1^2 + 13\right\}$$

Integer Equations in a Finite Field

To solve an equation modulo an integer, use the **msolve** command. The **msolve** command finds solutions for all variables. For more information, refer to the **?msolve** help page.

> $msolve\left(\{x^2 = 1\}, 13\right)$

$$\{x = 1\}, \{x = 12\}$$

Solving Linear Systems

To solve a linear system, use the **LinearAlgebra[LinearSolve]** command. The **LinearSolve** command returns the vector **x** that satisfies **A . x = B**. For more information, refer to the **?LinearAlgebra[LinearSolve]** help page.

For example, construct an augmented matrix using the **Matrix** palette (see *Creating Matrices and Vectors (page 159)*) in which the first four columns contain the entries of **A** and the final column contains the entries of **B**.

$$> \textit{linearsystem} := \begin{bmatrix} \frac{59}{10} & \frac{44}{25} & \frac{17}{2} & \frac{1}{100} & \frac{1}{2} \\ 1 & 0 & 7 & \frac{533}{100} & \frac{61}{50} \\ 98 & \frac{21}{10} & \frac{3}{10} & 7 & \frac{2178}{25} \\ 23 & 9 & 12 & \frac{51}{10} & \frac{786}{25} \end{bmatrix}:$$

$$> \textit{LinearAlgebra}[\textit{LinearSolve}](\textit{linearsystem})$$

$$\begin{bmatrix} \frac{31753441047}{41858667400} \\ \frac{16991806239}{8371733480} \\ -\frac{1489266217}{1674346696} \\ \frac{262603866}{209293337} \end{bmatrix}$$

For more information on using Maple to solve linear algebra problems, see *Linear Algebra (page 159)*.

Solving Recurrence Relations

To solve a recurrence relation, use the **rsolve** command. The **rsolve** command finds the general term of the function. For more information, refer to the **?rsolve** help page.

$$> \textit{rsolve}(\{f(n) = f(n-1) + f(n-2), f(0) = 1, f(1) = 1\}, \{f(n)\})$$

$$\left\{ f(n) = \left(-\frac{1}{10}\sqrt{5} + \frac{1}{2}\right)\left(-\frac{1}{2}\sqrt{5} + \frac{1}{2}\right)^n + \left(\frac{1}{10}\sqrt{5} + \frac{1}{2}\right)\left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^n \right\}$$

4.5 Units, Scientific Constants, and Uncertainty

In addition to manipulating exact symbolic and numeric quantities, Maple can perform computations with units and uncertainties.

Maple supports hundreds of units, for example, miles, coulombs, and bars, and provides facilities for adding custom units.

Maple has a library of hundreds of scientific constants with units, including element and isotope properties.

To support computations with uncertainties, Maple propagates errors through computations.

Units

The **Units** package in Maple provides a library of units, and facilities for using units in computations. It is fully extensible so that you can add units and unit systems as required.

Note: Some unit operations are available as task templates (see **Tools→Tasks→Browse**) and through context menus.

Overview of Units

A *dimension* is a measurable quantity, for example, length or force. The set of dimensions that are fundamental and independent are known as *base dimensions*.

In Maple, the base dimensions include length, mass, time, electric current, thermodynamic temperature, amount of substance, luminous intensity, information, and currency. For a complete list, enter and execute *Units*[*GetDimensions*]().

Complex dimensions (or composite dimensions) measure other quantities in terms of a combination of base dimensions. For example, the complex dimension force is a measurement of $\frac{mass \cdot length}{time^2}$.

Each dimension, base or complex, has associated units. (Base units measure a base dimension. Complex units measure a complex dimension.) Maple supports over 40 units of length, including feet, miles, meters, angstroms, microns, and astronomical units. A length must be measured in terms of a unit, for example, a length of 2 parsecs.

Table 4.4 lists some dimensions, their corresponding base dimensions, and example units.

Table 4.4: Sample Dimensions

Dimension	Base Dimensions	Example Units
Time	time	second, minute, hour, day, week, month, year, millennium, blink, lune
Energy	$\frac{length^2 \cdot mass}{time^2}$	joule, electron volt, erg, watt hour, calorie, Calorie, British thermal unit
Electric potential	$\frac{length^2 \cdot mass}{time^3 \cdot electric\ current}$	volt, abvolt, statvolt

For the complete list of units (and their contexts and symbols) available for a dimension, refer to the corresponding help page, for example, the **?Units/length** help page for the units of length.

Each unit has a *context*. The context differentiates between different definitions of the unit. For example, the standard and US survey miles are different units of length, and the second is a unit of time and of angle. You can specify the context for a unit by appending the context as an index to the unit, for example, **mile[US_survey]**. If you do not specify a context, Maple uses the default context.

Units are collected into systems, for example, the foot-pound-second (FPS) system and international system, or *système international*, (SI). Each system has a default set of units used for measurements. In the FPS system, the foot, pound, and second are used to measure the dimensions of length, mass, and time. The unit of speed is the foot/second. In SI, the meter, kilogram, and second are used to measure the dimensions of length, mass, and time. The units of speed, magnetic flux, and power are the meter/second, weber, and watt, respectively.

Unit Conversions

To convert a value measured in a unit to the corresponding value in a different unit, use the **Units Calculator**.

- From the Tools→Assistants menu, select **Units Calculator**.

The **Units Calculator** application (Figure 4.6) opens.

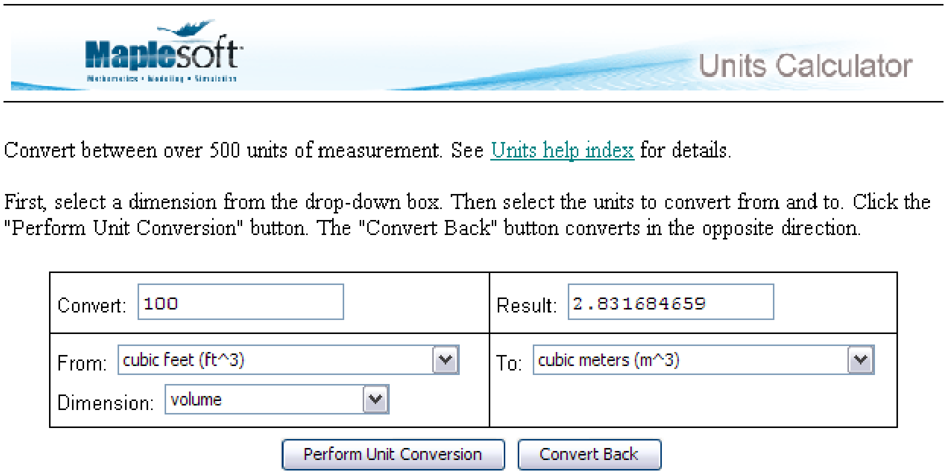


Figure 4.6: Units Calculator Assistant

To perform a conversion:

1. In the **Convert** text field, enter the numeric value to convert.

2. In the **Dimension** drop-down list, select the dimensions of the unit.

3. In the **From** and **To** drop-down lists, select the original unit and the unit to which to convert.

4. Click **Perform Unit Conversion**.

The same conversion can be done with the **convert/units** command.

> *convert*(1.0, '*units*', '*lbf ft*(*radius*)', '*N m*(*radius*)')

1.355817948

Using the **Units Calculator**, you can convert temperatures and temperature changes.

- To perform a *temperature* conversion, in the **Dimension** drop-down list, select **temperature(absolute)**.
- To perform a *temperature change* conversion, in the **Dimension** drop-down list, select **temperature(relative)**.

To convert temperature changes, the **Units Calculator** uses the **convert/units** command. For example, an increase of 32 degrees Fahrenheit corresponds to an increase of almost 18 degrees Celsius.

> *convert*(32.0, '*units*', '*degF*', '*degC*')

$$17.77777778$$

To convert absolute temperatures, the **Unit Converter** uses the **convert/temperature** command. For example, 32 degrees Fahrenheit corresponds to 0 degrees Celsius.

> *convert*(32, '*temperature*', '*degF*', '*degC*')

$$0$$

Applying Units to an Expression

To insert a unit, use the **Units** palettes. The **Units (FPS)** palette (Figure 4.7) contains important units from the foot-pound-second system of units. The **Units (SI)** palette (Figure 4.8) contains important units from the international system of units.

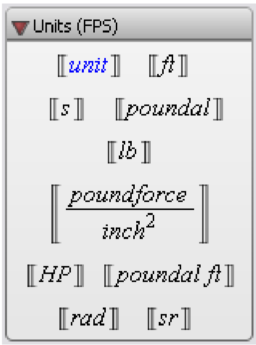


Figure 4.7: Units (FPS) Palette

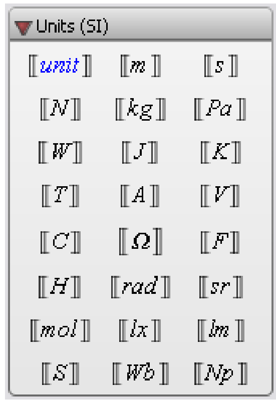


Figure 4.8: Units (SI) Palette

To insert a unit:

- In a **Units** palette, click a unit symbol.

> 3⟦*ft*⟧

$$3\,[\![ft]\!]$$

To insert a unit that is unavailable in the palettes:

1. In a **Units** palette, click the **unit** symbol ⟦*unit*⟧. Maple inserts a **Unit** object with the placeholder selected.

2. In the placeholder, enter the unit name (or symbol).

For example, to enter 0.01 standard (the default context) miles, you can specify the unit name, **mile**, or symbol, **mi**.

> $0.01 [\![mile]\!]$

$$0.01 [\![mi]\!]$$

The context of a unit is displayed only if it is not the default context.

Important: In 1-D Math input, the quantity and unit (entered using the top-level **Unit** command) are a product, not a single entity. The following calling sequences define different expressions.

```
> 1*Unit(m)/(2*Unit(s));
```

$$\frac{1}{2} \frac{[\![m]\!]}{[\![s]\!]}$$

```
> 1*Unit(m)/2*Unit(s);
```

$$\frac{1}{2} [\![m]\!] [\![s]\!]$$

Some units support prefixes. For example, SI units support prefixes to names and symbols. You can specify 1000 meters using **kilometer** or **km**. For more information, refer to the **?Units/prefixes** help page.

> $1.5 [\![km_{SI}]\!]$

$$1.5 [\![km]\!]$$

Performing Computations with Units

In the default Maple environment, you cannot perform computations with quantities that have units. You can perform only unit conversions. For more information about the default environment, refer to the **?Units/default** help page.

To compute with expressions that have units, you must load a **Units** environment, Natural or Standard. It is recommended that you use the Standard environment.

> $with(Units[Standard]):$

In the **Standard Units** environment, commands that support expressions with units return results with the correct units.

> $area := 3 [\![ft]\!] \cdot \frac{1}{8} [\![mile]\!]$

$$area := \frac{14370939}{78125} [\![m^2]\!]$$

> $\frac{(-12 \sin(x) + x^2) [\![m]\!]}{[\![s]\!]}$

$$(-12 \sin(x) + x^2) \left[\!\!\left[\frac{m}{s} \right]\!\!\right] \tag{4.15}$$

> $int($ **(4.15)** $, x [\![s]\!])$

$$\left(12 \cos(x) + \frac{1}{3} x^3\right) [\![m]\!] \tag{4.16}$$

> $diff($ **(4.16)** $, x [\![s]\!])$

$$(-12 \sin(x) + x^2) \left[\!\!\left[\frac{m}{s} \right]\!\!\right]$$

For information on differentiation and integration, see *Calculus (page 175)*.

Changing the Current System of Units

If a computation includes multiple units, all units are expressed using units from the current system of units.

> $132.25 [\![mile]\!]$

$$132.25 [\![mi]\!] \tag{4.17}$$

By default, Maple uses the SI system of units, in which length is measured in meters and time is measured in seconds.

> $\frac{\textbf{(4.17)}}{3 [\![hour]\!]}$

$$19.70701333 \left[\!\!\left[\frac{m}{s} \right]\!\!\right]$$

To view the name of the default system of units, use the **Units[UsingSystem]** command.

> *with*(*Units*) :

> *UsingSystem*( )

$$SI$$

To change the system of units, use the **Units[UseSystem]** command.

> *UseSystem*(*FPS*) :

> **(4.17)** $\cdot$ 3⟦*m*⟧ $\cdot$ 1.1⟦*kg*⟧

$$1.666720741\ 10^{7}\ ⟦ft^{2}\ lb⟧$$

Extensibility

You can extend the set of:

- Base dimensions and units
- Complex dimensions
- Complex units
- Systems of units

For more information, refer to the **?Units[AddBaseUnit]**, **?Units[AddDimension]**, **?Units[AddUnit]**, and **?Units[AddSystem]** help pages.

For more information about units, refer to the **?Units** help page.

Scientific Constants and Element Properties

Computations often require not only units (see *Units (page 128)*), but also the values of scientific constants, including properties of elements and their isotopes. Maple supports computations with scientific constants. You can use the built-in constants and add custom constants.

Overview of Scientific Constants and Element Properties

The **ScientificConstants** package provides the values of constant physical quantities, for example, the velocity of light and the atomic weight of sodium. The **ScientificConstants** package also provides the units for the constant values, allowing for greater understanding of the equation as well as unit-matching for error checking of the solution.

The quantities available in the **ScientificConstants** package are divided into two distinct categories.

- Physical constants
- Chemical element (and isotope) properties

Scientific Constants

List of Scientific Constants

You have access to scientific constants important in engineering, physics, chemistry, and other fields. Table 4.5 lists some of the supported constants. For a complete list of scientific constants, refer to the **?ScientificConstants/PhysicalConstants** help page.

Table 4.5: Scientific Constants

Name	Symbol
Newtonian_constant_of_gravitation	G
Planck_constant	h
elementary_charge	e
Bohr_radius	a[0]
deuteron_magnetic_moment	mu[d]
Avogadro_constant	N[A]
Faraday_constant	F

You can specify a constant using either its name or symbol.

Accessing Constant Definition

The **GetConstant** command in the **ScientificConstants** package returns the complete definition of a constant.

To view the definition of the Newtonian gravitational constant, specify the symbol **G** (or its name) in a call to the **GetConstant** command.

```
> with(ScientificConstants):
> GetConstant('G')
```

$$Newtonian\_constant\_of\_gravitation, symbol = G, value = 6.673\ 10^{-11}, uncertainty = 1.0\ 10^{-13}, units = \frac{m^3}{kg\, s^2}$$

For information on accessing a constant's value, units, or uncertainty, see *Value, Units, and Uncertainty (page 137)*.

Element Properties

Maple also contains element properties and isotope properties.

Elements

Maple supports all 117 elements of the periodic table. Each element has a unique name, atomic number, and chemical symbol. You can specify an element using any of these labels. For a complete list of supported elements, refer to the **?ScientificConstants/elements** help page.

Maple supports key element properties, including atomic weight (**atomicweight**), electron affinity (**electronaffinity**), and density. For a complete list of element properties, refer to the **?ScientificConstants/properties** help page.

Isotopes

Isotopes, variant forms of an element that contain the same number of protons but a different number of neutrons, exist for many elements.

To see the list of supported isotopes for an element, use the **GetIsotopes** command.

> *GetIsotopes*('*element*'='Li')

$$Li_{4}, Li_{5}, Li_{6}, Li_{7}, Li_{8}, Li_{9}, Li_{10}, Li_{11}, Li_{12}$$

Maple supports isotopes and has a distinct set of properties for isotopes, including abundance, binding energy (**bindingenergy**), and mass excess (**massexcess**). For a complete list of isotope properties, refer to the **?ScientificConstants/properties** help page.

Accessing an Element or Isotope Property Definition

The **GetElement** command in the **ScientificConstant**s package returns the complete definition of an element or isotope.

> *GetElement*('*Li*')

3, *symbol* = Li, *name* = *lithium*, *names* = {*lithium*}, *electronaffinity* = [*value* = 0.6180, *uncertainty* = 0.0005, *units* = *eV*], *atomicweight* = [*value* = 6.941, *uncertainty* = 0.002, *units* = *amu*], *boilingpoint* = [*value* = 1615., *uncertainty* = *undefined*, *units* = *K*], *ionizationenergy* = [*value* = 5.3917, *uncertainty* = *undefined*, *units* = *eV*], *density* = $\left[value = 0.534, uncertainty = undefined, units = \frac{g}{cm^3}\right]$, *electronegativity* = [*value* = 0.98, *uncertainty* = *undefined*, *units* = 1], *meltingpoint* = [*value* = 453.65, *uncertainty* = *undefined*, *units* = *K*]

> *GetElement*('*Li*[4]')

Li_4, *massexcess* = [*value* = 25320.173, *uncertainty* = 212.132, *units* = *keV*], *bindingenergy* = [*value* = 4618.058, *uncertainty* = 212.132, *units* = *keV*], *atomicmass* = [*value* = $4.027182329\ 10^6$, *uncertainty* = 227.733, *units* = *uamu*]

Value, Units, and Uncertainty

To use constants or element properties, you must first construct a **ScientificConstants** object.

To construct a scientific constant, use the **Constant** command.

> *G* := *Constant*('*G*') :

To construct an element (or isotope) property, use the **Element** command.

> *LiAtomicWeight* := *Element*('Li', *atomicweight*)

LiAtomicWeight := *Element*(Li, *atomicweight*)

Value

To obtain the value of a **ScientificConstants** object, use the **evalf** command.

> *evalf*(*G*)

$$1.068912061\ 10^{-9}$$

> *evalf*(*LiAtomicWeight*)

$$2.541006042\ 10^{-26}$$

Note: The value returned depends on the current system of units.

Units

To obtain the units for a **ScientificConstants** object, use the **GetUnit** command.

> *GetUnit*(*G*)

$$\left[\!\left[\frac{ft^3}{lb\, s^2} \right]\!\right]$$

> *GetUnit*(*LiAtomicWeight*)

$$[\![lb]\!]$$

For information on changing the default system of units, for example, from SI to foot-pound-second, see *Changing the Current System of Units (page 133)*.

Value and Units

If you are performing computations with units, you can access the value and units for a **ScientificConstants** object by specifying the **units** option when constructing the object, and then evaluating the object.

> *evalf*(*Constant*('*G*', *units*))

$$1.068912061\, 10^{-9} \left[\!\left[\frac{ft^3}{lb\, s^2} \right]\!\right]$$

> *evalf* (*Element*('Li[5]', *atomicmass*, *units*))

$$1.835022162\, 10^{-26} [\![lb]\!]$$

Uncertainty

The value of a constant is often determined by direct measurement or derived from measured values. Hence, it has an associated uncertainty. To obtain the uncertainty in the value of a **ScientificConstants** object, use the **GetError** command.

> *GetError*(*G*)

$$1.0\ 10^{-13}$$

> *GetError*(*LiAtomicWeight*)

$$3.321080400\ 10^{-30}$$

Performing Computations

You can use constant values in any computation. To use constant values with units, use a **Units** environment as described in *Performing Computations with Units (page 132)*. For information on computing with quantities that have an uncertainty, see the following section.

Modification and Extensibility

You can change the definition of a scientific constant or element (or isotope) property.

For more information, refer to the **?ScientificConstants[ModifyConstant]** and **?ScientificConstants[ModifyElement]** help pages.

You can extend the set of:

- Constants
- Elements (and isotopes)
- Element (or isotope) properties

For more information, refer to the **?ScientificConstants[AddConstant]**, **?ScientificConstants[AddElement]**, and **?ScientificConstants[AddProperty]** help pages.

For more information about constants, refer to the **?ScientificConstants** help page.

Uncertainty Propagation

Some computations involve uncertainties (or errors). Using the **ScientificErrorAnalysis** package, you can propagate the uncertainty in these values through the computation to indicate the possible error in the final result.

The **ScientificErrorAnalysis** package does not perform interval arithmetic. That is, the error of an object does not represent an interval in which possible values must be contained. (To perform interval arithmetic, use the **Tolerances** package. For more information, refer to the **?Tolerances** help page.) The quantities represent unknown values with a central tendency. For more information on central tendency, refer to any text on error analysis for the physical sciences or engineering.

Quantities with Uncertainty

Creating: To construct quantities with uncertainty, use the **Quantity** command. You must specify the value and uncertainty. The uncertainty can be defined absolutely, relatively, or in units of the last digit. For more information on uncertainty specification, refer to the **?ScientificErrorAnalysis[Quantity]** help page.

The output displays the value and uncertainty of the quantity.

```
> with(ScientificConstants):  with(ScientificErrorAnalysis):
```

```
> Quantity(105, 1.2)
```

Quantity(105, 1.2)

```
> Quantity(105, 0.03, 'relative')
```

Quantity(105, 3.15) (4.18)

To specify the error in units of the last digit, the value must be of floating-point type.

```
> Quantity(105.0, 12, 'uld')
```

Quantity(105.0, 1.2)

To access the value and uncertainty of a quantity with uncertainty, use the **evalf** and **ScientificErrorAnalysis[GetError]** commands.

```
> evalf((4.18))
```

105.

```
> GetError((4.18))
```

3.15

Rounding: To round the error of a quantity with uncertainty, use the **ApplyRule** command. For a description of the predefined rounding rules, refer to the **?ScientificErrorAnalysis/rules** help page.

> $GetError(ApplyRule(\textbf{(4.18)}, \text{'round[2]'}))$

$$3.2$$

Units: Quantities with errors can have units. For example, the scientific constants and element (and isotope) properties in the **ScientificConstants** packages are quantities with errors and units.

To construct a new quantity with units and an uncertainty, include units in the **Quantity** calling sequence.

For an absolute error, you must specify the units in both the value and error.

> $with(Units[Standard]) : with(ScientificErrorAnalysis) :$

> $Quantity(3.5 [\![m]\!], 0.1 [\![m]\!])$

$$ScientificErrorAnalysis{:-}Quantity(3.5 [\![m]\!], 0.1 [\![m]\!])$$

For a relative error, you can specify the units in only the value.

> $Quantity(3.5[\![m]\!], 0.1, \text{'}relative\text{'})$

$$ScientificErrorAnalysis{:-}Quantity(3.5 [\![m]\!], 0.35 [\![m]\!])$$

For information on the correlation between, variance of, and covariance between quantities with uncertainty, refer to the **?ScientificErrorAnalysis** help page.

Performing Computations with Quantities with Uncertainty

Many Maple commands support quantities with uncertainty.

> $q1 := Quantity(31., 2.):$

> $q2 := Quantity(20., 1.):$

Compute the value of the derivative of $q1 \cdot x^2 + sin(q2 \cdot x)$ at $x = sin(\pi/4)$.

> $d1 := diff\left(q1 \cdot x^2 + \sin(q2 \cdot x)\ , x\right)$

$$d1 := 2\, ScientificErrorAnalysis{:}\text{-}Quantity(31., 2.)\, x + \cos(ScientificErrorAnalysis{:}\text{-}Quantity(20., 1.)\, x)\, ScientificErrorAnalysis{:}\text{-}Quantity(20., 1.)$$

> $d2 := eval\left(d1, x = sin\left(\frac{\pi}{4}\right)\right)$:

To convert the solution to a single quantity with uncertainty, use the **combine/errors** command.

> $result := combine(d2, 'errors')$:

The value of the result is:

> $evalf(result)$

$$43.74124725$$

The uncertainty of the result is:

> $GetError(result)$

$$14.42690612$$

Additional Information

For information on topics including:

- Creating new rounding rules,
- Setting the default rounding rule, and
- Creating a new interface to quantities with uncertainty,

refer to the **?ScientificErrorAnalysis** help page.

4.6 Restricting the Domain

By default, Maple computes in the complex number system. Most computations are performed without any restrictions or assumptions on the variables. Maple often returns results that are extraneous or unsimplified when computing in the field of complex numbers. Using restrictions, you can more easily and efficiently perform computations in a smaller domain.

Maple has facilities for performing computations in the real number system and for applying assumptions to variables.

Real Number Domain

To force Maple to perform computations in the field of real numbers, use the **RealDomain** package.

The **RealDomain** package contains a small subset of Maple commands related to basic precalculus and calculus mathematics, for example, **arccos**, **limit**, and **log**, and the symbolic manipulation of expressions and formulae, for example, **expand**, **eval**, and **solve**. For a complete list of commands, refer to the **?RealDomain** help page.

After you load the **RealDomain** package, Maple assumes that all variables are real. Commands return simplified results appropriate to the field of real numbers.

> $with(RealDomain):$

> $simplify\left(\sqrt{x^2}\right)$

$$|x|$$

> $\ln(e^x)$

$$x$$

Some commands that generally return **NULL** instead return a numeric result when you use the **RealDomain** package.

> $(-32)^{\left(\frac{1}{5}\right)}$

$$-2$$

Complex return values are excluded or replaced by **undefined**.

> $solve(x^2 = -1)$

> $\arcsin(e^2)$

$$undefined$$

Assumptions on Variables

To simplify problem solving, it is recommended that you always apply any known assumptions to variables. You can impose assumptions using the **assume** command. To apply assumptions for a single computation, use the **assuming** command.

Note: The **assume** and **assuming** commands are not supported by the **RealDomain** package.

The assume Command

You can use the **assume** command to set variable properties, for example, **x::real**, and relationships between variables, for example, **x < 0** or **x < y**. For information on valid properties, refer to the **?assume** help page. For information on the double colon (**::**) operator, refer to the **?type** help page.

The **assume** command allows improved simplification of symbolic expressions, especially multiple-valued functions, for example, computing the square root.

To assume that **x** is a positive real number, use the following calling sequence. Then compute the square root of x^2 .

> $assume(0 < x)$: $\sqrt{x^2}$

$$x\sim$$

The trailing tilde (~) on the name **x** indicates that it carries assumptions.

When you use the **assume** command to place another assumption on **x**, all previous assumptions are removed.

> $assume(x < 0)$: $\sqrt{x^2}$

$$-x\sim$$

Displaying Assumptions: To view the assumptions on an expression, use the **about** command.

> *about*(x)

```
Originally x, renamed x~:
  is assumed to be: RealRange(-infinity,Open(0))
```

Imposing Multiple Assumptions: To simultaneously impose multiple conditions on an expression, specify multiple arguments in the **assume** calling sequence.

> *assume*$(0 < x, x < 2)$

To specify additional assumptions without replacing previous assumptions, use the **additionally** command. The syntax of the **additionally** calling sequence is the same as that of the **assume** command.

> *additionally*$(x :: \mathit{integer})$: *about*(x)

```
Originally x, renamed x~:
  is assumed to be: 1
```

The only integer in the open interval **(0, 2)** is **1**.

Testing Properties: To test whether an expression always satisfies a condition, use the **is** command.

> *assume*$(15 < x, 7 < y)$: *is*$(100 < x\,y)$

$$\mathit{true}$$

The following test returns **false** because there are values of **x** and **y** (**x = 0**, **y = 10**) that satisfy the assumptions, but do not satisfy the relation in the **is** calling sequence.

> *assume*$(x :: \mathit{nonnegint}, 10 \leq y)$: *is*$(10 < x + y)$

$$\mathit{false}$$

To test whether an expression can satisfy a condition, use the **coulditbe** command.

> *coulditbe*$(10 < x + y)$

$$\mathit{true}$$

Removing Assumptions: To remove all assumptions on a variable, unassign its name.

> *unassign* ('*x*', '*y*')

For more information, see *Unassigning Names (page 95)*.

For more information on the **assume** command, refer to the **?assume** help page.

The assuming Command

To perform a single evaluation under assumptions on the names in an expression, use the **assuming** command.

The syntax of the assuming command is *<expression>* **assuming** *<property or relation>*. Properties and relations are introduced in *The assume Command (page 144)*.

The **frac** command returns the fractional part of an expression.

> frac(x) assuming $x :: integer$

$$0$$

Using the **assuming** command is equivalent to imposing assumptions with the **assume** command, evaluating the expression, and then removing the assumptions.

> $about(x)$

```
x:
  nothing known about this object
```

If you do not specify the names to which to apply a property, it is applied to all names.

> $\sqrt{\left(\frac{a}{b}\right)^2}$ assuming $positive$

$$\frac{a}{b}$$

Assumptions placed on names using the **assume** command are ignored by the **assuming** command, unless you include the **additionally** option.

> $assume(x < 1)$

> $is(1 - x^2 > 0)$ assuming $x > -1$

$$false$$

> $is(1 - x^2 > 0)$ assuming $additionally, x > -1$

$$true$$

The **assuming** command does not affect variables inside procedures. (For information on procedures, see *Procedures (page 383)*.) You must use the **assume** command.

```
> f := proc(x) sqrt(a^2) + x end proc;
```

$$f := \textbf{proc}(x)\ \ \text{sqrt}(a\hat{}2) + x\ \textbf{end proc}$$

> $f(1)$ assuming $a > 0$

$$\sqrt{a^2} + 1$$

> $assume(a > 0)$: $f(1)$

$$a\sim + 1$$

For more information on the **assuming** command, refer to the **?assuming** help page.

5 Mathematical Problem Solving

This chapter focuses on solving problems in specific mathematical disciplines. The areas described below are not all that Maple provides, but represent the most commonly used packages. Examples are provided to teach you how to use the different methods of calculation available in Maple, including tutors, assistants, commands, task templates, plotting, and context menus.

The examples in this chapter assume knowledge of entering commands and mathematical symbols. For information, see *Entering Expressions (page 19)*. For information on basic computations, including integer operations and solving equations, see *Basic Computations (page 101)*.

5.1 In This Chapter

Section	**Topics**
Algebra (page 150) - Performing algebra computations	• Polynomial Algebra
Linear Algebra (page 159) - Performing linear algebra computations	• Creating Matrices and Vectors • Accessing Entries in Matrices and Vectors • Linear Algebra Computations • **Student LinearAlgebra** Package
Calculus (page 175) - Performing calculus computations	• Limits • Differentiation • Series • Integration • Differential Equations • Calculus Packages
Optimization (page 188) - Performing optimization computations using the **Optimization** package	• Point-and-Click Interface • Efficient Computation • MPS(X) File Support
Statistics (page 193) - Performing statistics computations using the **Statistics** package	• Probability Distributions and Random Variables • Statistical Computations • Plotting
Teaching and Learning with Maple (page 198) - Student and Instructor resources for using Maple in an academic setting	• Table of Student and Instructor Resources • **Student** Packages and Tutors

Section	Topics
Clickable Math Examples (page 213) - Solve math problems using some of the interactive methods available in Maple	• Step-by-Step examples

5.2 Algebra

Maple contains a variety of commands that perform integer operations, such as factoring and modular arithmetic, as described in *Integer Operations (page 106)*. In addition, it supports polynomial algebra.

For information on matrix and vector algebra, see *Linear Algebra (page 159)*.

Polynomial Algebra

A Maple polynomial is an expression in powers of an unknown. *Univariate* polynomials are polynomials in one unknown, for example, $x^3 - 2x + 13$. *Multivariate* polynomials are polynomials in multiple unknowns, such as $x^3y - \frac{3}{2}xy^2 + 7x$.

The coefficients can be integers, rational numbers, irrational numbers, floating-point numbers, complex numbers, variables, or a combination of these types.

> $a x^2 + 7x - \frac{b}{2}$

$$a x^2 + 7x - \frac{1}{2} b$$

Arithmetic

The polynomial arithmetic operators are the standard Maple arithmetic operators excluding the division operator (/). (The division operator accepts polynomial arguments, but does not perform *polynomial division*.)

Polynomial division is an important operation. The **quo** and **rem** commands find the quotient and remainder of a polynomial division. See Table 5.1. (The **iquo** and **irem** commands find the quotient and remainder of an integer division. For more information, see *Integer Operations (page 106)*.)

Table 5.1: Polynomial Arithmetic Operators

Operation	Operator	Example
Addition	$+$	> $(x^2+1)+(3x^3-5x+2)$ x^2+3+3x^3-5x
Subtraction	$-$	> $(x^2+1)-(3x^3-5x+2)$ x^2-1-3x^3+5x
Multiplication[1]	*	> $(x^2+1)\cdot(3x^3-5x+2)$ $(x^2+1)(3x^3-5x+2)$
Division: Quotient and Remainder	**quo** **rem**	> $quo(2x^2+x-3, 3x+5, x)$ $\frac{2}{3}x-\frac{7}{9}$ > $rem(2x^2+x-3, 3x+5, x)$ $\frac{8}{9}$
Exponentiation[2]	^	> $(x^2+1)^3$ $(x^2+1)^3$

[1]You can specify multiplication explicitly by entering *****, which displays in 2-D Math as $\cdot$. In 2-D Math, you can also implicitly multiply by placing a space character between two expressions. In some cases, the space character is optional. For example, Maple interprets a number followed by a name as an implicit multiplication.

[2]In 2-D Math, exponents display as superscripts.

To expand a polynomial, use the **expand** command.

> $expand\left(3x^2 \cdot (3x+5) - (x^2-2)\right)$

$$9x^3 + 14x^2 + 2$$

If you need to determine whether one polynomial divides another, but do not need the quotient, use the **divide** command. The **divide** command tests for exact polynomial division.

> $divide\left(x^4y^2+x^3y^2-x^2y^2+13x^2+13x-13+y\cdot x^2+x\cdot y-y, x^2+x-1\right)$

$$true$$

Important: You must insert a space character or a multiplication operator ($\cdot$) between adjacent variables names. Otherwise, they are interpreted as a single variable.

For example, x does not divide the single variable xy.

> $divide(xy, x)$

$$false$$

But, x divides the product of x and y.

> $divide(x\ y, x);\ divide(x\cdot y, x)$

$$true$$

$$true$$

For information on polynomial arithmetic over finite rings and fields, refer to the **?mod** help page.

Sorting Terms

To sort the terms of a polynomial, use the **sort** command.

> $p1 := x^2 + x^3 - x + x^4$

$$p1 := x^2 + x^3 - x + x^4$$

> $sort(p1)$

$$x^4 + x^3 + x^2 - x$$

Note: The **sort** command returns the sorted polynomial, and updates the order of the terms in the polynomial.

The terms of **p1** are sorted.

> $p1$

$$x^4 + x^3 + x^2 - x$$

To specify the unknowns of the polynomial and their ordering, include a list of names.

> $sort(a^2 x^3 + x^2 + x\,a + a + b, [a])$

$$x^3\,a^2 + x\,a + a + x^2 + b$$

> $sort(a^2 x^3 + x^2 + x\,a + a + b, [x, b])$

$$a^2\,x^3 + x^2 + a\,x + b + a$$

By default, the **sort** command sorts a polynomial by decreasing *total degree* of the terms.

> $p2 := x^3 + y^3 + x^2 y^2:$

> $sort(p2, [x, y])$

$$x^2\,y^2 + x^3 + y^3$$

The first term has total degree 4. The other two terms have total degree 3. The order of the final two terms is determined by the order of their names in the list.

To sort the terms by *pure lexicographic order*, that is, first by decreasing order of the first unknown in the list option, and then by decreasing order of the next unknown in the list option, specify the **'plex'** option.

> *sort*(*p2*, [*x*,*y*],'*plex*')

$$x^3 + x^2 y^2 + y^3$$

For information on enclosing keywords in right single quotes ('), see *Delaying Evaluation (page 366)*.

The first term contains x to the power 3; the second, x to the power 2; and the third, x to the power 0.

Using context menus, you can perform operations, such as sorting, for polynomials and many other Maple objects.

To sort a polynomial:

1. Right-click (**Control**-click, for Macintosh) the polynomial.

2. The context menu displays. From the **Sorts** menu, select:

- **Single-variable**, and then the unknown
- **Two-variable** (or **Three-variable**), **Pure Lexical** or **Total Degree**, and then the sort priority of the unknowns.

See Figure 5.1.

> $x^3 + y^3 + x^2y^2$

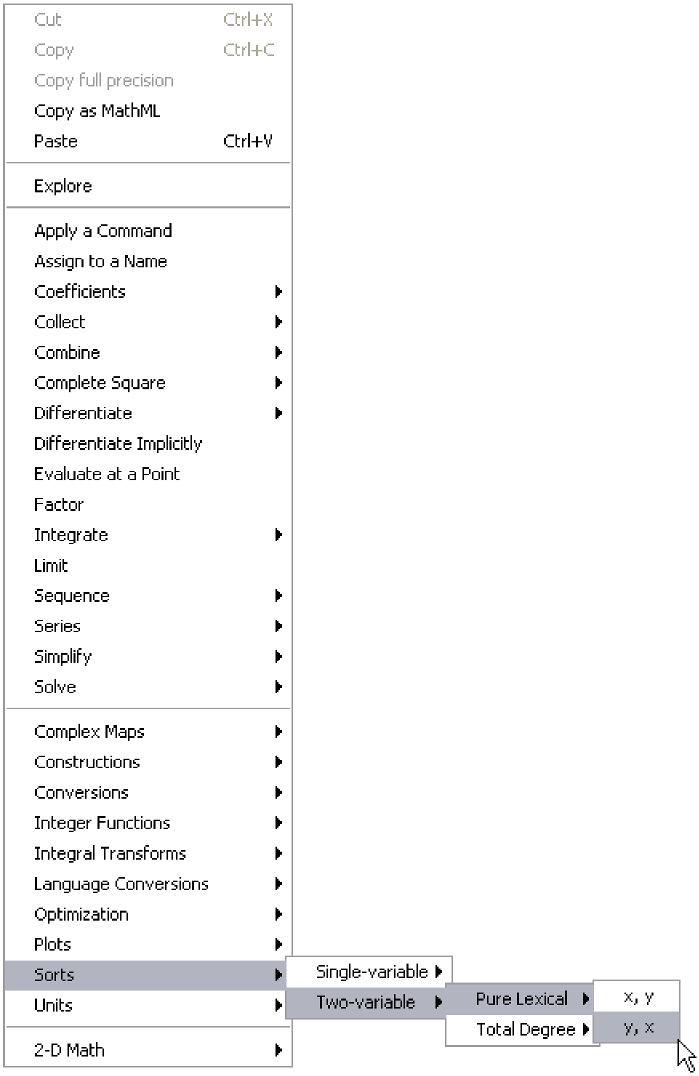


Figure 5.1: Sorting a Polynomial Using a Context Menu

Maple sorts the polynomial.

In Worksheet mode, Maple inserts the calling sequence that performs the sort followed by the sorted polynomial.

> $x^3 + y^3 + x^2y^2$:

> *sort*(x ^ 3+y ^ 3+x ^ 2 * y ^ 2 , [y , x], plex)

$$y^3 + y^2 x^2 + x^3$$

You can use context menus to perform operations on 2-D Math content including output. For more information, see *Context Menus (page 70)* (for Document mode) or *Context Menus (page 89)* (for Worksheet mode).

Collecting Terms

To collect the terms of polynomial, use the **collect** command.

> $collect\left(2\,a\,x\,y + c\,x^2\,y - z\,y^2 + a\,z - 13\,b\,y + \frac{3\,y^2}{x}, y\right)$

$$\left(-z + \frac{3}{x}\right) y^2 + \left(2\,a\,x + c\,x^2 - 13\,b\right) y + a\,z$$

Coefficients and Degrees

Maple has several commands that return coefficient and degree values for a polynomial. See Table 5.2.

Table 5.2: Polynomial Coefficient and Degree Commands

Command	Description	Example
coeff	Coefficient of specified degree term	> $coeff\left(\frac{1}{2}x^3 - 2\,x + 5, x^3\right)$ $\frac{1}{2}$
lcoeff	Leading coefficient	> $lcoeff\left(\frac{1}{2}x^3 - 2\,x + 5\right)$ $\frac{1}{2}$
tcoeff	Trailing coefficient	> $tcoeff\left(\frac{1}{2}x^3 - 2\,x + 5\right)$ 5

Command	Description	Example
coeffs	Sequence of all coefficients, in one-to-one correspondence with the terms **Note:** It does not return zero coefficients	> $coeffs\left(\frac{1}{2}x^3 - 2x + 5\right)$ $5, -2, \frac{1}{2}$
degree	(Highest) degree	> $degree\left(\frac{1}{2}x^3 - 2x + 5\right)$ 3
ldegree	Lowest degree term with a non-zero coefficient	> $ldegree\left(\frac{1}{2}x^3 - 2x\right)$ 1

Factorization

To express a polynomial in fully factored form, use the **factor** command.

> $factor(x^4 - 1)$

$$(x-1)\,(x+1)\,(x^2+1)$$

The **factor** command factors the polynomial over the ring implied by the coefficients, for example, integers. You can specify an algebraic number field over which to factor the polynomial. For more information, refer to the **?factor** help page. (The **ifactor** command factors an integer. For more information, see *Integer Operations (page 106)*.)

To solve for the roots of a polynomial, use the **solve** command. For information on the **solve** command, see *Solving Equations and Inequations (page 112)*. (The **isolve** command solves an equation for integer solutions. For more information, see *Integer Equations (page 126)*.)

Other Commands

Table 5.3 lists other commands available for polynomial operations.

Table 5.3: Select Other Polynomial Commands

Command	Description
content	Content (multivariate polynomial)
compoly	Decomposition

Command	Description
discrim	Discriminant
gcd	Greatest common divisor (of two polynomials)
gcdex	Extended Euclidean algorithm (for two polynomials)
CurveFitting[PolynomialInterpolation] See also the **CurveFitting Assistant** (**Tools → Assistants → Curve Fitting**)	Interpolating polynomial (for list of points)
lcm	Least common multiple (of two polynomials)
norm	Norm
EPROM	Pseudo-remainder (of two multivariate polynomials)
primpart	Primitive part (multivariate polynomial)
randpoly	Random polynomial
PolynomialTools[IsSelfReciprocal]	Determine whether self-reciprocal
resultant	Resultant (of two polynomials)
roots	Exact roots (over algebraic number field)
sqrfree	Square-free factorization (multivariate polynomial)

Additional Information

Table 5.4: Additional Polynomial Help

Topic	Resource
General polynomial information	**?polynom** help page
PolynomialTools package	**?PolynomialTools** package overview help page
Algebraic manipulation of numeric polynomials	**?SNAP** (Symbolic-Numeric Algorithms for Polynomials) package overview help page
Efficient arithmetic for sparse polynomials	**?SDMPolynom** (Sparse Distributed Multivariate Polynomial data structure) help page
Polynomial information and commands	Maple Help System Table of Contents: **Mathematics→Algebra→Polynomials** section

5.3 Linear Algebra

Linear algebra operations act on Matrix and Vector data structures.

You can perform many linear algebra operations using task templates. In the **Task Browser** (**Tools** → **Tasks** → **Browse**), expand the **Linear Algebra** folder.

Creating Matrices and Vectors

Creating Matrices

You can create a Matrix using

- The Matrix command
- The angle bracket shortcut notation
- The Matrix palette (see Figure 5.2).

When creating a Matrix using the Matrix command, there are several input formats available. For example, enter a list of lists. The dimensions of the matrix are inferred from the number of entries given.

$$> \mathit{Matrix}\left(\left[[1, \pi, 0], \left[e^2, \sin(t), \frac{87}{2}\right], [0, 0, 5e]\right]\right)$$

$$\begin{bmatrix} 1 & \pi & 0 \\ e^2 & \sin(t) & \frac{87}{2} \\ 0 & 0 & 5\,e \end{bmatrix}$$

Alternatively, use the angle bracket shortcut, <>. Separate items in a column with commas, and separate columns with vertical bars, |.

$$> \left\langle 1, \pi, 0 \middle| e^2, \sin(t), \frac{87}{2} \middle| 0, 0, 5e \right\rangle$$

$$\begin{bmatrix} 1 & e^2 & 0 \\ \pi & \sin(t) & 0 \\ 0 & \frac{87}{2} & 5\,e \end{bmatrix}$$

For information on the Matrix command options, see *Creating Matrices and Vectors with Specific Properties (page 164)*.

Use the Matrix palette to interactively create a matrix without commands:

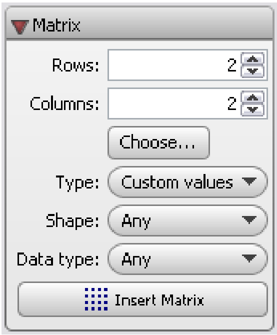


Figure 5.2: Matrix Palette

In the **Matrix** palette, you can specify the matrix size (see Figure 5.3) and properties. To insert a matrix, click the **Insert Matrix** button.

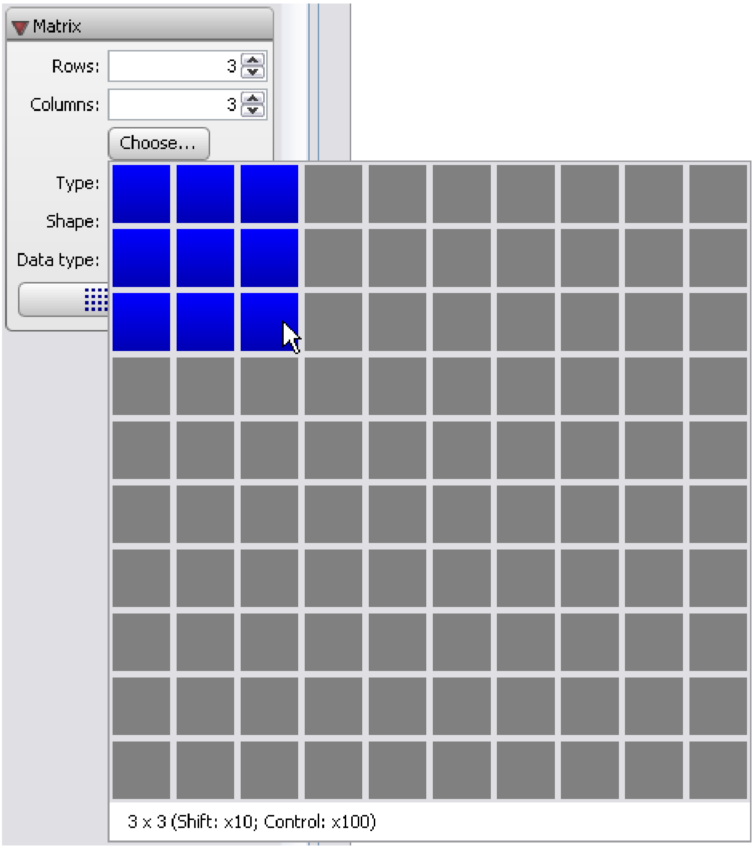


Figure 5.3: Matrix Palette: Choosing the Size

After inserting the matrix:

1. Enter the values of the entries. To move to the next entry placeholder, press **Tab**.

2. After specifying all entries, press **Enter**.

$$> \begin{bmatrix} 1 & e^2 & 0 \\ \pi & \sin(t) & 0 \\ 0 & \frac{87}{2} & 5e \end{bmatrix} :$$

Creating Vectors

You can create a Vector using angle brackets (< >).

To create a column vector, specify a comma-delimited sequence, **<a, b, c>**. The number of elements is inferred from the number of expressions.

> $\langle 1, 2, 3 \rangle$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

To create a row vector, specify a vertical-bar-delimited (|) sequence, **<a | b | c>**. The number of elements is inferred from the number of expressions.

> $\langle 1 \mid 2 \mid 3 \rangle$

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

For information on the Vector command options, refer to the **?Vector** help page.

You can also create vectors using the **Matrix** palette. If either the number of rows or number of columns specified is 1, then you have the option of inserting a matrix, or inserting a vector of the appropriate type. See Figure 5.4.

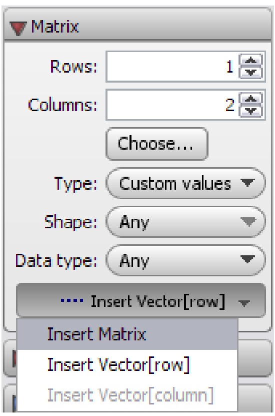


Figure 5.4: Insert Matrix or Insert Vector

Viewing Large Matrices and Vectors

Matrices 10×10 and smaller, and vectors with 10 or fewer elements, display in the document. Larger objects are displayed as a placeholder.

For example, insert a 15×15 matrix.

In the Matrix palette:

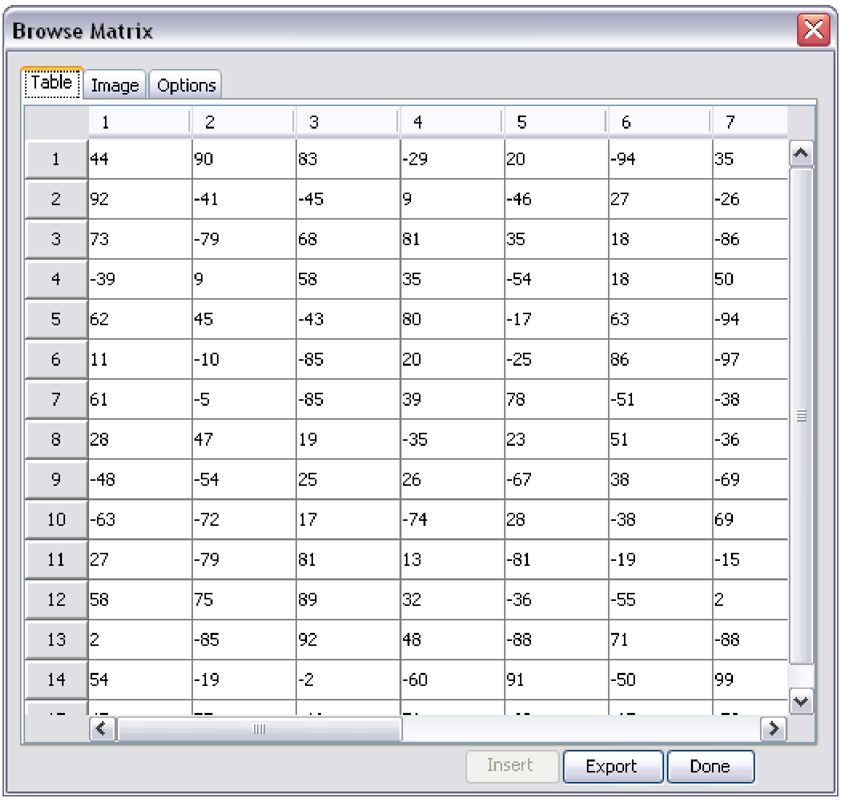
1. Specify the dimensions: 15 rows and 15 columns.

2. In the **Type** drop-down list, select a matrix type, for example, **Random**.

3. Click **Insert Matrix**. Maple inserts a placeholder.

$$> \left[\begin{array}{c} \textit{15 x 15 Matrix} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{array}\right]:$$

To edit or view a large matrix or vector, double-click the placeholder. This launches the Matrix Browser. See Figure 5.5.



	1	2	3	4	5	6	7
1	44	90	83	-29	20	-94	35
2	92	-41	-45	9	-46	27	-26
3	73	-79	68	81	35	18	-86
4	-39	9	58	35	-54	18	50
5	62	45	-43	80	-17	63	-94
6	11	-10	-85	20	-25	86	-97
7	61	-5	-85	39	78	-51	-38
8	28	47	19	-35	23	51	-36
9	-48	-54	25	26	-67	38	-69
10	-63	-72	17	-74	28	-38	69
11	27	-79	81	13	-81	-19	-15
12	58	75	89	32	-36	-55	2
13	2	-85	92	48	-88	71	-88
14	54	-19	-2	-60	91	-50	99

Figure 5.5: Matrix Browser

To modify the entries using the Matrix Browser:

1. Select the **Table** tab.

2. Double-click an entry, and then edit its value. Press **Enter**.

3. Repeat for each entry to edit.

4. When you have finished updating entries, click **Done**.

You can view the matrix or vector as a table or as an image, which can be inserted into the document. For more information, refer to the **?MatrixBrowser** help page.

To set the maximum dimension of matrices and vectors displayed inline:

- Use the **interface** command with the **rtablesize** option.

For example, **interface(rtablesize = 15)**.

For more information, refer to the **?interface** help page.

Creating Matrices and Vectors with Specific Properties

By default, matrices and vectors can store any values. To increase the efficiency of linear algebra computations, create matrices and vectors with properties. You must specify the properties, for example, the matrix shape or data type, when defining the object.

The **Matrix** palette (Figure 5.2) supports several properties.

To specify the matrix type:

- Use the **Shape** and **Type** drop-down lists.

To specify the data type:

- Use the **Data type** drop-down list.

For example, define a diagonal matrix with small integer coefficients.

In the Matrix palette:

1. Specify the size of the matrix, for example, 3×3 .

2. In the **Shapes** drop-down list, select **Diagonal**.

3. In the **Data type** drop-down list, select **integer[1]**.

4. Click the **Insert Matrix** button.

5. Enter the values in the diagonal entries.

> $\begin{bmatrix} -23 & 0 & 0 \\ 0 & 17 & 0 \\ 0 & 0 & 32 \end{bmatrix}$:

You cannot specify properties when defining vectors using the angle-bracket notation. You must use the **Vector** constructor.

To define a column vector using the Vector constructor, specify:

- The number of elements. If you explicitly specify all element values, this argument is not required.
- A list of expressions that define the element values.
- Parameters such as **shape**, **datatype**, and **fill** that set properties of the vector.

The following two calling sequences are equivalent.

> *Vector*([0, 0, 0])

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

> *Vector*(3, '*shape*' = '*zero*')

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

To create a row vector using the **Vector** constructor, include **row** as an index.

> *Vector*[*row*](3,'*fill*'= 1)

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

> *Vector*[*row*]([127, 0, 34], '*datatype*' = '*integer*[1]')

$$\begin{bmatrix} 127 & 0 & 34 \end{bmatrix}$$

The **Matrix** palette does not support some properties. To set all properties, use the **Matrix** constructor.

To define a matrix using the Matrix constructor, specify:

- The number of rows and columns. If you explicitly specify all element values, these arguments are not required.
- A list of lists that define the element values row-wise.
- Parameters such as **shape**, **datatype**, and **fill** that set properties of the matrix.

For example:

> *Matrix*([[1, 2, 3], [4, 5, 6]])

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

The **Matrix** palette cannot fill the matrix with an arbitrary value. Use the **fill** parameter.

> *Matrix*(3, 4, [[1, 2, 3], [4, 5, 6]], '*fill*' = 2 + *I*)

$$\begin{bmatrix} 1 & 2 & 3 & 2+\mathrm{I} \\ 4 & 5 & 6 & 2+\mathrm{I} \\ 2+\mathrm{I} & 2+\mathrm{I} & 2+\mathrm{I} & 2+\mathrm{I} \end{bmatrix}$$

For more information on the constructors, including other calling sequence syntaxes and parameters, refer to the **?storage**, **?Matrix**, and **?Vector** help pages.

See also *Numeric Computations (page 174)*.

Accessing Entries in Matrices and Vectors

Matrices

To select an entry in a Matrix, enter the matrix name with a sequence of two non-zero integer indices, row first.

> $M := \langle -4.3, -6.7, 1.9 | 2.9, -1.2, 9.6 | 9.3, -8.0, -9.2 \rangle$

$$M := \begin{bmatrix} -4.3 & 2.9 & 9.3 \\ -6.7 & -1.2 & -8.0 \\ 1.9 & 9.6 & -9.2 \end{bmatrix}$$

> $M[1, 3]$

$$9.3$$

To select an entire row, enter a single index; to select an entire column, enter first the entire range of rows, $1 .. -1$, then the column index.

> $M[2]$

$$\begin{bmatrix} -6.7 & -1.2 & -8.0 \end{bmatrix}$$

> $M[1 .. -1, 1]$

$$\begin{bmatrix} -4.3 \\ -6.7 \\ 1.9 \end{bmatrix}$$

Similarly, you can access submatrices. Enter the indices as a list or range.

> $M[2 .. 3, 1 .. 2]$

$$\begin{bmatrix} -6.7 & -1.2 \\ 1.9 & 9.6 \end{bmatrix}$$

Vectors

To select an entry in a vector, enter the vector name with a non-zero integer index.

> $a := <85.3, 47.1, 59.9, 38.1>$

$$a := \begin{bmatrix} 85.3 \\ 47.1 \\ 59.9 \\ 38.1 \end{bmatrix}$$

> $a[1]$

$$85.3$$

Negative integers select entries from the end of the vector.

> $a[-1]$

$$38.1$$

To create a Vector consisting of multiple entries, specify a list or range of integers in the index. For more information, refer to the **?set** and **?range** help pages.

> $a[[1, 2]]$

$$\begin{bmatrix} 85.3 \\ 47.1 \end{bmatrix}$$

> $a[2..4]$

$$\begin{bmatrix} 47.1 \\ 59.9 \\ 38.1 \end{bmatrix}$$

Linear Algebra Computations

Maple has extensive support for linear algebra. You can perform many matrix and vector computations using context menus. Matrix operations such as multiplication and inverses can be done with the basic matrix arithmetic operators. The **LinearAlgebra** package provides the full range of Maple commands for linear algebra and vector space computations, queries, and linear system solving.

Matrix Arithmetic

The matrix and vector arithmetic operators are the standard Maple arithmetic operators up to the following two differences.

- The scalar multiplication operator is the asterisk (*), which displays in 2-D Math as $\cdot$. The noncommutative matrix and vector multiplication operator is the period (.).
- There is no division operator (/) for matrix algebra. (You can construct the inverse of a matrix using the exponent -1 .)

Table 5.5 lists the basic matrix operators.

> $A := \begin{bmatrix} 93 & 43 \\ 19 & 37 \end{bmatrix} : B := \begin{bmatrix} 48 & 20 \\ 19 & 37 \end{bmatrix} : C := \langle 23, 6 \rangle :$

Table 5.5: Matrix and Vector Arithmetic Operators

Operation	Operator	Example
Addition	$+$	> $A + B$ $\begin{bmatrix} 141 & 63 \\ 38 & 74 \end{bmatrix}$
Subtraction	$-$	> $A - B$ $\begin{bmatrix} 45 & 23 \\ 0 & 0 \end{bmatrix}$
Multiplication	$\cdot$	> $A.C$ $\begin{bmatrix} 2397 \\ 659 \end{bmatrix}$

Operation	Operator	Example
Scalar Multiplication[1]	*	> $12\,A$ $$\begin{bmatrix} 1116 & 516 \\ 228 & 444 \end{bmatrix}$$ > $4 \cdot C$ $$\begin{bmatrix} 92 \\ 24 \end{bmatrix}$$
Exponentiation[2]	^	> A^3 $$\begin{bmatrix} 986548 & 613868 \\ 271244 & 187092 \end{bmatrix}$$ > B^{-1} $$\begin{bmatrix} \frac{37}{1396} & -\frac{5}{349} \\ -\frac{19}{1396} & \frac{12}{349} \end{bmatrix}$$

[1]You can specify scalar multiplication explicitly by entering *, which displays in 2-D Math as $\cdot$. In 2-D Math, you can also implicitly multiply a scalar and a matrix or vector by placing a space character between them. In some cases, the space character is optional. For example, Maple interprets a number followed by a name as an implicit multiplication.

[2]In 2-D Math, exponents display as superscripts.

A few additional matrix and vector operators are listed in Table 5.6.

Define two column vectors.

> $d := \langle 1, 2, 3 \rangle : e := \langle 4, 5, 6 \rangle :$

Table 5.6: Select Matrix and Vector Operators

Operation	Operator	Example
Transpose	**^%T**[1]	> $d^{\%T}$ $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$
Hermitian Transpose	**^%H**[1]	> $\begin{bmatrix} I & -2I \\ 3+4I & 2-I \end{bmatrix}^{\%H}$ $\begin{bmatrix} -I & 3-4I \\ 2I & 2+I \end{bmatrix}$
Cross Product (3-D vectors only)	**&x**[2]	> *with*(*LinearAlgebra*): > *d* &x *e* $\begin{bmatrix} -3 \\ 6 \\ -3 \end{bmatrix}$

[1]Exponential operators display in 2-D Math as superscripts.

[2]After loading the **LinearAlgebra** package, the cross product operator is available as the infix operator **&x** . Otherwise, it is available as the **LinearAlgebra[CrossProduct]** command.

For information on matrix arithmetic over finite rings and fields, refer to the **?mod** help page.

Point-and-Click Interaction

Using context menus, you can perform many matrix and vector operations.

Matrix operations available in the context menu include the following.

- Perform standard operations: determinant, inverse, norm (1, Euclidean, infinity, or Frobenius), transpose, and trace
- Compute eigenvalues, eigenvectors, and singular values
- Compute the dimension or rank
- Convert to the Jordan form, or other forms
- Perform Cholesky decomposition and other decompositions

For example, compute the infinity norm of a matrix. See Figure 5.6.

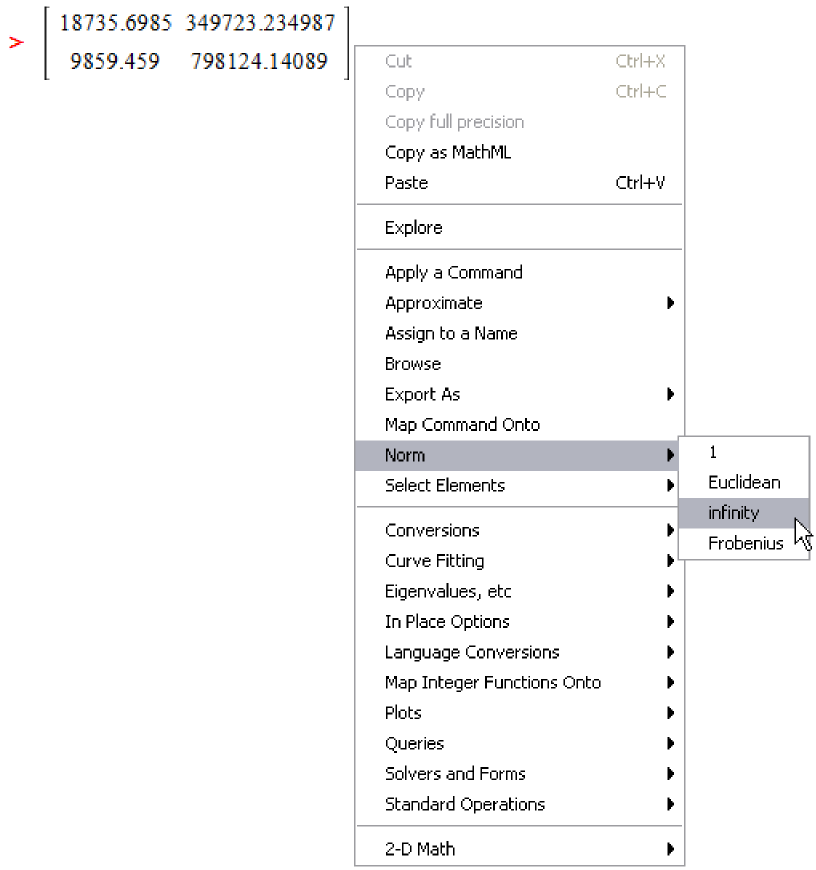


Figure 5.6: Computing the Infinity Norm of a Matrix

In Document mode, Maple inserts a right arrow and the name of the computation performed, followed by the norm.

$$\begin{bmatrix} 18735.6985 & 349723.234987 \\ 9859.459 & 798124.14089 \end{bmatrix} \xrightarrow{\text{infinity-norm}} 8.0798359990\ 10^{5}$$

Vector operations available in the context menu include the following.

- Compute the dimension
- Compute the norm (1, Euclidean, and infinity)
- Compute the transpose

- Select an element

For more information on context menus, see *Context Menus (page 70)* (for Document mode) or *Context Menus (page 89)* (for Worksheet mode).

LinearAlgebra Package Commands

The **LinearAlgebra** package contains commands that construct and manipulate matrices and vectors, compute standard operations, perform queries, and solve linear algebra problems.

Table 5.7 lists some **LinearAlgebra** package commands. For a complete list, refer to the **?LinearAlgebra/Details** help page.

Table 5.7: Select LinearAlgebra Package Commands

Command	Description
Basis	Return a basis for a vector space
CrossProduct	Compute the cross product of two vectors
DeleteRow	Delete a row or rows of a matrix
Dimension	Determine the dimension of a matrix or a vector
Eigenvalues	Compute the eigenvalues of a matrix
Eigenvectors	Compute the eigenvectors of a matrix
FrobeniusForm	Reduce a matrix to Frobenius form
GaussianElimination	Perform Gaussian elimination on a matrix
HessenbergForm	Reduce a square matrix to Hessenberg form
HilbertMatrix	Construct a generalized Hilbert matrix
IsOrthogonal	Test if a matrix is orthogonal
LeastSquares	Compute the least-squares approximation to **A . x = b**
LinearSolve	Solve the linear system **A . x = b**
MatrixInverse	Compute the inverse of a square matrix or pseudo-inverse of a non-square matrix
QRDecomposition	Compute the QR factorization of a matrix
RandomMatrix	Construct a random matrix
SylvesterMatrix	Construct the Sylvester matrix of two polynomials

For information on arithmetic operations, see *Matrix Arithmetic (page 169)*.

For information on selecting entries, subvectors, and submatrices, see *Accessing Entries in Matrices and Vectors (page 166)*.

Example: Determine a basis for the space spanned by the set of vectors **{(2, 13, -15), (7, -2, 13), (5, -4, 9)}**. Express the vector **(25, -4, 9)** with respect to this basis.

> *with*(*LinearAlgebra*):

> $v1 := <2, 13, -15>: v2 := <7, -2, 13>: v3 := <5, -4, 9>:$

Find a basis for the vector space spanned by these vectors, and then construct a matrix from the basis vectors.

> $basis := Matrix(Basis([v1, v2, v3]));$

$$basis := \begin{bmatrix} 2 & 7 & 5 \\ 13 & -2 & -4 \\ -15 & 13 & 9 \end{bmatrix}$$

To express **(25, -4, 9)** in this basis, use the **LinearSolve** command**.**

> $LinearSolve(basis, <25, -4, 9>)$

$$\begin{bmatrix} \frac{170}{91} \\ -\frac{285}{91} \\ \frac{786}{91} \end{bmatrix}$$

Numeric Computations

You can very efficiently perform computations on large matrices and vectors that contain floating-point data using the built-in library of numeric linear algebra routines. Some of these routines are provided by the Numerical Algorithms Group (NAG®). Maple also contains portions of the CLAPACK and optimized ATLAS libraries.

For information on performing efficient numeric computations using the **LinearAlgebra** package, refer to the **?EfficientLinearAlgebra** help page.

See also *Creating Matrices and Vectors with Specific Properties (page 164)* and *Reading from Files (page 414)*.

Student LinearAlgebra Package

The **Student** package contains subpackages that help instructors teach concepts and allow students to visualize and explore ideas. These subpackages also contain computational commands.

In the **Student[LinearAlgebra]** subpackage, the environment differs from that of the **LinearAlgebra** package in that floating-point computations are generally performed using software precision, instead of hardware precision, and symbols are generally assumed to represent real, rather than complex, quantities. These defaults, and others, can be controlled using the **SetDefault** command. For more information, refer to the **?Student[LinearAlgebra][SetDefault]** help page.

For information on using Maple as a teaching and learning tool, see *Teaching and Learning with Maple (page 198)*.

5.4 Calculus

The **Task Browser** (**Tools→Tasks→Browse**) contains numerous calculus task templates. For a list of tasks, navigate to one of the related folders, such as **Calculus**, **Differential Equations**, **Multivariate Calculus**, or **Vector Calculus**.

This section describes the key Maple calculus commands, many of which are used in task templates or available in the context menus.

For a complete list of calculus commands, refer to the **Mathematics** (including **Calculus**, **Differential Equations**, **Power Series**, and **Vector Calculus** subfolders) and **Student Package** sections of the Maple Help System Table of Contents.

Limits

To compute the limit of an expression as the independent variable approaches a value:

1. In the **Expression** palette, click the limit item $\lim_{x \to a} f$.

2. Specify the independent variable, limit point, and expression, and then evaluate it. Press **Tab** to move to the next placeholder.

For example:

$$> \lim_{x \to 0} \left(\frac{x}{\sin(x)} \right)$$

$$1$$

The limit Command

By default, Maple searches for the real bidirectional limit (unless the limit point is ∞ or -∞). To specify a direction, include one of the options **left**, **right**, **real**, or **complex** in a call to the **limit** command. See Table 5.8.

Table 5.8: Limits

Limit	Command Syntax	Output
$\lim_{x \to 0} \left(\frac{1}{x}\right)$	> $limit\left(\frac{1}{x}, x=0\right)$	*undefined*
$\lim_{x \to 0+} \left(\frac{1}{x}\right)$	> $limit\left(\frac{1}{x}, x=0, 'right'\right)$	∞
$\lim_{x \to 0-} \left(\frac{1}{x}\right)$	> $limit\left(\frac{1}{x}, x=0, 'left'\right)$	$-\infty$

Using the **limit** command, you can also compute multidimensional limits.

> $limit\left(\frac{x^2}{y}, \{x=1, y=\infty\}\right)$

$$0$$

For more information on multidimensional limits, refer to the **?limit/multi** help page.

Numerically Computing a Limit

To numerically compute a limit:

- Use the **evalf(Limit(arguments))** calling sequence.

Important: Use the inert **Limit** command, not the **limit** command. For more information, refer to the **?limit** help page.

The **Limit** command accepts the same arguments as the **limit** command.

For example:

> $evalf\left(Limit\left(\frac{sin(x)}{cos(x) + tan(x)}, x=1.225\right)\right)$

$$0.3020605357$$

For information on the **evalf** command, see *Numerical Approximation (page 361)*.

The **Limit** command does not compute the limit. It returns an unevaluated limit.

$> \mathit{Limit}\left(\frac{\mathit{sin}(x)}{\mathit{cos}(x)+\mathit{tan}(x)}, x=1.225\right)$

$$\lim_{x\to 1.225} \frac{\sin(x)}{\cos(x)+\tan(x)}$$

For more information on the **Limit** command, refer to the **?Limit** help page.

Differentiation

Maple can perform symbolic and numeric differentiation.

To differentiate an expression:

1. In the **Expression** palette, click the differentiation item $\frac{d}{dx} f$ or the partial differentiation item $\frac{\partial}{\partial x} f$.

2. Specify the expression and independent variable, and then evaluate it.

For example, to differentiate $x \sin(a x)$ with respect to x:

$> \frac{d}{dx}(x \sin(a x))$

$$\sin(a x) + x \cos(a x)\, a$$

You can also differentiate using context menus. For more information, see *Context Menus (page 39)*.

To calculate a higher order or partial derivative, edit the derivative symbol inserted. For example, to calculate the second derivative of $x \sin(a x) + x^2$ with respect to x:

$> \frac{d^2}{dx^2}\left(x \sin(a x) + x^2\right)$

$$2 \cos(a x)\, a - x \sin(a x)\, a^2 + 2$$

To calculate the mixed partial derivative of $x \sin(3 y) + y x^5$:

$$> \frac{\partial^2}{\partial y \, \partial x} \left(x \sin(3y) + y x^5 \right)$$

$$3\cos(3y) + 5x^4$$

Note: To enter another ∂ symbol, you can copy and paste the existing symbol, or enter d and use symbol completion.

The diff Command

Maple computes derivatives using the **diff** command. To directly use the **diff** command, specify the expression to differentiate and the variable.

$$> x\sin(ax) + x^2$$

$$x\sin(ax) + x^2 \qquad (5.1)$$

$$> \mathit{diff}(\mathbf{(5.1)}, x)$$

$$\sin(ax) + x\cos(ax)\,a + 2x \qquad (5.2)$$

For information on equation labels such as (5.1), see *Equation Labels (page 97)*.

You can calculate a higher order derivative by specifying a sequence of differentiation variables. Maple recursively calls the **diff** command.

$$> \mathit{diff}(\mathbf{(5.1)}, x, x)$$

$$2\cos(ax)\,a - x\sin(ax)\,a^2 + 2 \qquad (5.3)$$

To calculate a partial derivative, use the same syntax. Maple assumes that the derivatives commute.

$$> \mathit{diff}\left(x \sin(3y) + y\sqrt{x}, x, y\right)$$

$$3\cos(3y) + \frac{1}{2\sqrt{x}}$$

To enter higher order derivatives, it is convenient to use the syntax **diff(f, x$n)**. This syntax can also be used to compute the symbolic nth order derivative.

For example:

> $diff(\cos(t), t\$n)$

$$\cos\left(t + \frac{1}{2} n \pi\right)$$

Differentiating an Operator

You can also specify a mathematical function as a *functional operator* (a mapping). For a comparison of operators and other expressions, see *Distinction between Functional Operators and Other Expressions (page 344)*.

To find the derivative of a functional operator:

- Use the **D** operator.

The **D** operator returns a functional operator.

For example, find the derivative of an operator that represents the mathematical function $F : x \to x\cos(x)$.

First, define the operator *F*.

1. In the **Expression** palette, click the single-variable function definition item $f := a \to y$.

2. Enter placeholder values.

- To move to the next placeholder, press the **Tab** key. **Note:** If pressing the **Tab** key inserts a tab, click the Tab icon in the toolbar.

> $F := x \to x\cos(x):$

Now, define the operator, *G*, that maps x to the derivative of $x\cos(x)$.

> $G := D(F)$

$$G := x \to \cos(x) - x\sin(x)$$

F and *G* evaluated at $\frac{\pi}{2}$ return the expected values.

> $F\left(\frac{\pi}{2}\right); G\left(\frac{\pi}{2}\right)$

$$0$$

$$-\frac{1}{2}\pi$$

For more information on the **D** operator, refer to the **?D** help page. For a comparison of the **diff** command and **D** operator, refer to the **?diffVersusD** help page.

Directional Derivative

To compute and plot a directional derivative, use the **Directional Derivative Tutor**. The tutor computes a floating-point value for the directional derivative.

To launch the tutor:

From the **Tools** menu, select **Tutors**, **Calculus - Multi-Variable**, and then **Directional Derivatives**. Maple launches the **Directional Derivative Tutor**. See Figure 5.7.

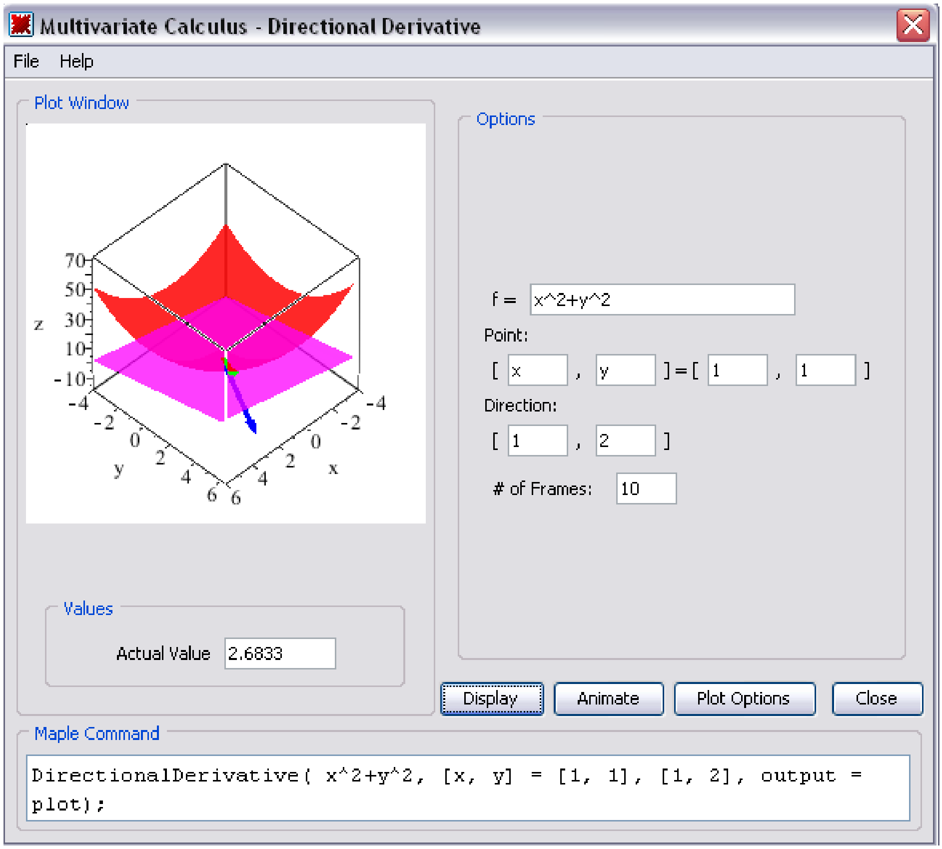


Figure 5.7: Directional Derivative Tutor

To compute a symbolic value for the directional derivative, use the **Student[Multivariate-Calculus][DirectionalDerivative]** command. The first list of numbers specifies the point at which to compute the derivative. The second list of numbers specifies the direction in which to compute the derivative.

For example, at the point $[1, 2]$, the gradient of $x^2 + y^2$ points in the direction $[2, 4]$, which is the direction of greatest increase. The directional derivative in the orthogonal direction $[-2, 1]$ is zero.

> $with(Student[MultivariateCalculus])$:

> $DirectionalDerivative(x^2+y^2, [x,y]=[1,2], [1,2]);$

$$2\sqrt{5}$$

> $DirectionalDerivative(x^2+y^2, [x,y]=[1,2], [-2,1]);$

$$0$$

Series

To generate the Taylor series expansion of a function about a point, use the **taylor** command.

> $taylor(\sin(4x)\cos(x), x=0)$

$$4x - \frac{38}{3}x^3 + \frac{421}{30}x^5 + O(x^6)$$

Note: If a Taylor series does not exist, use the **series** command to find a general series expansion.

For example, the cosine integral function does not have a taylor series expansion about 0. For more information, refer to the **?Ci** help page.

> $taylor(Ci(x), x=0)$

```
Error, does not have a taylor expansion, try series()
```

To generate a truncated series expansion of a function about a point, use the **series** command.

> $series(Ci(x), x=0)$

$$\gamma + \ln(x) - \frac{1}{4}x^2 + \frac{1}{96}x^4 + O(x^6)$$

By default, Maple performs series calculations up to order 6. To use a different order, specify a non-negative integer third argument.

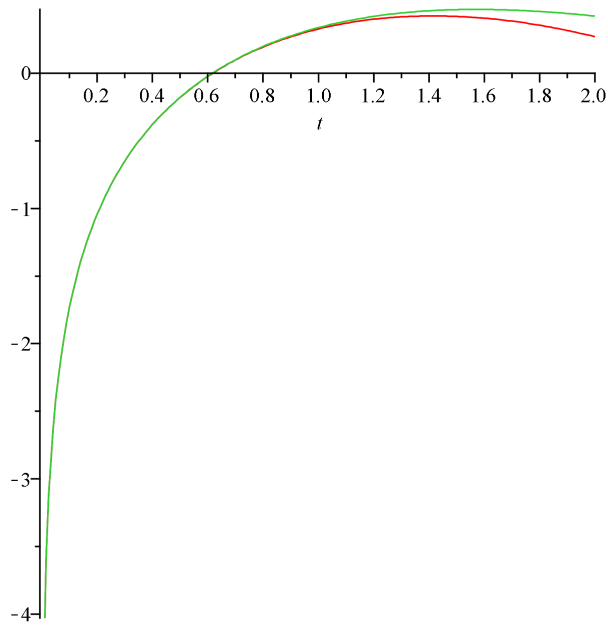
> $expansion := series(Ci(t), t=0, 4)$

$$expansion := \gamma + \ln(t) - \frac{1}{4}t^2 + O(t^4)$$

To set the order for all computations, use the **Order** environment variable. For information about the **Order** variable and the $O(t^4)$ term, refer to the **?Order** help page.

The expansion is of type **series**. Some commands, for example, **plot**, do not accept arguments of type **series**. To use the expansion, you must convert it to a polynomial using the **convert/polynom** command.

> $plot\left(\{Ci(t), convert(expansion, polynom)\}, t = \frac{1}{100}..2\right)$



For information on Maple types and type conversions, see *Maple Expressions (page 337)*.

For information on plotting, see *Plots and Animations (page 237)*.

Integration

Maple can perform symbolic and numeric integration.

To compute the *indefinite* integral of an expression:

1. In the **Expression** palette, click the indefinite integration item .
2. Specify the integrand and variable of integration, and then evaluate it.

For example, to integrate $x \sin(a\,x)$ with respect to x:

> $\int x \sin(a\,x)\,\mathrm{d}x$

$$\frac{\sin(a\,x) - x\cos(a\,x)\,a}{a^2}$$

Recall that you can also enter symbols, including $\int$ and d, using symbol completion.

- Enter the symbol name (or part of the name), for example, **int** or **d**, and then press the completion shortcut key.

For more information, see *Symbol Names (page 29)*.

You can also compute an indefinite integral using context menus. For more information, see *Context Menus (page 39)*.

To compute the definite integral of an expression:

1. In the **Expression** palette, click the definite integration item $\int_a^b f\,\mathrm{d}x$.

2. Specify the endpoints of the interval of integration, integrand expression, and variable of integration, and then evaluate it.

For example, to integrate $\mathrm{e}^{-a\,t}\ln(t)$ over the interval $(0, \infty)$:

> $\int_0^\infty \mathrm{e}^{-a\,t}\ln(t)\,\mathrm{d}t$

$$\lim_{t\to\infty}\left(-\frac{\mathrm{e}^{-a\,t}\ln(t) + \mathrm{Ei}(1, a\,t) + \gamma + \ln(a)}{a}\right)$$

Maple treats the parameter **a** as a complex number. As described in *Assumptions on Variables (page 144)*, you can compute under the assumption that **a** is a positive, real number using the **assuming** command.

> $\int_0^\infty \mathrm{e}^{-a\,t}\ln(t)\,\mathrm{d}t$ assuming $a > 0$

$$-\frac{\gamma + \ln(a)}{a}$$

To compute iterated integrals, line integrals, and surface integrals, use the task templates (**Tools** → **Tasks** → **Browse**) in the **Multivariate** and **Vector Calculus** folders.

The int Command

$\int f\,dx$ and $\int_a^b f\,dx$ use the **int** command. To use the **int** command directly, specify the following arguments.

- Expression to integrate
- Variable of integration

> $x \sin(a x)$

$$x \sin(a x) \tag{5.4}$$

> $int(\mathbf{(5.4)}, x)$

$$\frac{\sin(a x) - x \cos(a x)\, a}{a^2} \tag{5.5}$$

For a definite integration, set the variable of integration equal to the interval of integration.

> $int\left(\mathbf{(5.4)}, x = 0 .. \frac{\pi}{a}\right)$

$$\frac{\pi}{a^2} \tag{5.6}$$

Numeric Integration

To perform numeric integration:

- Use the **evalf(Int(arguments))** calling sequence.

Important: Use the inert **Int** command, not the **int** command. For more information, refer to the **?int** help page.

In addition to the arguments accepted by the **int** command, you can include optional arguments such as **method**, which specifies the numeric integration method.

> $evalf\left(Int\left(\frac{1}{\Gamma(x)}, x=0..2, 'method'=\_Dexp\right)\right)$

$$1.626378399$$

Note: To enter an underscore character (_) in 2-D Math, enter _.

For information on the **evalf** command, see *Numerical Approximation (page 361)*.

For information on numeric integration, including iterated integration and controlling the algorithm, refer to the **?evalf/Int** help page.

Differential Equations

Maple has a powerful set of solvers for ordinary differential equations (ODEs) and partial differential equations (PDEs), and systems of ODEs and PDEs.

For information on solving ODEs and PDEs, see *Other Specialized Solvers (page 121)*.

Calculus Packages

In addition to top-level calculus commands, Maple contains calculus packages.

VectorCalculus Package

The **VectorCalculus** package contains commands that perform multivariate and vector calculus operations on **VectorCalculus** vectors (vectors with an additional coordinate system attribute) and vector fields (vectors with additional coordinate system and **vectorfield** attributes), for example, **Curl**, **Flux**, and **Torsion**.

> $with(VectorCalculus)$:

> $BasisFormat(false)$:

> $SetCoordinates('cartesian[x, y, z]')$:

> $VectorField1 := VectorField(\langle -y, x, z \rangle)$

$$VectorField1 := \begin{bmatrix} -y \\ x \\ z \end{bmatrix}$$

Note: For information on changing the display format in the VectorCalculus package, see the **?VectorCalculus[BasisFormat]** help page.

Find the curl of **VectorField1**.

> *Curl*(*VectorField1*);

$$\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

Find the flux of **VectorField1** through a sphere of radius **r** at the origin.

> *Flux*(*VectorField1*, *Sphere*(⟨0, 0, 0⟩, *r*))

$$\frac{4}{3} r^3 \pi$$

Compute the torsion of a space curve. The curve must be a vector with parametric function components.

> *simplify*(*Torsion*(⟨*t*, *t*^2, *t*^3⟩, *t*)) *assuming t*::*real*

$$\frac{3}{9 t^4 + 9 t^2 + 1}$$

For information on the **assuming** command, see *The assuming Command (page 146)*.

For more information on the **VectorCalculus** package, including a complete list of commands, refer to the **?VectorCalculus** help page.

To find other calculus packages, such as **VariationalCalculus**, refer to the **?index/package** help page.

Student Calculus Packages

The **Student** package contains subpackages that help instructors teach concepts and allow students to visualize and explore ideas. These subpackages also contain computational commands. The **Student** calculus subpackages include **Calculus1**, **MultivariateCalculus**, and **VectorCalculus**. The **Student[VectorCalculus]** package provides a simple interface to a limited subset of the functionality available in the **VectorCalculus** package.

For information on using Maple as a teaching and learning tool, and some computational examples, see *Teaching and Learning with Maple (page 198)*.

5.5 Optimization

Using the **Optimization** package, you can numerically solve optimization problems. The package uses fast Numerical Algorithms Group (NAG) algorithms to *minimize* or *maximize* an objective function.

The **Optimization** package solves constrained and unconstrained problems.

- Linear programs
- Quadratic programs
- Nonlinear programs
- Linear and nonlinear least-squares problems

The **Optimization** package contains local solvers. In addition, for univariate finitely-bounded nonlinear programs with no other constraints, you can compute global solutions using the **NLPSolve** command. To find global solutions generally, purchase the **Global Optimization Toolbox**. For more information, visit **http://www.maplesoft.com/products/toolboxes**.

Point-and-Click Interface

The primary method for solving optimization problems is the **Optimization Assistant**.

To launch the Optimization Assistant:

- From the **Tools** menu, select **Assistants**, and then **Optimization**.

Maple launches the **Optimization Assistant**. See Figure 5.8.

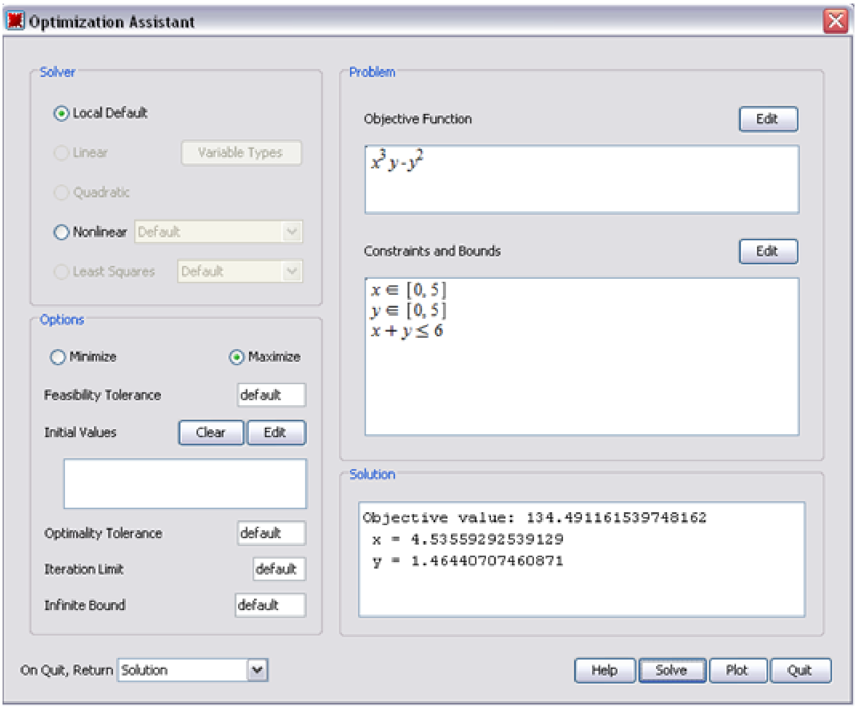


Figure 5.8: Optimization Assistant

To solve a problem:

1. Enter the objective function, constraints, and bounds.

2. Select the **Minimize** or **Maximize** radio button.

3. Click the **Solve** button. The solution is displayed in the **Solution** text box.

You can also enter the problem (objective function, constraints, and bounds) in the calling sequence of the **Optimization[Interactive]** command.

For example, find the maximum value of $x^3y - y^2$ subject to the constraints $x + y \leq 6, x \in [0,5], y \in [0,5]$.

> $Optimization[Interactive]\left(x^3\, y - y^2, \{x + y \leq 6, x = 0\,..5, y = 0\,..5\}\right)$

$$[134.491161539748162, [x = 4.53559292539129189, y = 1.46440707460870746]]$$

- When the Optimization Assistant opens, select **Maximize**, then **Solve**.

After finding a solution, you can plot it. To plot a solution:

- In the **Optimization Assistant** window, click the **Plot** button. The **Optimization Plotter** window is displayed. See Figure 5.9.

Note: When you close the **Optimization Assistant**, you can choose to return the solution, problem, command used, plot, or nothing, using the drop-down in the bottom right corner of the assistant window.

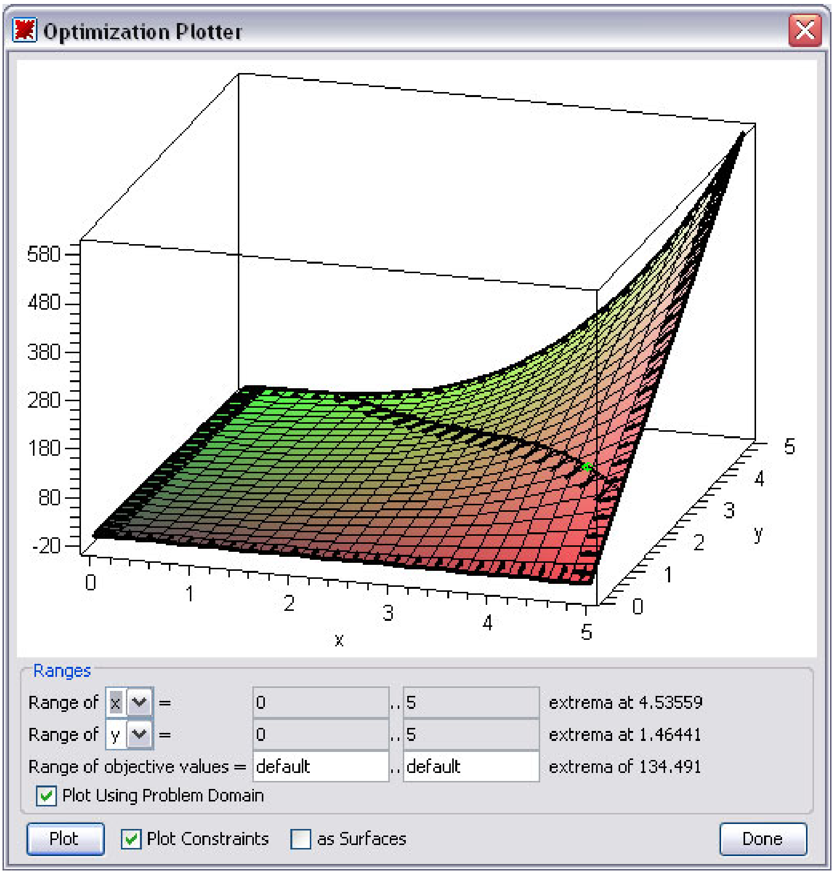


Figure 5.9: Optimization Assistant Plotter Window

For information on the algorithms used to solve optimization problems, refer to the **?Optimization/Methods** help page.

Large Optimization Problems

The **Optimization Assistant** accepts input in an algebraic form. You can specify input in other forms, described in the **?Optimization/InputForms** help page, in command calling sequences.

The Matrix form, described in the **?Optimization/MatrixForm** help page, is more complex but offers greater flexibility and efficiency.

For example, solve the linear program:

Maximize $c^T x$ subject to $Ax \leqslant b$, where x is the vector of problem variables.

1. Define the column vector, **c**, of the linear objective function.

> $with(LinearAlgebra)$:

> $c := RandomVector[column](20, outputoptions = ['datatype' = 'float'])$:

2. Define the matrix **A**, the coefficient matrix for the linear inequality constraints.

> $A := RandomMatrix(19, 20, outputoptions = ['datatype' = 'float'])$:

3. Define the column vector **b**, the linear inequality constraints.

> $b := RandomVector[column](19, outputoptions = ['datatype' = 'float'])$:

4. The **QPSolve** command solves quadratic programs.

> $Optimization[LPSolve](c, [A, b], maximize, assume = nonnegative)$

$$\left[43.2673034492019, \left[\begin{array}{c} \textit{1 .. 20 Vector}_{\textit{column}} \\ \textit{Data Type: float}_8 \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{array} \right] \right]$$

This example uses a random data set to demonstrate the problem. You could also read data from an external file as Matrices, and use that data. For details and an example, see *Reading from Files (page 414)*.

Note: For information on creating matrices and vectors (including how to use the **Matrix** palette to easily create matrices), see *Linear Algebra (page 159)*.

For additional information on performing efficient computations, refer to the **?Optimization/Computation** help page.

MPS(X) File Support

To import linear programs from a standard MPS(X) data file, use the **ImportMPS** command.

Optimization Package Commands

Each **Optimization** package command solves the problem using a different optimization method. These are described in Table 5.9, along with the general input form for each command.

Table 5.9: Optimization Package Commands

Command	Description
LPSolve	Solve a linear program (LP), which involves computing the minimum (or maximum) of a linear objective function subject to linear constraints; input is in equation or Matrix form
LSSolve	Solve a least-squares (LS) problem, which involves computing the minimum of a real-valued objective function having the form $\frac{1}{2}\left(f_1(x)^2 + f_2(x)^2 + \ldots + f_q(x)^2\right)$, where x is a vector of problem variables, possibly subject to constraints; input is in equation or Matrix form
Maximize	Compute a local maximum of an objective function, possibly subject to constraints
Minimize	Compute a local minimum of an objective function, possibly subject to constraints
NLPSolve	Solve a non-linear program (NLP), which involves computing the minimum (or maximum) of a real-valued objective function, possibly subject to constraints; input is in equation or Matrix form
QPSolve	Solve a quadratic program (QP), which involves computing the minimum (or maximum) or a quadratic objective function, possibly subject to linear constraints; input is in equation or Matrix form

For a complete list of commands and other **Optimization** package information, refer to the **?Optimization** help page.

5.6 Statistics

The **Statistics** package provides tools for mathematical statistics and data analysis. The package supports a wide range of common statistical tasks including quantitative and graphical data analysis, simulation, and curve fitting.

In addition to standard data analysis tools, the **Statistics** package provides a wide range of symbolic and numeric tools for computing with random variables. The package supports over 35 major probability distributions and can be extended to include new distributions.

Probability Distributions and Random Variables

The **Statistics** package supports:

- Continuous distributions, which are defined along the real line by probability density functions. Maple supports many continuous distributions, including the normal, Student-t, Laplace, and logistic distributions.
- Discrete distributions, which have nonzero probability only at discrete points. A discrete distribution is defined by a probability function. Maple supports many discrete distributions, including the Bernoulli, geometric, and Poisson distributions.

For a complete list of distributions, refer to the **?Statistics/Distributions** help page.

You can define random variables by specifying a distribution in a call to the **RandomVariable** command.

> $with(Statistics)$:

> $X := RandomVariable(Poisson(\lambda))$:

Find the probability distribution function for **X**. (For information on statistics computations, see *Statistical Computations (page 194)*.)

> $PDF(X, t)$

$$\sum_{k=0}^{\infty} \frac{\lambda^k \mathrm{e}^{-\lambda} \mathrm{Dirac}(t-k)}{k!}$$

Adding Custom Distributions

To add a new distribution, specify a probability distribution in a call to the **Distribution** command.

> $U := Distribution\left(PDF = \left(t \to \begin{cases} 0 & t < 0 \\ \frac{1}{3} & t < 3 \\ 0 & otherwise \end{cases}\right)\right):$

To construct a piecewise-continuous function in 1-D Math, use the **piecewise** command, for example, `t -> piecewise(t < 0, 0, t < 3, 1/3, 0)`.

Define a new random variable with this distribution.

> $Z := RandomVariable(U): PDF(Z, t)$

$$\begin{cases} 0 & t < 0 \\ \frac{1}{3} & t < 3 \\ 0 & otherwise \end{cases}$$

Calculate the mean value of the random variable.

> $Mean(Z)$

$$\frac{3}{2}$$

Statistical Computations

In addition to basic functions, like mean, median, standard deviation, and percentile, the **Statistics** package contains commands that compute, for example, the interquartile range and hazard rate.

Example 1 - Interquartile Range

Compute the average absolute range from the interquartile of the Rayleigh distribution with scale parameter 3.

> $InterquartileRange(Rayleigh(3))$

$$\sqrt{36}\,\sqrt{\ln(2)} - \sqrt{-18\ln\left(\frac{3}{4}\right)}$$

To compute the result numerically:

- Specify the **'numeric'** option.

> $InterquartileRange(Rayleigh(3), 'numeric')$

$$2.719744818$$

Example 2 - Hazard Rate

Compute the hazard rate of the Cauchy distribution with location and scale parameters **a** and **b** at an arbitrary point **t**.

> $HazardRate(Cauchy(a, b), t)$

$$\frac{1}{\pi\, b \left(1 + \frac{(t-a)^2}{b^2}\right) \left(\frac{1}{2} - \frac{\arctan\left(\frac{t-a}{b}\right)}{\pi}\right)}$$

You can specify a value for the point **t**.

> $HazardRate\left(Cauchy(a, b), \frac{1}{2}\right)$

$$\frac{1}{\pi\, b \left(1 + \frac{\left(\frac{1}{2}-a\right)^2}{b^2}\right) \left(\frac{1}{2} - \frac{\arctan\left(\frac{\frac{1}{2}-a}{b}\right)}{\pi}\right)}$$

You can also specify that Maple compute the result numerically.

> $HazardRate\left(Cauchy(10, 1), \frac{1}{2}, 'numeric'\right)$

$$0.003608801460$$

For more information, refer to the **?Statistics/DescriptiveStatistics** help page.

Plotting

You can generate statistical plots using the visualization commands in the **Statistics** package. Available plots include:

- Bar chart
- Frequency plot

- Histogram
- Pie chart
- Scatter plot

For example, create a scatter plot for a distribution of points that vary from $\sin\left(\frac{2\pi x}{200}\right)$ by a small value determined by a normally distributed sample.

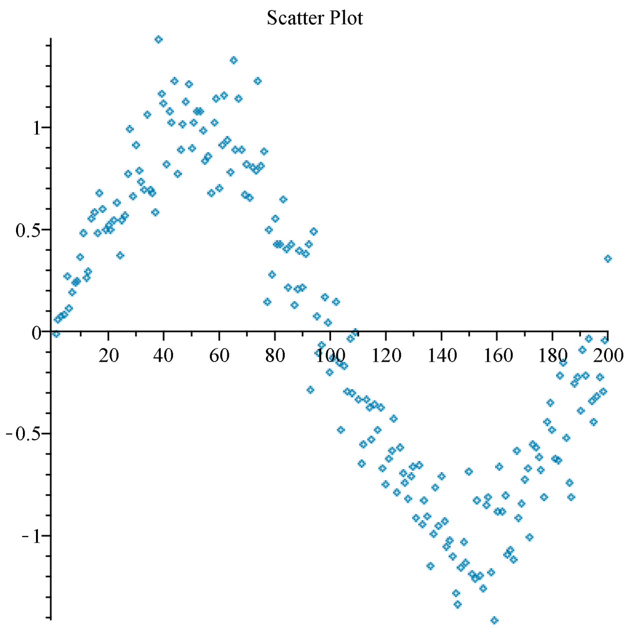
> $N := 200$:

> $U := Sample(Normal(0,1), N)$:

> $X := <seq(x, x = 1..N)>$:

> $Y := <seq\left(\sin\left(\frac{2\pi x}{N}\right) + \frac{U[x]}{5}, x = 1..N\right)>$:

> $ScatterPlot(X, Y, 'title' = $ "Scatter Plot");



To fit a curve to the data points, include the optional **fit** equation parameter.

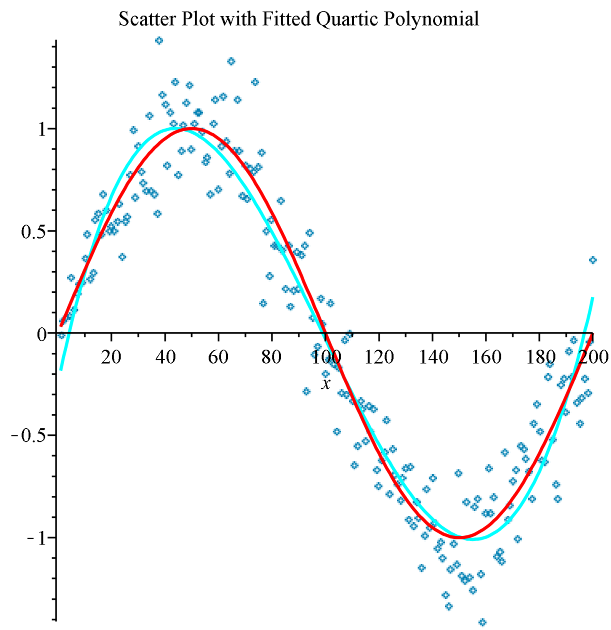
Using the **plots[display]** command, create a plot that contains:

- a scatter plot of the data points
- a quartic polynomial fitted to the data points: $f(x) = a\,x^4 + b\,x^3 + c\,x^2 + d\,x + e$
- the function $\sin\left(\frac{2\,\pi\,x}{N}\right)$

> $P := ScatterPlot\left(X, Y, fit = \left[a\,x^4 + b\,x^3 + c\,x^2 + d\,x + e, x\right], thickness = 2\right):$

> $Q := plot\left(sin\left(\frac{2\,\pi\,x}{N}\right), x = 1\,..N, thickness = 2, color = red\right):$

> $plots[display](P, Q, 'title' = $"Scatter Plot with Fitted Quartic Polynomial")



For more information on statistical plots, refer to the **?Statistics/Visualization** help page.

For an overview of plotting, see *Plots and Animations (page 237)*.

Additional Information

For more information on the **Statistics** package, including regression analysis, estimation, data manipulation, and data smoothing, refer to the **?Statistics** help page.

The Data Analysis Assistant provides an interactive way to perform data analysis. For more information, refer to the **?Statistics[InteractiveDataAnalysis]** help page.

5.7 Teaching and Learning with Maple

Table 5.10 lists the available resources for instructors and students. For additional resources, see *Available Resources (page 58)*.

Table 5.10: Student and Instructor Resources

Resource	Description
Student Packages and Tutors	The **Student** package contains computational and visualization (plotting and animation) functionality, and point-and-click interfaces for explaining and exploring concepts (**Tools→Tutors**). For more information, refer to the **?Student** help page.
Maple Portal	The Maple Portal includes material designed for all Maple users as well as specific portals for students and educators. The Maple Portal includes: • How Do I... topics that give quick answers to essential questions • Tutorials that provide an overview of topics from getting started to plotting and working with matrices • Navigation to portals with specialized information for students, math educators, and engineers Access the portal from the **Help** menu (**Help → Manuals, Resources, and More → Maple Portal)**.
Mathematics and Engineering Dictionary	The Maple Help System has an integrated dictionary of over 5000 mathematics and engineering terms. You can search the dictionary by entering a term in the Help System search field.
Maple Application Center	The Maple Application Center contains tutorials and applications that help instructors begin using Maple and use Maple in the classroom. Browse the many resources in the **Education** and **Education PowerTools** categories. (**http://www.maplesoft.com/applications**)

Resource	Description
Student Help Center	The Maple Student Help Center contains tutorials and applications that help students learn how to use Maple, explore mathematical concepts, and solve problems. Available resources include: • Study guides - Complete lessons with examples for academic courses, including precalculus and calculus. For example, the Interactive Precalculus Study Guide contains worked problems, each solved as in a standard textbook, using Maple commands and custom Maplet graphical interfaces. • Free course lessons for many subjects including precalculus to vector calculus; high school, abstract, and linear algebra; engineering; physics; differential equations; cryptography; and classical mechanics. • Applications for students, written by students, providing examples in many subject areas. • Student FAQs with answers from experts. (**http://www.maplesoft.com/academic/students**)
Teacher Resource Center	The Maple Teacher Resource Center contains resources and tips for teachers using Maplesoft products to help in the classroom. Available resources include: • Training videos • E-books (**http://www.maplesoft.com/teachercenter**)

Student Packages and Tutors

The **Student** package is a collection of subpackages for teaching and learning mathematics and related subjects. The **Student** package contains packages for a variety of subjects, including precalculus, calculus, and linear algebra.

Instructors can:

- Teach concepts without being distracted by the mechanics of the computations.
- Create examples and quickly update them during a lesson to demonstrate different cases or show the effect of the variation of a parameter.
- Create plots and animations to visually explain concepts, for example, the geometric relationship between a mathematical function and its derivatives (**Tools→Tutors→Calculus - Single Variable→Derivatives**). See Figure 5.10.

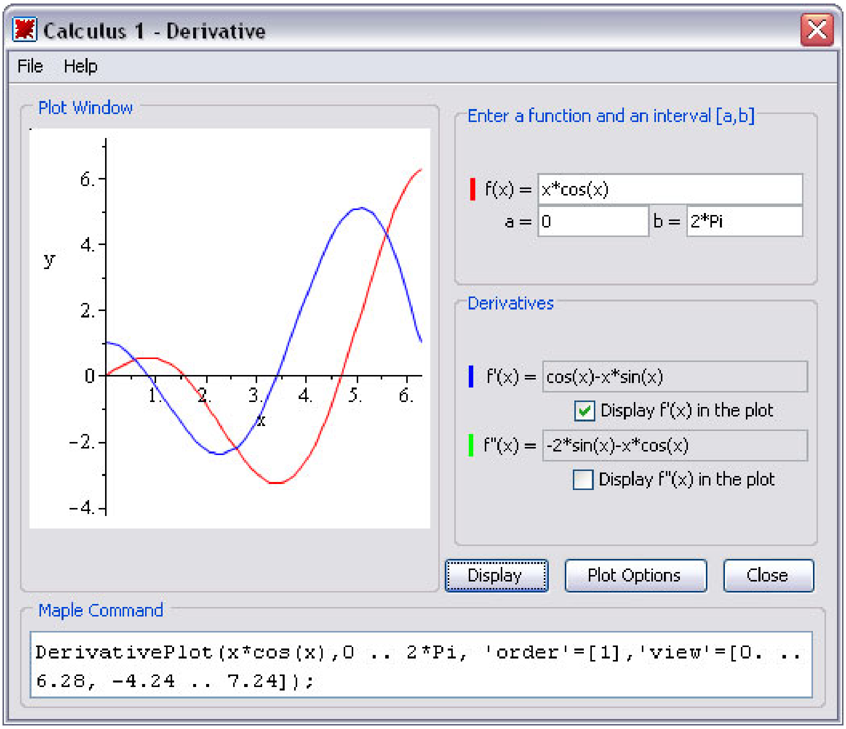


Figure 5.10: Student[Calculus1] Derivatives Tutor

Students can:

- Perform step-by-step computations, for example, compute a derivative by applying differentiation rules using commands or a tutor (**Tools→Tutors→Calculus - Single Variable→Differentiation Methods**). See Figure 5.11.
- Perform computations.
- Visually explore concepts.

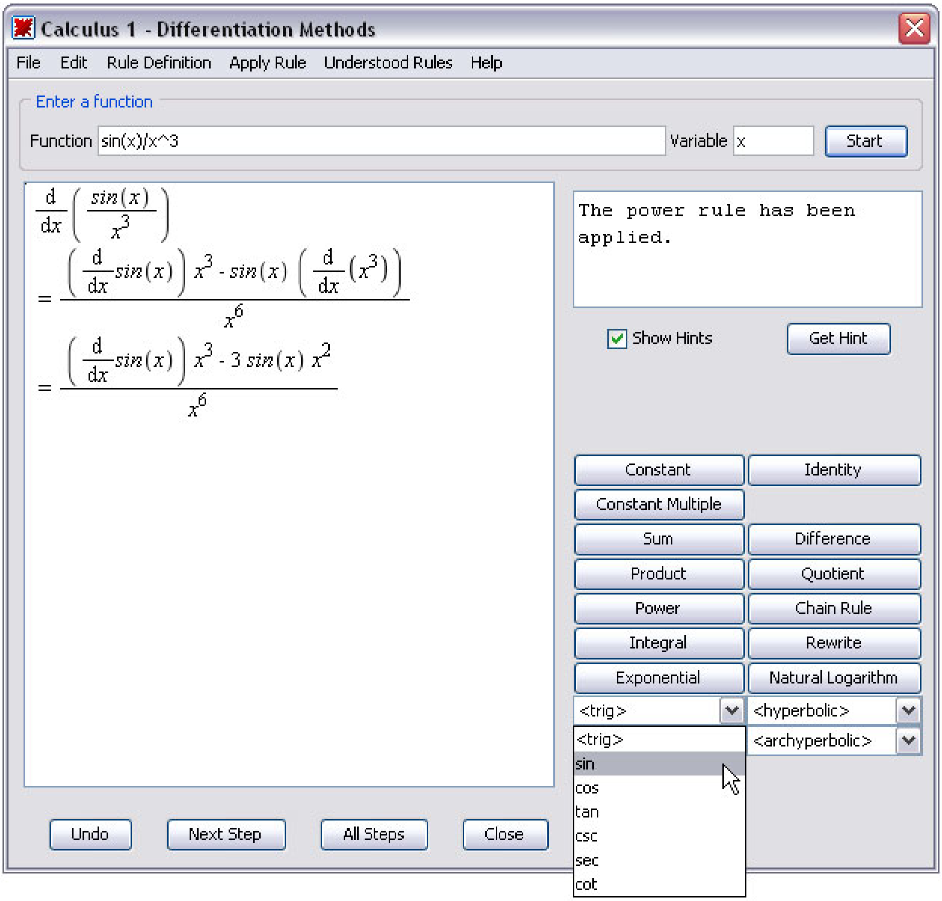


Figure 5.11: Student[Calculus1] Differentiation Methods Tutor

Tutors provide point-and-click interfaces to the **Student** package functionality.

To launch a tutor:

1. From the **Tools** menu, select **Tutors**.

2. Select a subject, for example, **Calculus - Multi-Variable**.

3. Select a tutor, for example, **Gradients**.

Maple inserts the *Student*[*MultivariateCalculus*][*GradientTutor*]() calling sequence (in Worksheet mode), and launches the **Multivariate Calculus Gradient Tutor**.

By rotating the three-dimensional plot, you can show that the gradient points in the direction of greatest increase of the surface (see Figure 5.12) and show the direction of the gradient vector in the x-y plane by rotating the plot (see Figure 5.13).

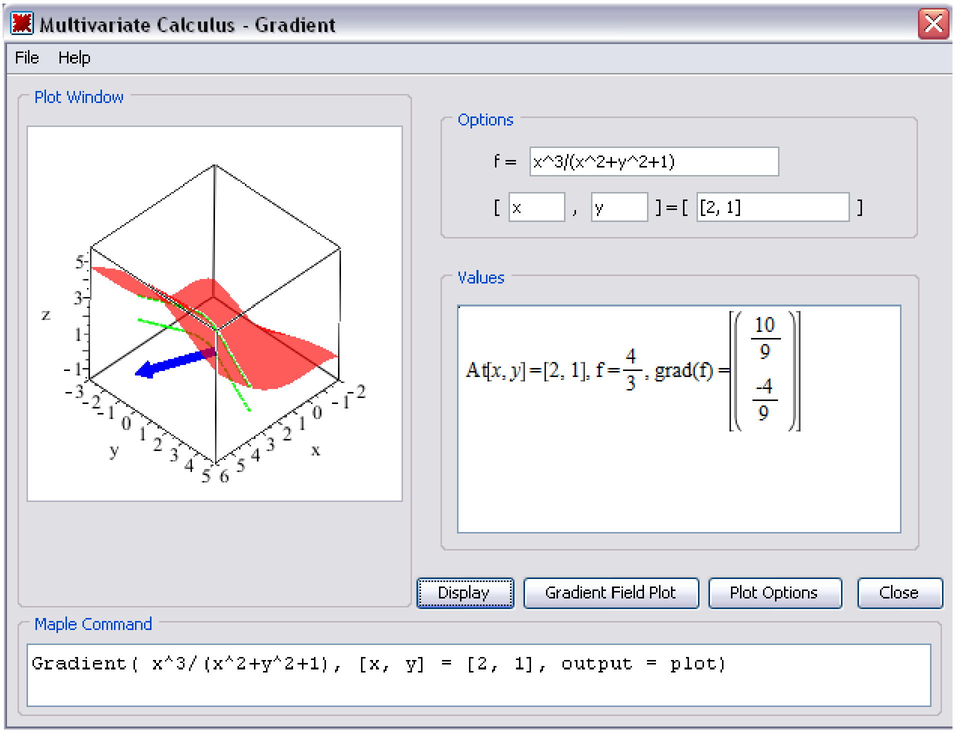


Figure 5.12: Multivariate Calculus Gradient Tutor

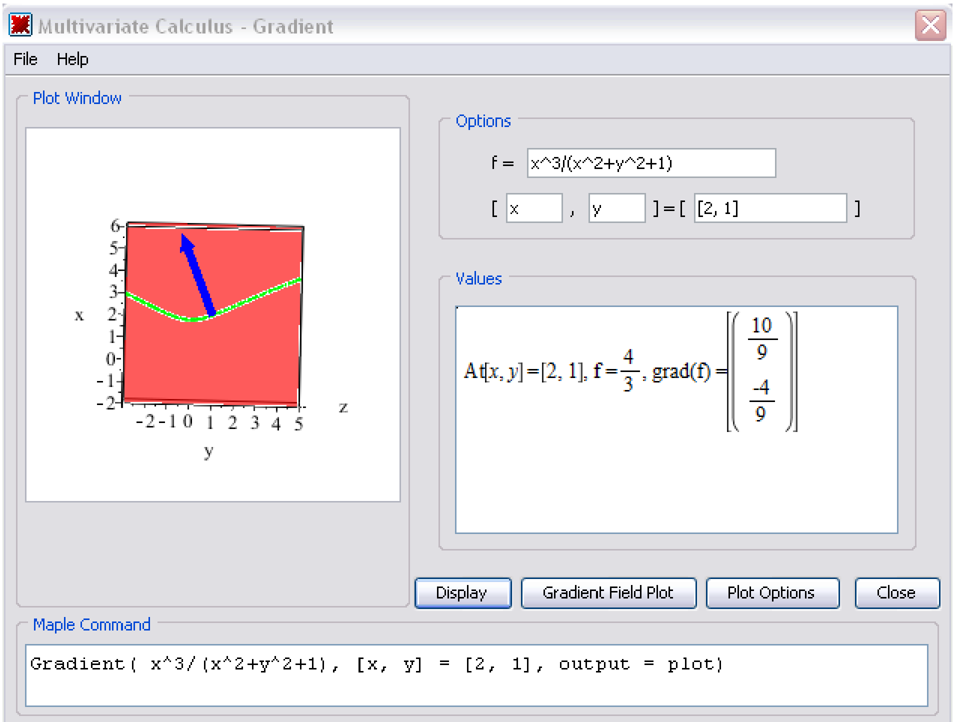
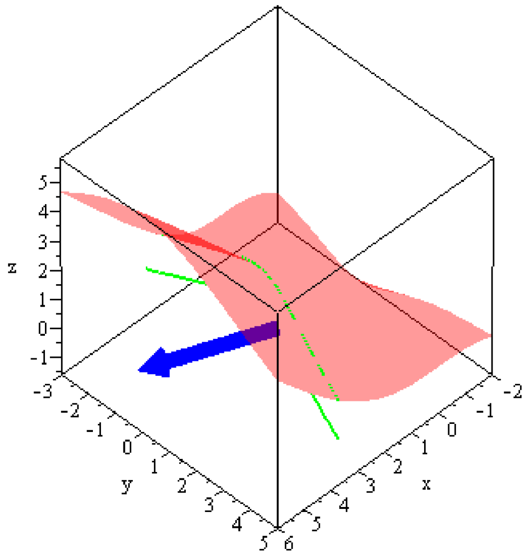


Figure 5.13: Multivariate Calculus Gradient Tutor Showing x-y Plane

When you close the tutor, Maple inserts the 3-D plot.

> *Student*[*MultivariateCalculus*][*GradientTutor*]( );



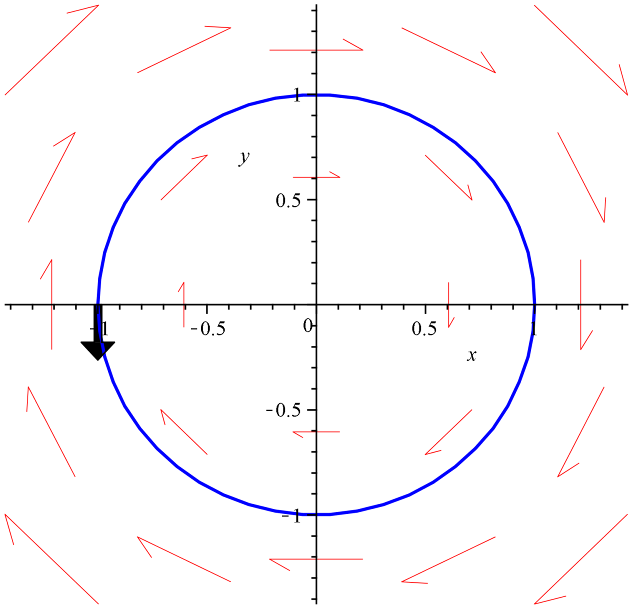
Many **Student** package commands can return a value, mathematical expression, plot, or animation. This allows you to compute the final answer, see the general formula applied to a specific problem, or visualize the underlying concepts.

For example, the **Student[VectorCalculus][LineInt]** (line integral) command can return the following.

- Plot that visually indicates the vector field, path of integration, and tangent vectors to the path
- Unevaluated line integral
- Numeric value of the line integral

> *with*(*Student*[*VectorCalculus*]):

> *LineInt*(*VectorField*(< *y*, −*x*>), *Circle*(< 0, 0 >, 1), '*output*' = '*plot*')



> *LineInt*(*VectorField*(< *y*, −*x*>), *Circle*(< 0, 0 >, 1), '*output*' = '*integral*')

$$\int_0^{2\pi} \left(-\sin(t)^2 - \cos(t)^2\right) dt \tag{5.7}$$

To evaluate the integral returned by the **output = integral** calling sequence, use the **value** command.

> *value*(**(5.7)**)

$$-2\pi \tag{5.8}$$

By default, the **LineInt** command returns the value of the integral.

> *LineInt*(*VectorField*(< *y* − *x*, −*x* −*y*>), *Circle*(< 0, 0 >, *r*))

$$-2\pi r^2$$

For more information on the **Student** package, refer to the **?Student** help page.

Calculus Problem Solving Examples

Maple is a powerful application with many resources to guide you. The following examples provide you with scenarios to learn about using Maple resources and the Maple program.

When using Maple to solve a problem, consider the following process.

1. Formulate your problem.

2. Obtain Maple resources that allow you to solve it.

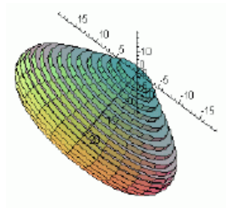
Problem

Scenario A:

Your company is designing a bottle for its new spring water product. The bottle must contain 18 ounces of water and the height is fixed. The design includes an undulating curved surface. You know the amplitude and equation of the curve, but you must find the radius. You require the **Volume of Revolution**.

Scenario B:

You want to teach your students the concept of a **Volume of Revolution**. Specifically, you want to plot and compute the volume of a solid generated by rotating $f(x), a \leq x \leq b$, about an axis or a line parallel to an axis.



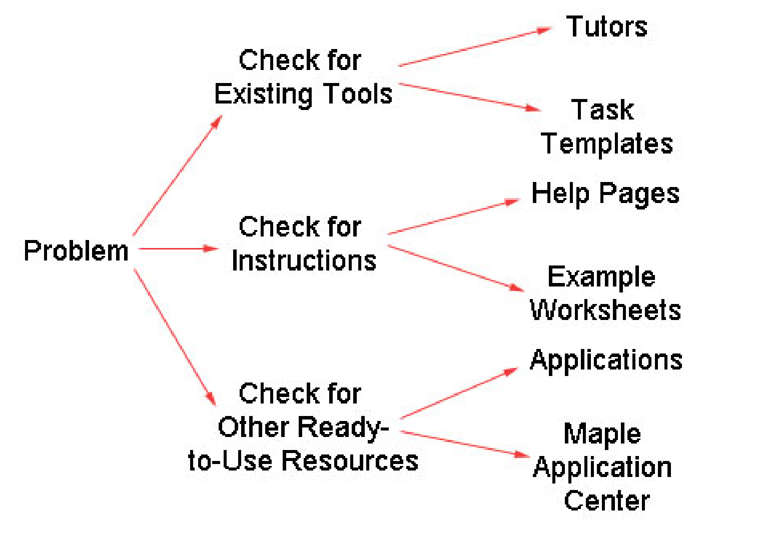


Figure 5.14: Flowchart of solving a problem

Check for Existing Tools: Tutor

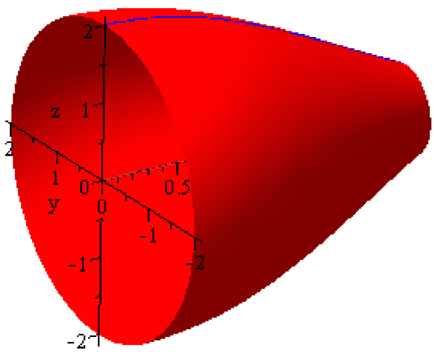
Begin by examining the **Tools** menu for a **Tutor** to a Volume of Revolution problem.

To access a Tutor for the Volume of Revolution:

1. From the **Tools** menu, select **Tutors**, and then **Calculus-Single Variable**. Notice that a Volume of Revolution tutor exists.

2. Click the **Volume of Revolution** menu item. The following Maple command is entered in your document.

> *Student*[*Calculus1*][*VolumeOfRevolutionTutor*]( );



The **Volume of Revolution Tutor** is displayed. See Figure 5.15. Use this tutor to enter a function and an interval, view and manipulate the corresponding plot, and view the full Maple command associated with your entries and selections.

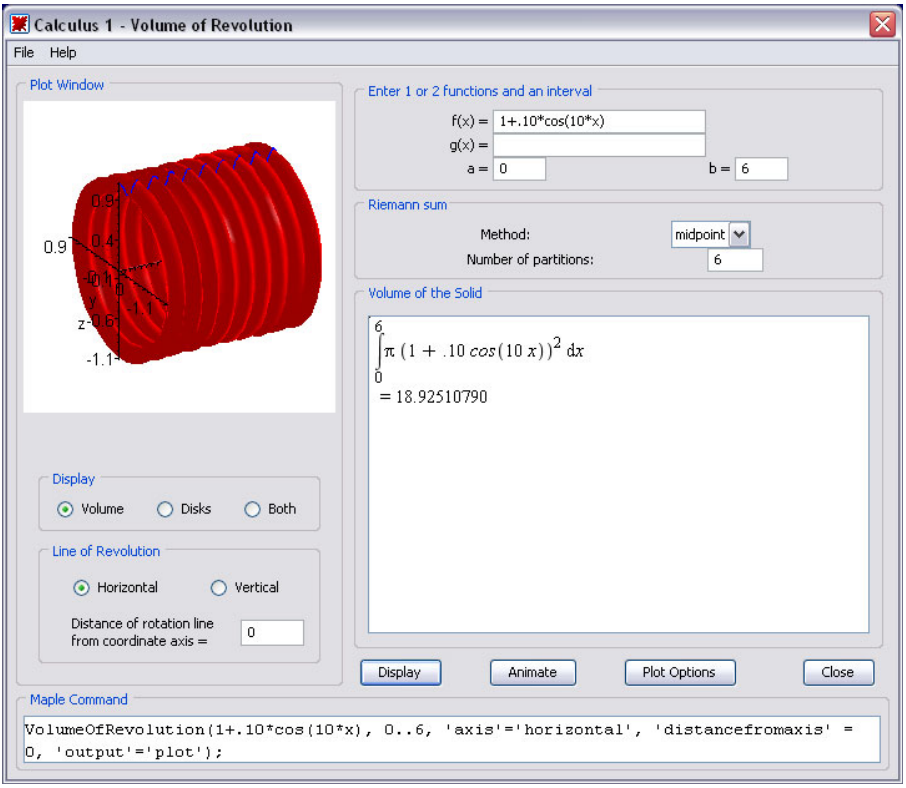


Figure 5.15: Volume of Revolution Tutor

After you **Close** the tutor, the plot is inserted into your worksheet.

Check for Existing Tools: Task Template

1. From the **Tools** menu, select **Tasks**, and then **Browse**. The **Browse Tasks** dialog opens, displaying a list of tasks in the left pane. The tasks are sorted by subject to help you quickly find the desired task.

2. Expand the **Calculus - Integral→ Applications → Solids of Revolution** folder.

3. From the displayed list, select **Volume**. The **Volume of Revolution** task is displayed in the right pane of the **Browse Tasks** dialog.

4. Select the **Insert into New Worksheet** check box.

5. Click **Insert Default Content**. Before inserting a task, Maple checks whether the task variables have assigned values in your worksheet. If any task variable is assigned, the **Task Variables** dialog opens allowing you to modify the names. Maple uses the edited variable names for all variable instances in the inserted task. The content is inserted into your document. See Figure 5.16.

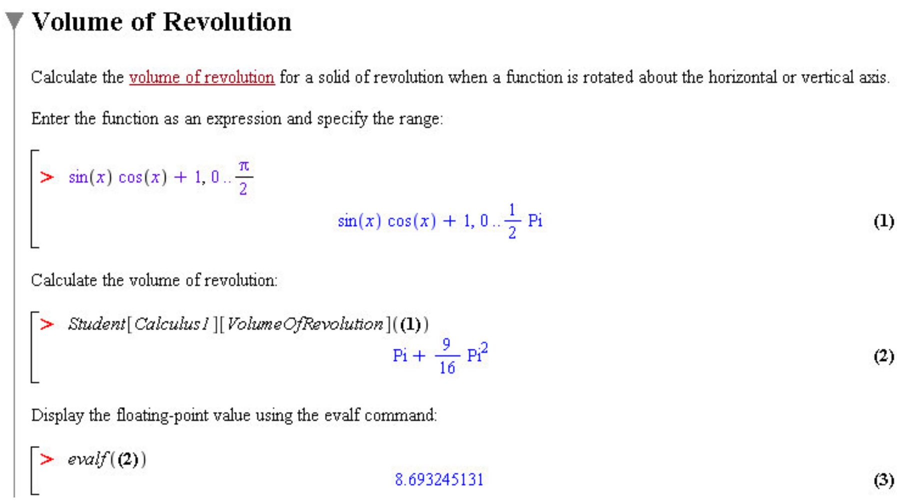


Figure 5.16: Inserted Task Template

6. When a Task Template is inserted, parameters are marked as placeholders, denoted by purple font. To navigate between placeholders, press the **Tab** key. After updating any parameters, execute the command by pressing **Enter**.

Check for Instructions: Help Page and Example Worksheet

The help system provides command syntax information.

To access a help page:

1. From the **Help** menu, select **Maple Help**.

2. In the search field, enter **volume of revolution** and click **Search**. The search results include the command help page, the dictionary definition, and the associated tutor help page.

3. Review the calling sequence, parameters, and description in the **Student[Calculus1][VolumeOfRevolution]** help page.

4. Copy the examples into your worksheet: from the help system **Edit** menu, select **Copy Examples**.

5. Close the Help Navigator.

6. In your document, from the **Edit** menu, select **Paste**. The examples are pasted into your document.

7. Execute the examples and examine the results.

To access an example worksheet:

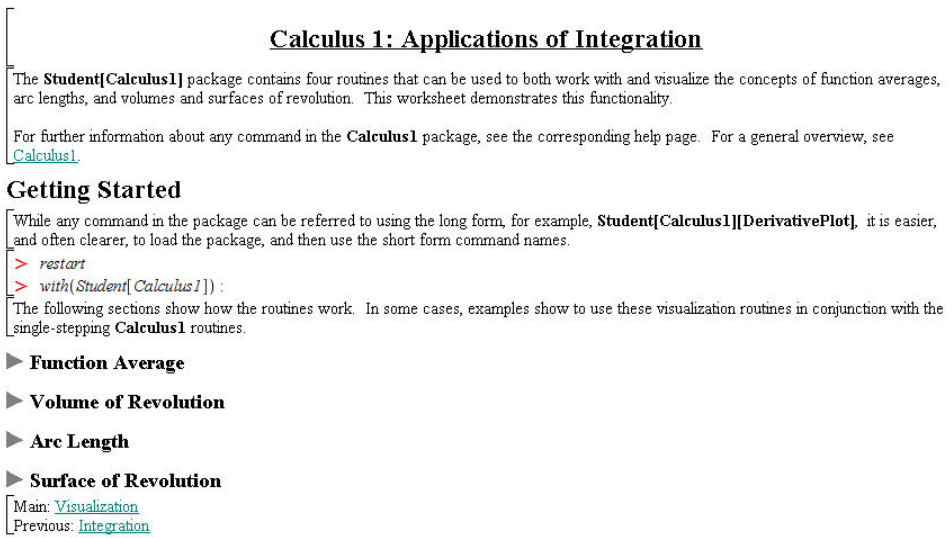
1. In the worksheet, enter ?**index/examples**. The **Example Worksheet Index** opens.

2. Expand the **Calculus** topic.

3. Click the **examples/Calculus1IntApps** link. The **Calculus1: Applications of Integration** worksheet opens. See Figure 5.17.

4. Expand the **Volume of Revolution** topic.

5. Examine and execute the examples.


<u>**Calculus 1: Applications of Integration**</u>

The **Student[Calculus1]** package contains four routines that can be used to both work with and visualize the concepts of function averages, arc lengths, and volumes and surfaces of revolution. This worksheet demonstrates this functionality.

For further information about any command in the **Calculus1** package, see the corresponding help page. For a general overview, see Calculus1.

Getting Started

While any command in the package can be referred to using the long form, for example, **Student[Calculus1][DerivativePlot]**, it is easier, and often clearer, to load the package, and then use the short form command names.

```
> restart
> with(Student[Calculus1]) :
```

The following sections show how the routines work. In some cases, examples show to use these visualization routines in conjunction with the single-stepping **Calculus1** routines.

▶ **Function Average**

▶ **Volume of Revolution**

▶ **Arc Length**

▶ **Surface of Revolution**

Main: Visualization
Previous: Integration

Figure 5.17: Example Worksheet

Check for Other Ready-To-Use Resources: Application Center

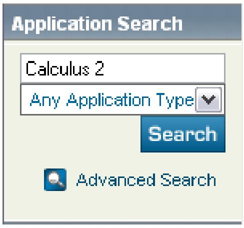
The Maple Application Center contains free user-contributed applications related to mathematics, education, science, engineering, computer science, statistics and data analysis, finance, communications, graphics, and more.

To access a free application for volume of revolution:

1. Go to the Maplesoft web site, **http://www.maplesoft.com**.

2. In the menu of the main web page, click **Community**, and then **Application Center**.

3. In the **Application Search** section, enter **Calculus 2** in the **Keyword or phrase** field.

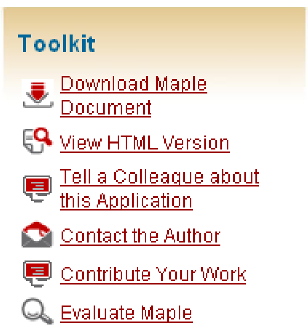


4. Click **Search**.

5. From the search results page, under **Displaying applications**, click the **Click here** link.

6. From the list of archived applications**,** select **Calculus II: Complete Set of Lessons**.

7. Click on the **Download Maple Document** link.



8. Log in as a guest or Maplesoft Member.

9. Download the **.zip** file.

10. Extract the **L2-volumeRevolution.mws** file.

11. Execute the worksheet and examine the results.

5.8 Clickable Math Examples

Maple has incorporated several features that eliminate the learning curve for new users. With drag-and-drop functionality, context menus, built-in tutors, command completion, and over 20 palettes, using Maple has never been so easy. This chapter is designed to show several ways to solve the same problem in Maple. Throughout these examples, you will need to insert new document block regions. This is done through the **Format** menu, by selecting **Create Document Block**. Also, these examples only use the keyboard keys needed for a Windows operating system. Refer to *Shortcut Keys by Platform (page xiv)* for the keys needed for your operating system.

Example 1 - Graph a Function and its Derivatives

On the interval $[-\pi, \pi]$, graph f, f', and f'' for $f(x) = x\cos(x)$.

We solve this problem using the following methods:

- *Solution by Context Menus (page 213)*
- *Solution by Tutor (page 215)*
- *Access the Tutor from a Task Template (page 217)*

Solution by Context Menus

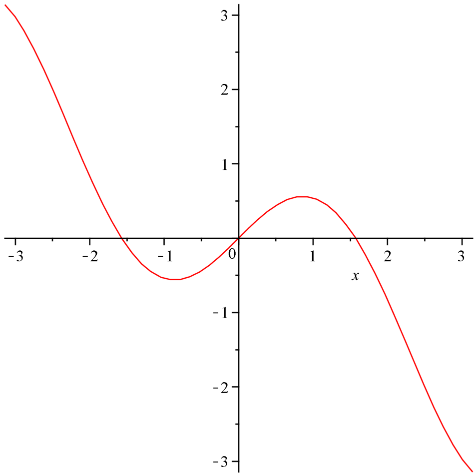
Action	Result in Document
1. Enter the expression $x\cos(x)$.	$x\cos(x)$
Make a copy of the expression and calculate the derivative: 2. Insert a new document block region by selecting from the **Format** menu **Create Document Block**. 3. Highlight the original expression. **Ctrl** + drag the expression to the new document block. 4. Right-click the expression and select **Differentiate → x**.	$x\cos(x) \xrightarrow{\text{differentiate w.r.t. x}}$ $\cos(x) - x\sin(x)$
Make a copy of the derivative and calculate the second derivative: 5. Insert a new document block, and **Ctrl** + drag the derivative to the document block. 6. Right-click the derivative and select **Differentiate → x.**	$\cos(x) - x\sin(x) \xrightarrow{\text{differentiate w.r.t. x}}$ $-2\sin(x) - x\cos(x)$

<table>
<tr><th>Action</th><th>Result in Document</th></tr>
<tr><td>Plot the original expression:

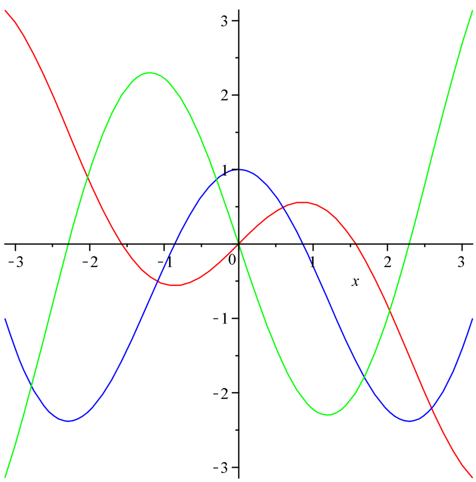
7. Insert a new document block, and Ctrl + drag the original expression to the new block.

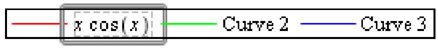
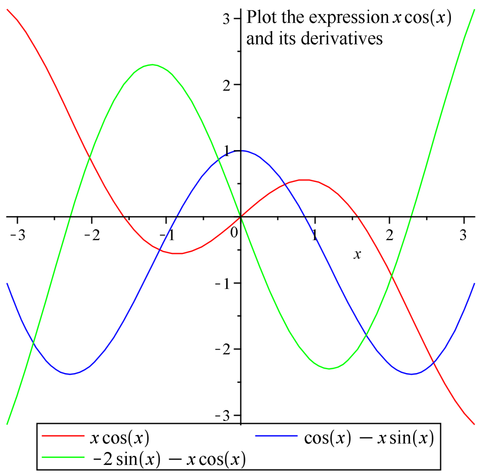
8. Right-click the expression and select Plots → Plot Builder.

9. In the Interactive Plot Builder: Select Plot Type dialog, change the x Axis range to -Pi to Pi.</td><td>$x\cos(x)$

→
</td></tr>
<tr><td>Add the first and second derivatives to the plot:

11. Select and then Ctrl + drag the derivative of the expression onto the plot region. Do the same for the second derivative.</td><td>$x\cos(x)$

→
</td></tr>
</table>

Action	**Result in Document**
Enhance the plot by adding a legend using context menus: 12. Right-click in the plot region and select **Legend → Show Legend**. 13. In the legend, double-click **Curve 1**. Notice that the **Text** icon is selected in the toolbar, Text. Delete the text and select the **Math** icon in the toolbar, Math. This allows you to enter 2-D Math in a text region. Enter the original expression, $x\cos(x)$. 14. Repeat for Curve 2 and Curve 3.	
Add a title: 15. To enter a title, click the **Drawing** icon in the toolbar, Drawing. If the **Drawing** icon is not accessible, click in the plot region. 16. Click **T** in the **Drawing** toolbar, T. 17. Click the plot region and a text region appear. Notice that the toolbar has changed once again with the **Text** icon selected. Enter "**Plot the expression** ". Click the **Math** icon, and enter the expression $x\cos(x)$. Click the **Text** icon once again and enter " **and its derivatives**". 18. Click the text region and the border becomes highlighted. You can now reshape the text region and move it around the plot region using the mouse.	$x\cos(x)$ $\rightarrow$ 

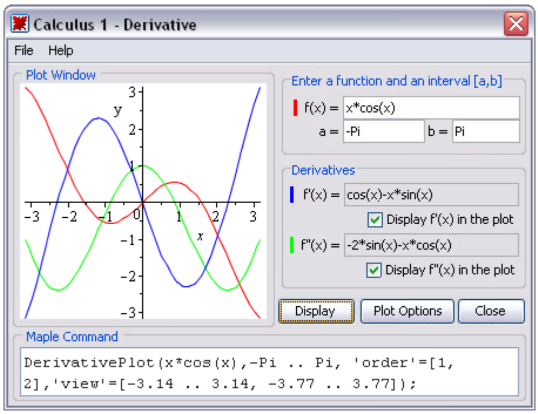
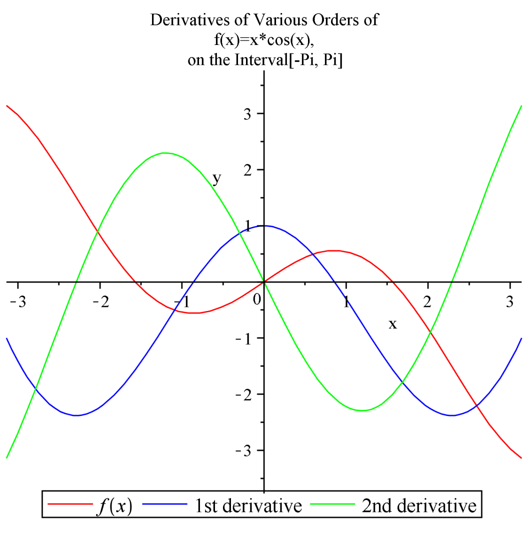
Solution by Tutor

The **Student Calculus 1** package contains a tutor called **Derivatives**, which displays a plot of the expression along with its derivatives. In this example, we solve the same problem as previously, using this tutor

<table>
<tr><th>Action</th><th>Result in Document</th></tr>
<tr><td>1. Load the Student Calculus 1 package. From the Tools menu, select Load Package → Student Calculus 1.

2. Ctrl + drag the expression $x\cos(x)$ to a blank document block region.</td><td>Loading Student:-Calculus1</td></tr>
<tr><td>3. Right-click the expression and select Tutors → Calculus - Single Variable → Derivatives. Note: The Tutors menu is now available in the context menu because we loaded the Student Calculus 1 package in step 1.

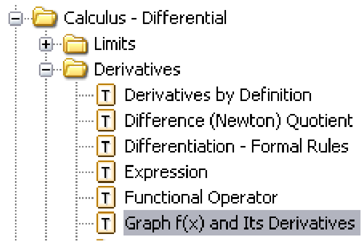
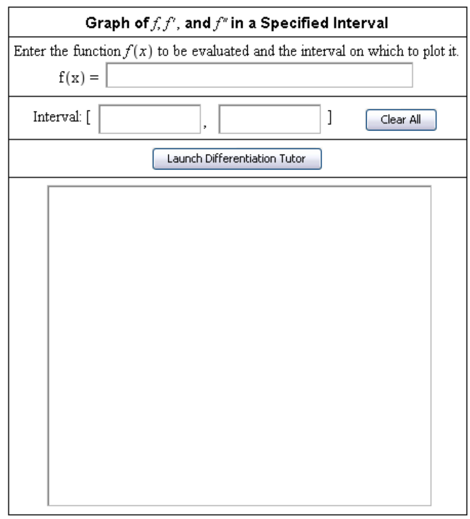
In the Derivative Tutor, the color swatch shown beside the original expression is the color used for the curve in the plot region. Similarly for $f'(x)$ and $f''(x)$.

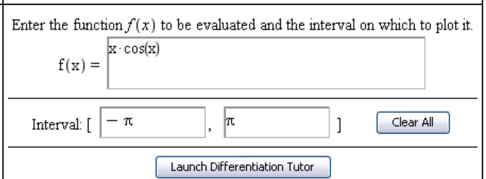
4. Change the lower endpoint to -Pi. Select the check box to display $f''(x)$ in the plot. Click Display to make these changes take effect.</td><td>
</td></tr>
<tr><td>5. You can change the expression and modify plot options from within this tutor. For each change made, click Display to view the altered plot. When complete, click Close to display the resulting plot in the document. The title and legend are automatically added.</td><td>$x\cos(x) \xrightarrow{\text{derivative tutor}}$

</td></tr>
</table>

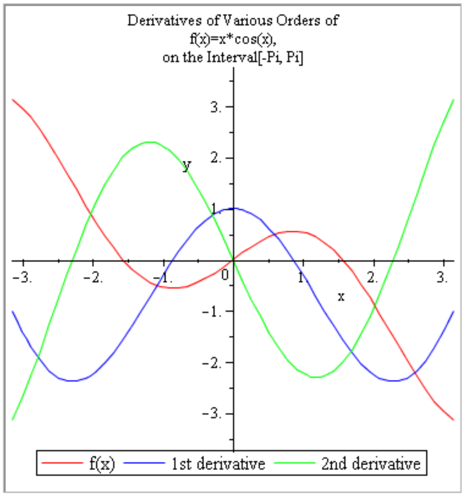
Access the Tutor from a Task Template

Maple also comes with a Task Template to solve this problem without using any commands.

<table>
<tr><th>Action</th><th>Result in Document</th></tr>
<tr><td>1. Launch the Task Template Browser by selecting Tools → Tasks → Browse.

2. In the table of contents of the Task Browser dialog, select Calculus -Differential→ Derivatives → Graph $f(x)$ and its Derivatives.</td><td>
</td></tr>
<tr><td>3. Click Insert Minimal Content at the top of the dialog to insert the task template into the current document.</td><td>
</td></tr>
<tr><td>4. Enter the new expression $x*\cos(x)$ in the f(x) region.

5. Enter the interval $[-\pi, \pi]$. To insert the symbol for pi, you can use command completion or select π from the Common Symbols palette.</td><td>
</td></tr>
</table>

Action	Result in Document
6. Click **Launch Differentiation Tutor** to launch the same tutor as in the previous solution. 7. When complete, click **Close**. A plot of the expression and its derivatives displays in the plot region of the inserted task template.	

Example 2 - Solve for x in a Quadratic Equation

Solve for x in the equation $(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$.

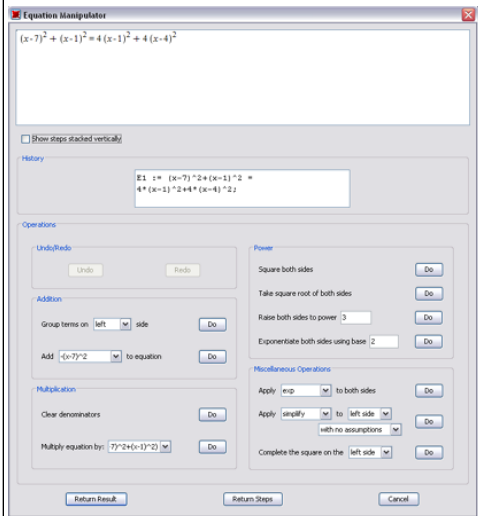
We solve this problem using the following methods:

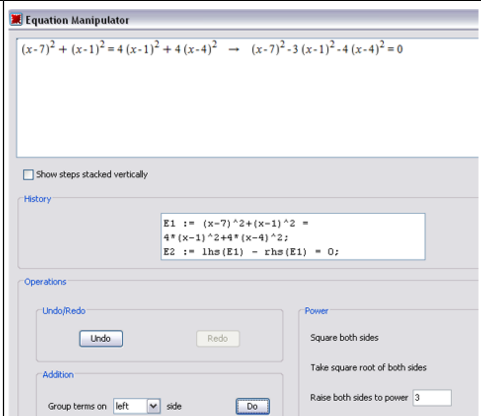
- *Solution through Equation Manipulator (page 218)*
- *Instant Solution (page 220)*
- *Step-by-step Interactive Solution (page 220)*
- *Graphical Solution (page 221)*

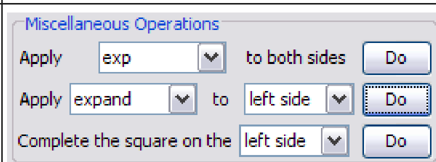
Solution through Equation Manipulator

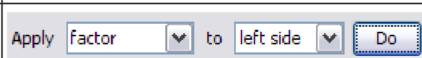
Maple provides a dialog that allows you to single-step through the process of manipulating an expression. This manipulator is available from the context menu.

Action	Result in Document
1. Enter the equation $(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$ in a new document block region.	$(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$

<table>
<tr><th>Action</th><th>Result in Document</th></tr>
<tr><td>2. Right-click this equation and select Manipulate Equation. The Manipulate Equation dialog displays.</td><td>
</td></tr>
<tr><td>Group all of the terms to the left:

3. In the Addition region, the Group terms row allows you to group terms on a specified side. With the left side already selected, click Do.</td><td>
</td></tr>
<tr><td>Expand the left side of the equation:

4. In the Miscellaneous Operations region, we can manipulate the equation by applying a command from the drop-down menus. Since we want to expand the left side of the equation only, click the first drop-down menu in the second row and select expand. Click Do.</td><td>
</td></tr>
<tr><td>Factor the equation:

5. From the same drop-down menu, select factor and click Do.</td><td>
</td></tr>
</table>

Action	**Result in Document**
6. Click **Return Steps** to close the dialog and return all of the steps to the Maple document.	$(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$ $\xrightarrow{\text{manipulate equation}}$ $(x-7)^2+(x-1)^2=4\,(x-1)^2+4\,(x-4)^2$ $(x-7)^2-3\,(x-1)^2-4\,(x-4)^2=0$ $-6x^2+24\,x-18=0$ $-6\,(x-1)\,(x-3)=0$
7. **Ctrl** + drag the factored form of the original equation to a new document block region. 8. Right-click and select **Solve** → **Obtain Solutions for** → **x**.	$-6\,(x-1)\,(x-3)=0 \xrightarrow{\text{solutions for x}} 1, 3$

Instant Solution

To apply an instant solution to this problem, use context menus.

Action	**Result in Document**
1. **Ctrl** + drag the equation $(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$ to a new document block region.	$(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$
2. Right-click the expression and select **Solve** → **Obtain Solutions for** → **x**.	$(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$ $\xrightarrow{\text{solutions for x}} 1, 3$

Step-by-step Interactive Solution

This equation can also be solved interactively in the document, by applying context-menu operations or commands one step at a time.

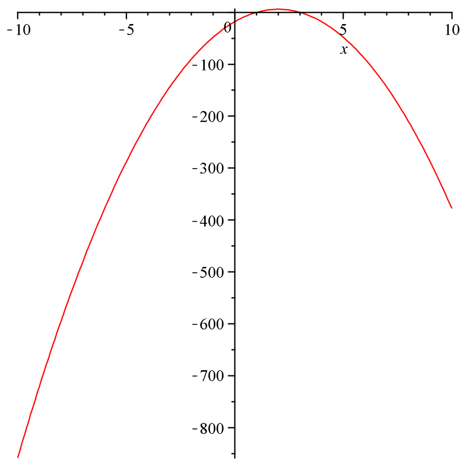
Action	Result in Document
1. **Ctrl** + drag the equation $(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$ to a blank document block region.	$(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$

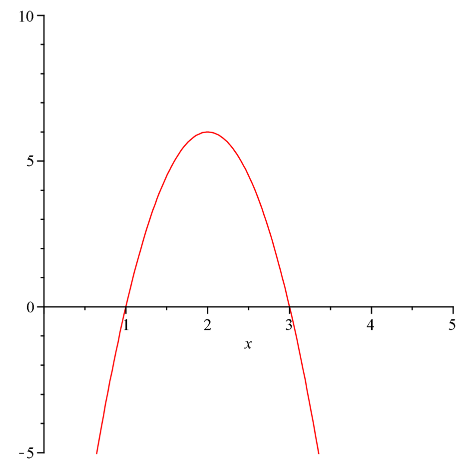
Group all terms on the right: 2. Right-click this equation and from the context menu select **Move to Right**.	$(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$ $\xrightarrow{\text{move to right}}$ $0=3\,(x-1)^2+4\,(x-4)^2-(x-7)^2$
Expand the expression on the right-hand side: 3.Right-click on the result and from the context menu select **Expand**.	$0=3\,(x-1)^2+4\,(x-4)^2-(x-7)^2$ $\overset{\text{expand}}{=}\quad 0=6\,x^2-24\,x+18$
Use Maple's factor command on the resulting right-hand side: 4. Right-click on the result and select **Right-hand Side**. 5. Right-click on the result and select **Factor**.	$0=6\,x^2-24\,x+18 \xrightarrow{\text{right hand side}}$ $6\,x^2-24\,x+18 \overset{\text{factor}}{=} 6\,(x-1)\,(x-3)$
Solve for x: 6. Right-click on the result and select **Solve** → **Obtain Solutions for** → **x**.	$6\,(x-1)\,(x-3) \xrightarrow{\text{solutions for x}} 1, 3$

Graphical Solution

Now that we have seen several methods to solve this problem, we can check the answer by plotting the expression.

Action	Result in Document
1. **Ctrl** + drag the equation $(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$ to a new document block region and press **Enter**.	$(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$ $(x-7)^2+(x-1)^2=4\,(x-1)^2+4\,(x-4)^2$
First, manipulate the equation to become an expression: 2. Right-click the output and select **Move to Left**. Note the difference in the alignment when using context menus on output rather than input. The result is centered in the document with the self-documenting arrow positioned at the left.	$(x-7)^2+(x-1)^2=4\,(x-1)^2+4\,(x-4)^2$ $\xrightarrow{\text{move to left}}$ $(x-7)^2-3\,(x-1)^2-4\,(x-4)^2=0$

3. Right-click the output and select **Left-hand Side**.	$(x-7)^2 - 3(x-1)^2 - 4(x-4)^2 = 0$ $\xrightarrow{\text{left hand side}}$ $(x-7)^2 - 3(x-1)^2 - 4(x-4)^2$
4. Right-click the output and select **Expand**.	$(x-7)^2 - 3(x-1)^2 - 4(x-4)^2$ $\overset{\text{expand}}{=}$ $-6x^2 + 24x - 18$
Now that the equation is in its simplest form, plot the result: 5. **Ctrl** + drag the output to a new document block. 6. Right-click the expression and select **Plots** → **2-D Plot**.	$-6x^2 + 24x - 18 \quad \rightarrow$ 

Change the x and y axis ranges using context menus: 7. By default, plots generated using the context menus have an x -axis range of -10 to 10. To change the range, right-click the plot and select **Axes** → **Properties**. In the **Horizontal** tab of the **Axes Properties** dialog, de-select **Use data extents** and change the **Range min** and `**Range max** to **0** and **5**, respectively. Click the **Vertical** tab and de-select **Use data extents**. Change the **Range min** and **Range max** to **-5** and **10**, respectively. 8. Click **OK** to apply the changes and return to the plot. The interception points of this graph with the x -axis are 1 and 3, the same solutions that we found previously.	

Example 3 - Solve a Quadratic Trig Equation

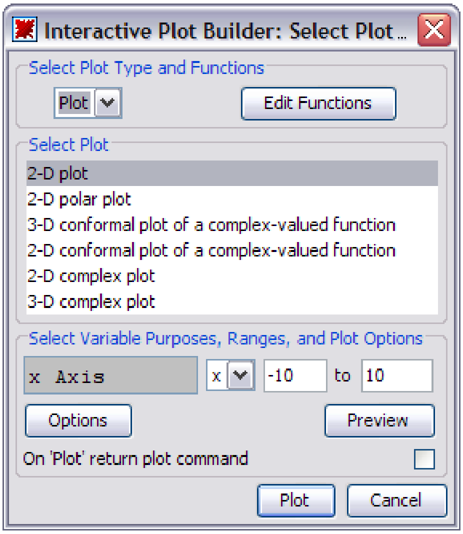
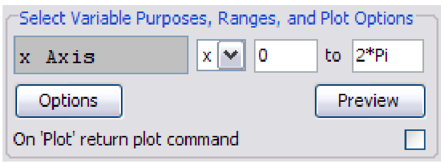
Find all of the solutions to the equation $6\cos^2(x) - \cos(x) - 2 = 0$ in the interval $[0, 2\pi]$.

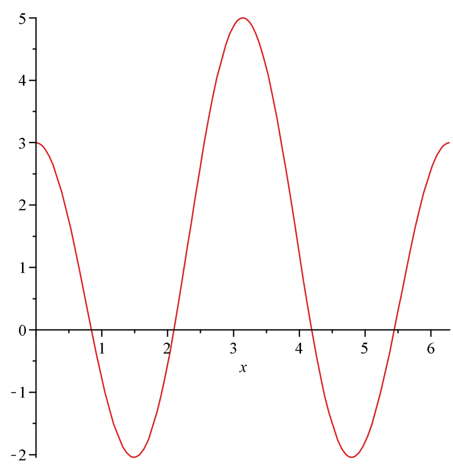
We solve this problem using the following methods:

- *Graphical Solution (page 223)*
- *Solution by Task Template (page 225)*
- *Analytic Solution (page 225)*

Graphical Solution

Action	**Result in Document**
1. **Ctrl** + drag the equation $6\cos^2(x) - \cos(x) - 2 = 0$ to a blank document block and press **Enter**.	$6\cos^2(x) - \cos(x) - 2 = 0$ $6\cos(x)^2 - \cos(x) - 2 = 0$
2. Right-click the output and select **Left-hand Side**.	$6\cos(x)^2 - \cos(x) - 2 = 0 \xrightarrow{\text{left hand side}}$ $6\cos(x)^2 - \cos(x) - 2$

<table>
<tr><th>Action</th><th>Result in Document</th></tr>
<tr><td>3. Right-click the output and select Plots → Plot Builder.</td><td>
</td></tr>
<tr><td>4. Modify the plot range to $x=0$ to $2*\mathrm{Pi}$.</td><td>
</td></tr>
<tr><td>5. Click Plot to display the plot in the document.

6. From the graph, we can see all of the solutions within the interval $[0, 2\pi]$. To approximate the values, click on the plot, then use the Point Probe tool to view the coordinates of the cursor in the toolbar.</td><td>
</td></tr>
</table>

Solution by Task Template

Action	Result in Document
1. From the **Format** menu, select **Tasks** → **Browse**. Expand the **Algebra** folder and select **Solve Analytically in a Specified Interval**.	Algebra Complete the Square Complex Arithmetic Conic - Analysis and Graph ⋮ Solve a Set of Equations Symbolically Solve an Equation Numerically Solve an Equation Symbolically Solve an Inequality Solve Analytically in Specified Interval
2. Click **Insert Minimal Content**.	**Solve Analytically in a Specified Interval** Enter an expression: $> 12\sin^2(x) - 5\sin(x) - 3$ $12\sin(x)^2 - 5\sin(x) - 3$ (15) Find the roots in a specified interval: $> Student[Calculus1][Roots]((15), 0..2\pi)$ $\left[\arcsin\left(\frac{3}{4}\right), -\arcsin\left(\frac{3}{4}\right) + \pi, \arcsin\left(\frac{1}{3}\right) + \pi, -\arcsin\left(\frac{1}{3}\right) + 2\pi\right]$ (16) Express the roots in floating-point form: $> evalf((16))$ [0.8480620790, 2.293530575, 3.481429564, 5.943348398] (17)
3. Replace the current equation with the one from this example, $6\cos^2(x) - \cos(x) - 2 = 0$, and then execute the commands. Notice that equation labels are used to reference the results.	**Solve Analytically in a Specified Interval** Enter an expression: $> 6\cos^2(x) - \cos(x) - 2 = 0$ $6\cos(x)^2 - \cos(x) - 2 = 0$ (15) Find the roots in a specified interval: $> Student[Calculus1][Roots]((15), 0..2\pi)$ $\left[\arccos\left(\frac{2}{3}\right), \frac{2}{3}\pi, \frac{4}{3}\pi, -\arccos\left(\frac{2}{3}\right) + 2\pi\right]$ (16) Express the roots in floating-point form: $> evalf((16))$ [0.8410686706, 2.094395103, 4.188790204, 5.442116637] (17)

Analytic Solution

Action	Result in Document
1. **Ctrl** + drag the equation $6\cos^2(x) - \cos(x) - 2 = 0$ to a blank document block region.	$6\cos^2(x) - \cos(x) - 2 = 0$
2. Right-click the expression and select **Left-hand Side**.	$6\cos^2(x) - \cos(x) - 2 = 0 \xrightarrow{\text{left hand side}}$ $6\cos(x)^2 - \cos(x) - 2$

Action	Result in Document
3. Right-click the output and select **Factor**.	$6\cos(x)^2 - \cos(x) - 2 \overset{\text{factor}}{=} (2\cos(x)+1)(3\cos(x)-2)$
4. **Ctrl** + drag the first factor to a blank document block region. 5. Right-click and select **Solve** → **Solve**.	$(2\cos(x)+1) \xrightarrow{\text{solve}} \left\{x = \frac{2}{3}\pi\right\}$
6. **Ctrl** + drag the second factor to a blank document block region. 7. Right-click and select **Solve** → **Solve**. Notice that you have not found all of the solutions, as with the above methods. These are all of the solutions in the interval $[0, \pi]$.	$(3\cos(x)-2) \xrightarrow{\text{solve}} \left\{x = \arccos\left(\frac{2}{3}\right)\right\}$

Example 4 - Find the Inverse Function

If $f(x) = x^2 + 1, x \geq 0$, find and graph the rule for $f^{-1}(x)$, its functional inverse.

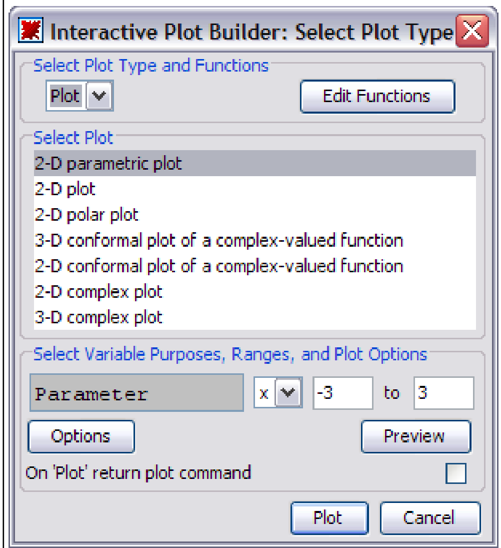
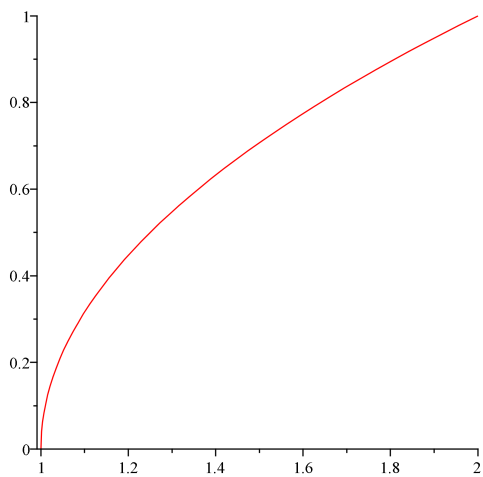
We solve this problem using the following methods:

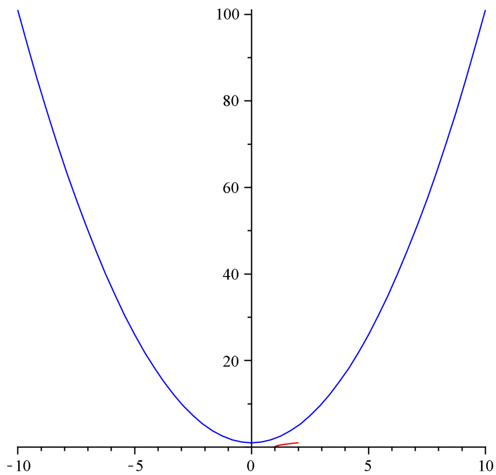
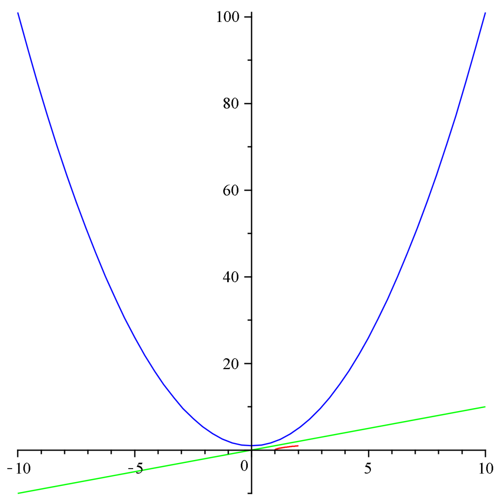
- *Implement the Definition Graphically (page 226)*
- *Solution by Tutor (page 229)*

Implement the Definition Graphically

The graph of the inverse function is the set of ordered pairs formed by interchanging the ordinates and abscissas.

Action	Result in Document
1. In a blank document block, enter $[x^2 + 1, x]$ and press **Enter**.	$[x^2 + 1, x]$ $[x^2 + 1, x]$

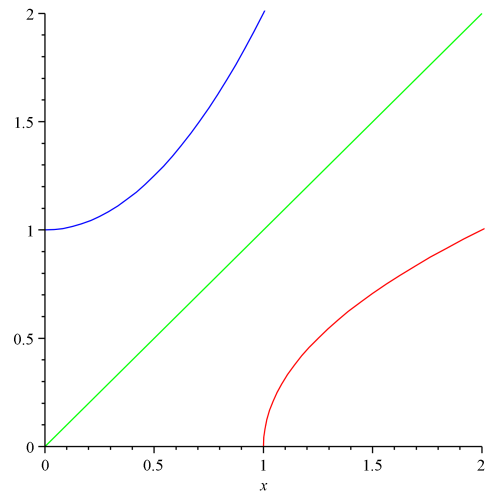
Action	Result in Document
2. Right-click the output and select **Plots** → **Plot Builder**.	
3. In the **Plot Builder : Select Plot Type** dialog, ensure that **2-D parametric plot** is selected in the **Select Plot** region. 4. Adjust the domain for x to the interval $[0, 1]$. 5. Click **Plot** to return the plot to the document.	→ 

Action	**Result in Document**
6. **Ctrl** + drag the expression $x^2 + 1$ onto this graph. Notice that the axis ranges alter.	
7. **Ctrl** + drag the expression onto this graph. The resulting graph shows $f(x), f^{-1}(x)$, and the line $y = x$.	

<table>
<tr><th>Action</th><th>Result in Document</th></tr>
<tr><td>Adjust the x and y axis ranges:

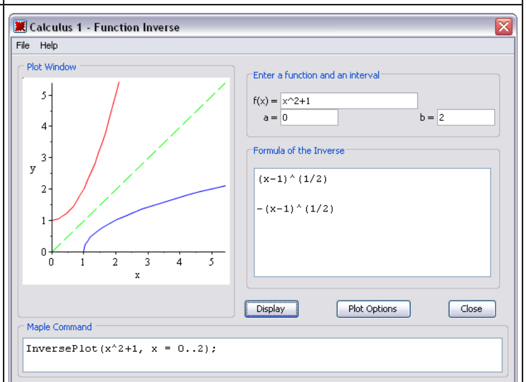
8. Right-click the plot and select Axes → Properties.

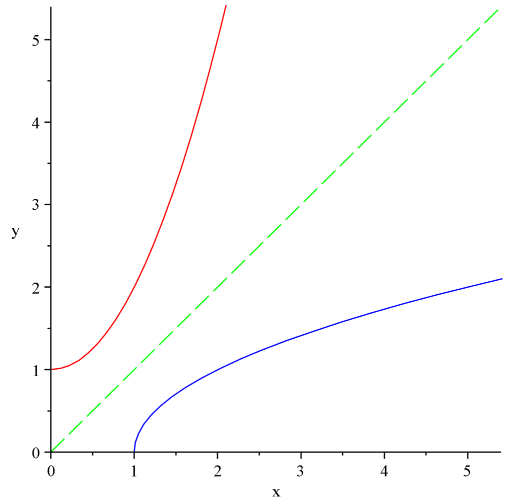
9. In the Axis Properties dialog, de-select Use data extents and change the range to 0 to 2.

10. Click the Vertical tab and repeat step 9. Click OK to apply these settings and close the dialog.</td><td>→

</td></tr>
</table>

Solution by Tutor

<table>
<tr><th>Action</th><th>Result in Document</th></tr>
<tr><td>1. Load the Student Calculus 1 package. From the Tools menu, select Load Package → Student Calculus 1.</td><td>Loading Student:-Calculus1</td></tr>
<tr><td>2. Enter the expression $x^2 + 1$ in a blank document block.</td><td>$x^2 + 1$</td></tr>
<tr><td>3. Right-click and select Tutors → Calculus - Single Variable → Function Inverse. The Function Inverse Tutor displays.

4. Adjust the domain to $[0, 2]$.</td><td>
</td></tr>
</table>

Action	Result in Document
5. When you are finished, click **Close**. The plot of the function, its inverse, and the line $y = x$ is returned to the document.	$x^2 + 1 \xrightarrow{\text{inverse tutor}}$ 

Example 5 - Methods of Integration - Trig Substitution

Evaluate the integral $\int \frac{1}{\sqrt{4 - x^2}}\, dx$ by making the substitution $x = 2\sin(u)$.

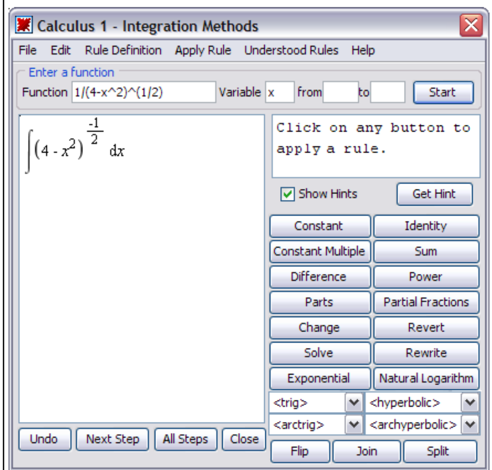
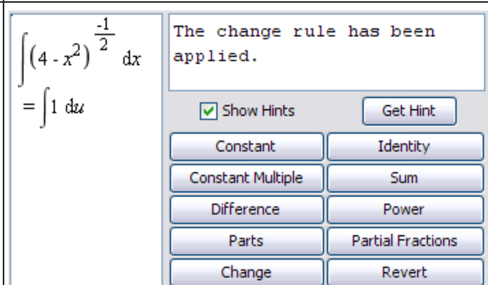
We solve this problem using the following methods:

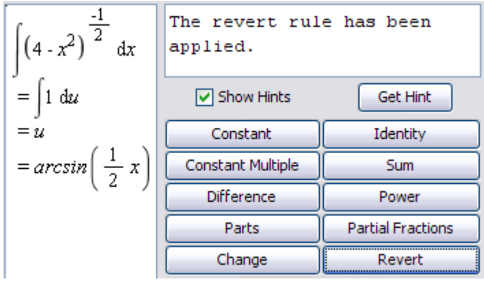
- *Immediate Evaluation of the Integral (page 230)*
- *Solution by Integration Methods Tutor (page 231)*
- *Solution by First Principles (page 232)*

Immediate Evaluation of the Integral

Action	Result in Document
1. Enter the integral $\int \frac{1}{\sqrt{4 - x^2}}\, dx$ in a blank document block region.	$\int \frac{1}{\sqrt{4 - x^2}}\, dx$
2. Right-click the expression and select **Evaluate and Display Inline**.	$\int \frac{1}{\sqrt{4 - x^2}}\, dx = \arcsin\left(\frac{1}{2}x\right)$

Solution by Integration Methods Tutor

Action	Result in Document
1. Load the **Student Calculus 1** package. From the **Tools** menu, select **Load Package** → **Student Calculus 1**.	Loading Student:-Calculus1
2. **Ctrl** + drag the integrand $\frac{1}{\sqrt{4-x^2}}$ to a blank document block region.	$\frac{1}{\sqrt{4-x^2}}$
3. Right-click the expression and select **Tutors** → **Calculus Single Variable** → **Integration Methods**. The **Integration Methods Tutor** displays.	
4. Perform a change of variables by selecting **Change** and entering **x = 2*sin(u)**.	

Action	**Result in Document**
5. Apply the constant rule by clicking **Constant**. 6. To revert back to the original variable, click **Revert**.	
7. Now that the integral has been evaluated, click **Close** to close the tutor and return the evaluated integral to the document.	$\frac{1}{\sqrt{4-x^2}} \xrightarrow{\text{integration methods tutor}}$ $\int \frac{1}{\sqrt{4-x^2}}\,dx$

Solution by First Principles

Action	**Result in Document**
1. **Ctrl** + drag the integrand $\frac{1}{\sqrt{4-x^2}}$ to a blank document block region and press **Enter**.	$\frac{1}{\sqrt{4-x^2}}$ $\frac{1}{\sqrt{4-x^2}}$
Perform trig substitution: 2. Right-click the output and select **Evaluate at a point**. In the dialog that displays, enter **2*sin(u)**.	$\xrightarrow{\text{evaluate at point}}$ $\frac{1}{\sqrt{4-4\sin(u)^2}}$
3. Right-click the output and select **Simplify → Symbolic**.	$\xrightarrow{\text{simplify symbolic}}$ $\frac{1}{2\cos(u)}$ (5.9)

Action	Result in Document
Calculate $\frac{\mathbf{d}\boldsymbol{u}}{\mathbf{d}\boldsymbol{x}}$: 4. In a blank document block, enter the substitution equation: $x=2\sin(u)$ and press **Enter**. 5. Right-click the output and select **Differentiate Implicitly**. In the dialog that displays, change the **Independent Variable** to **u**.	$x=2\sin(u)$ $x=2\sin(u)$ $\xrightarrow{\text{implicit differentiation}}$ $2\cos(u)$ (5.10)
Calculate the integral in terms of $\boldsymbol{u}$: 6. Referencing the results by their equation labels, multiply the original simplified expression by this derivative.	(5.9)·(5.10) 1 (5.11)
7. Integrate the resulting expression.	$\int$(5.11) $\mathrm{d}u$ u (5.12)
Revert the substitution: 8. Place the equation $x=2\sin(u)$ in a blank document block. Delete u and insert the equation label for the previous result, the value of the integral in terms of u. Press **Enter**. 9. Right-click the output and select **Solve → Solve for Variable → u**. The solution is $\arcsin\left(\frac{1}{2}x\right)$.	$x=2\sin($(5.12)$)$ $x=2\sin(u)$ $\xrightarrow{\text{solve for u}}$ $\left[\left[u=\arcsin\left(\frac{1}{2}x\right)\right]\right]$

Example 6 - Initial Value Problem

Solve and plot the solution of the initial value problem

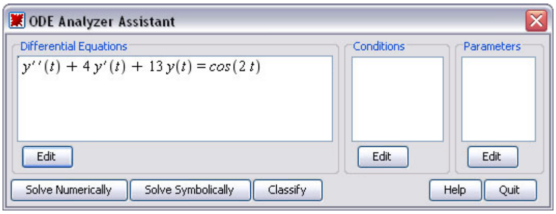
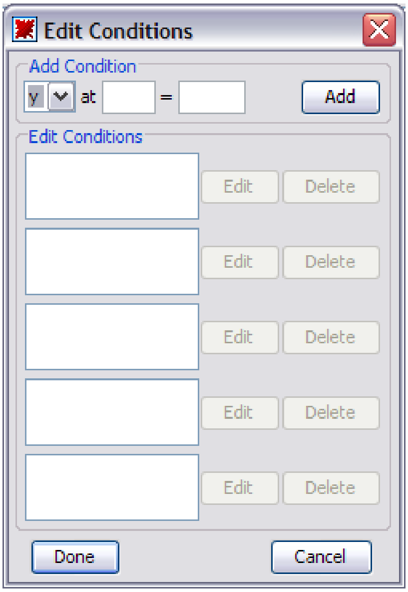
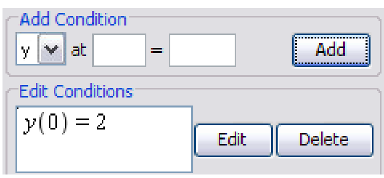
$$y''(t)+4y'(t)+13y(t)=\cos(2t)$$

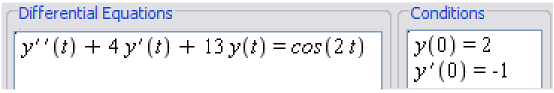
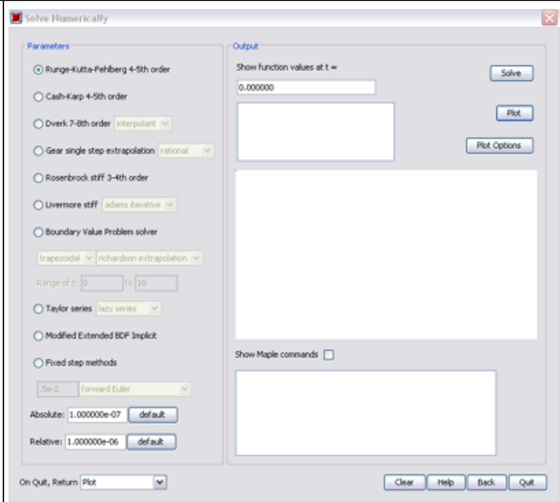
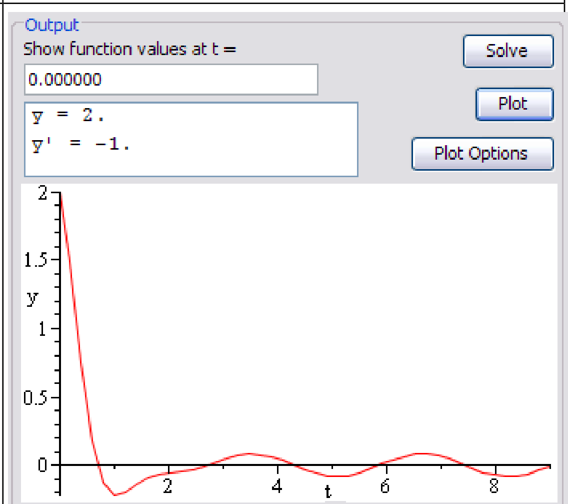
$$y(0)=2$$

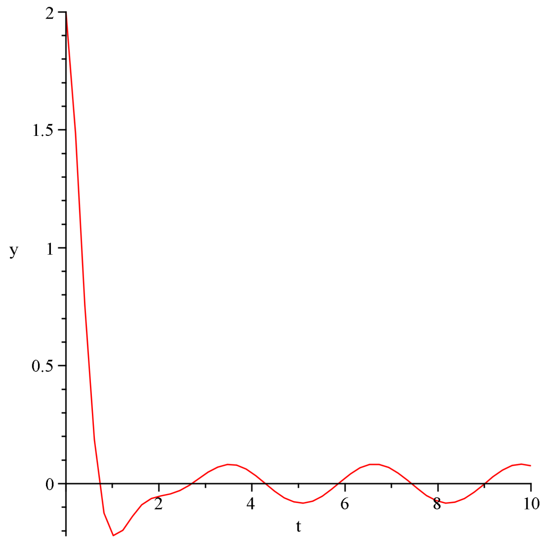
$$y'(0)=-1$$

Solution by ODE Analyzer Assistant

The ODE Analyzer Assistant lets you solve ODEs numerically or symbolically and displays a plot of the solution.

Action	Result in Document
1. Enter the ODE in a blank document block region.	$y''(t) + 4\,y'(t) + 13\,y(t) = \cos(2\,t)$
2. Right-click the equation and select **Solve DE Interactively**. The **ODE Analyzer Assistant** displays with the ODE automatically inserted.	
To insert the initial conditions: 3. In the **Conditions** region, click **Edit**. The **Edit Conditions** dialog opens.	
4. In the **Add Condition** region, with **y** selected in the drop-down menu, enter **0** in the first text field to the right and **2** in the second text field. Click **Add**. Your entry should match the one shown to the right.	

Action	Result in Document
5. To enter the initial condition for y' , select **y'** from the drop-down menu. In the text fields, enter **0** and **-1**. Click **Add**. Click **Done** to close this dialog and return to the main dialog. Notice that the initial conditions are in the **Conditions** section.	
6. Click **Solve Numerically**. A new dialog appears.	
7. Click **Solve** to solve the initial value problem. 8. Click **Plot** to plot the solution of the DE.	

Action	**Result in Document**
9. Click the **Plot Options** button to modify the default graph, if desired. 10. Click **Quit** to close the ODE Analyzer and return a plot of the solution to the document.	$y''(t) + 4\,y'(t) + 13\,y(t) = \cos(2\,t)$ $\xrightarrow{\text{solve DE interactively}}$ 

6 Plots and Animations

Maple can generate many forms of plots, allowing you to visualize a problem and further understand concepts.

- Maple accepts explicit, implicit, and parametric forms to display 2-D and 3-D plots and animations.
- Maple recognizes many coordinate systems.
- All plot regions in Maple are active; therefore, you can drag expressions to and from a plot region.
- Maple offers numerous plot options, such as axis styles, title, colors, shading options, surface styles, and axis ranges, which give you complete control to customize your plots.

For a reference to the types of plots available in Maple, see the **?Plotting Guide**.

6.1 In This Chapter

Section	**Topics**
Creating Plots (page 238) - Interactive and command-driven methods to display 2-D and 3-D plots	• **Interactive Plot Builder** • Context Menu • Dragging to a Plot Region • The **plot** and **plot3d** Commands • The **plots** Package • Multiple Plots in the Same Plot Region
Customizing Plots (page 264) - Methods for applying plot options before and after a plot displays	• **Interactive Plot Builder** Options • Context Menu Options • The **plot** and **plot3d** Command Options
Analyzing Plots (page 269) - Plot analyzing tools	• Point Probe • Rotate • Pan • Zoom
Creating Animations (page 270) - Interactive and command-driven methods to display animations	• **Interactive Plot Builder** • The **plots[animate]** Command • The **plot3d[viewpoint]** Command
Playing Animations (page 275) - Tools to run animations	• Animation Context Bar

Section	Topics
Customizing Animations (page 276) - Methods for applying plot options before and after an animation displays	• **Interactive Plot Builder** Animation Options • Context Menu Options • The **animate** Command Options
Exporting (page 279) - Methods for exporting plots	• Saving Plots to File Formats
Code for Color Plates (page 279) - Information on color plates	• Accessing Code for the Color Plates

6.2 Creating Plots

Maple offers several methods to easily plot an expression. These methods include:

- The **Interactive Plot Builder**
- Context menus
- Dragging to a plot region
- Commands

Each method offers a unique set of advantages. The method you use depends on the type of plot to display, as well as your personal preferences.

Interactive Plot Builder

The **Interactive Plot Builder** is a point-and-click interface to the Maple plotting functionality. The interface displays plot types based on the expression you specify. The available plot types include plots, interactive plots, animations, or interactive animations. Depending on the plot type you select, you can create a:

- 2-D / 3-D plot
- 2-D polar plot
- 2-D / 3-D conformal plot of a complex-valued function
- 2-D / 3-D complex plot
- 2-D density plot
- 2-D gradient vector-field plot
- 2-D implicit plot

Using the **Interactive Plot Builder**, you can:

1. Specify the plotting domain before you display the graph

2. Specify the endpoints of the graph as symbolic, such as Pi or sqrt(2)

3. Select different kinds of graphs, such as animations or interactive plots with slider control of a parameter; that is, customize and display a plot by selecting from the numerous plot types and applying plot options without any knowledge of plotting command syntax

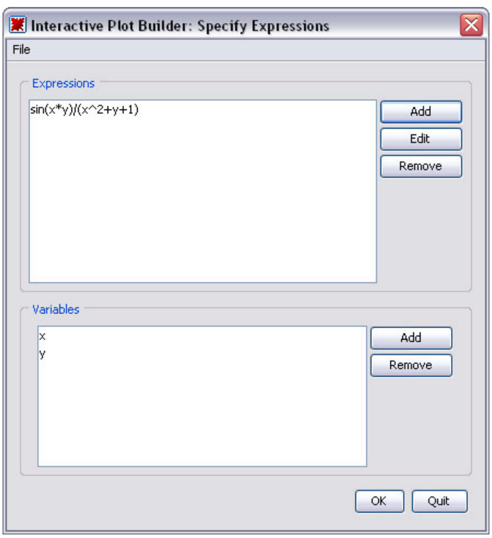
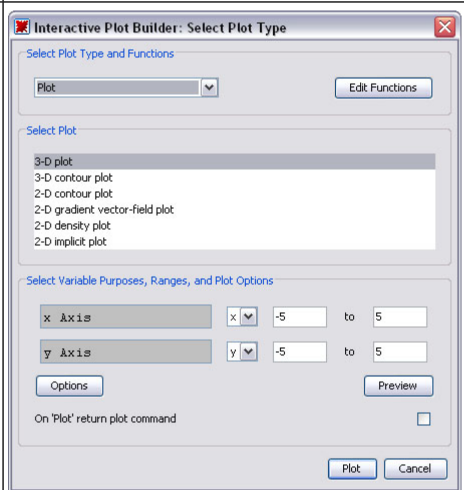
4. Apply the **discont=true** option for a discontinuous graph

The output from the **Interactive Plot Builder** is a plot of the expression or the command used to generate the plot in the document.

To launch the **Interactive Plot Builder**:

- From the **Tools** menu, select **Assistants**, and then **Plot Builder**. **Note:** The **Tools** menu also offers tutors to easily generate plots in several academic subjects. For more information, see *Teaching and Learning with Maple (page 198)*.

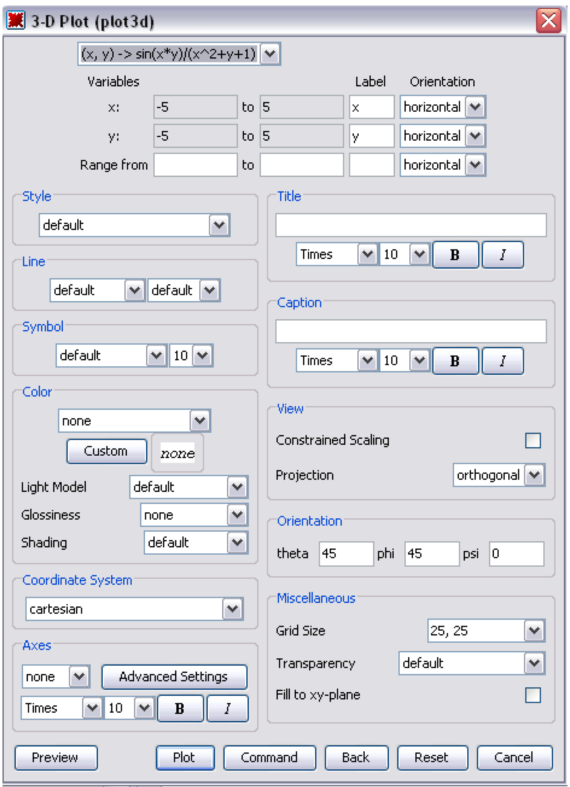
Table 6.1: Windows of the Interactive Plot Builder

1. Specify Expressions window	**2. Select Plot Type window**
	

1. **Specify Expressions window** - Add, edit, or remove expressions and variables. Once finished, you can advance to the **Select Plot Type** window.

2. **Select Plot Type window** - Select the plot type and corresponding plot, and edit the ranges. Once finished, you can display the plot or advance to the **Plot Options** window.

3. Plot Options window



3. **Plot Options window** - Apply plot options. Once finished, you can display the plot or return the command that generates the plot to the document.

Example 1 - Display a plot of a single variable expression

Maple can display two-dimensional graphs and offers numerous plot options such as color, title, and axis styles to customize the plot.

Launch the Interactive Plot Builder:

1. Make sure that the cursor is in a Maple input region.

2. From the **Tools** menu, select **Assistants**, and then **Plot Builder**.

Notes: 1. In worksheet mode, Maple inserts **plots[interactive]();** in the Maple document.

Entering this command at the Maple prompt also opens the Plot Builder.

2. Interaction with the document is disabled while the **Plot Builder** is running.

Enter an expression:

3. In the **Specify Expressions** window:

   a. Add the expression, **sin(x)/x**.

   b. Click **OK** to proceed to the **Select Plot Type** window.

Plot the expression:

4. In the **Select Plot Type** window, notice the default setting of a 2-D plot type and an **x** axis range, **-10** .. **10**. Notice also the various plot types available for this expression.

5. Click **Plot**.

To see the Maple syntax used to generate this plot, see *Maple commands from Creating Plots: Interactive Plot Builder (page 250)*

Example 2 - Display a plot of multiple expressions in 1 variable

Maple can display multiple expressions in the same plot region to compare and contrast. The **Interactive Plot Builder** accepts multiple expressions.

Launch the Interactive Plot Builder and enter the expressions:

1. Launch the **Interactive Plot Builder**. The **Plot Builder** accepts expressions in 1-D Math and performs basic calculations on expressions. For example, entering **diff(sin(x^2), x)** in the **Specify Expression** window performs the calculation and displays the expression as **2*cos(x^2)*x** in the **Expression** group box.
2. In the **Specify Expressions** window:
    a. In three separate steps, add the expressions **sin(x^2)**, **diff(sin(x^2),x)**, and **int(sin(x^2), x)**.

Change the x-axis range:

3. In the **Select Plot Type** window:
    a. Change the **x Axis** range to **-3** .. **3**.
    b. Click **Options** to proceed to the **Plot Options** window.

Launch the Plot Options window and return the plot command syntax to the document:

4. Click **Command**.

Display the actual plot:

5. Execute the inserted command to display the plot by using the context menu item **Evaluate.**

> *plots*[*interactive*]( );

By default, Maple displays each plot in a plot region using a different color. You can also apply a line style such as solid, dashed, or dotted for each expression in the graph. For more information, refer to the **?plot/options** help page. To see the Maple syntax used to generate this plot, see *Maple commands from Creating Plots: Interactive Plot Builder (page 250)*

Example 3 - Display a plot of a multi-variate expression

Maple can display three-dimensional plots and offers numerous plot options such as light models, surface styles, and shadings to allow you to customize the plot.

Launch the Interactive Plot Builder and enter an expression:

1. Add the expression **(1+sin(x*y))/(x^2+y^2)**.

In the Select Plot Type window:

2. Notice the available plot types for an expression with 2 variables, as well as the plot objects for each type.
3. Click **Options**.

In the Plot Options window:

4. From the **Variables** column at the top of the dialog, change the **Range from** field to **0** .. **0.05**.
5. From the **Label** column, enter **z**.
6. From the **Style** group box, select **surface**.
7. From the **Color** group box, in the **Light Model** drop-down menu, select **green-red**.
8. From the **Color** group box, in the **Shading**, drop-down menu, select **z (grayscale)**.
9. From the **Miscellaneous** group box, in the **Grid Size** drop-down menu, select **40, 40**.

Plot the expression:

10. Click **Plot**.

To see the Maple syntax used to generate this plot, see *Maple commands from Creating Plots: Interactive Plot Builder (page 250)*

Example 4 - Display a conformal plot

Maple can display a conformal plot of a complex expression mapped onto a two-dimensional grid or plotted on the Riemann sphere in 3-D.

Launch the Interactive Plot Builder and enter an expression:

1. Add the expression **z^3**.

In the Select Plot Type window:

2. From the **Select Plot** group box, select **2-D conformal plot of a complex-valued function**.
3. Change the range of the **z** parameter to **0** .. **2+2*I**.

In the Plot Options window:

4. From the **Axes** group box, select **normal**.

5. From the **Miscellaneous** group box, select the **Grid Size** drop-down menu option **30, 30**.

Plot the expression:

6. Click **Plot**.

Example 5 - Display a plot in polar coordinates

Cartesian (ordinary) coordinates is the Maple default. Maple also supports numerous other coordinate systems, including hyperbolic, inverse elliptic, logarithmic, parabolic, polar, and rose in two-dimensions, and bipolar cylindrical, bispherical, cylindrical, inverse elliptical cylindrical, logarithmic cosh cylindrical, Maxwell cylindrical, tangent sphere, and toroidal in three-dimensional plots. For a complete list of supported coordinate systems, refer to the **?coords** help page.

Launch the Interactive Plot Builder and enter an expression:

1. Add the expression **1+4*cos(4*theta)**.

Change the x-axis range:

2. In the **Select Plot Type** window:

    a. With 2-D polar plot selected, change the **Angle** of theta to **0** .. **8*Pi**.

In the Plot Options window:

3. From the **Color** group box, select **Magenta**.

Plot the expression:

4. Click **Plot**.

To see the Maple syntax used to generate this plot, see *Maple commands from Creating Plots: Interactive Plot Builder (page 250)*

Example 6 - Interactive Plotting

Using the **Interactive Plot Builder**, you can plot an expression with several of its variables set to numeric values. The **Interactive Parameter** window allows you to interactively adjust these numeric values within specified ranges to observe their effect. To access this window, enter an expression with two or more variables and select **Interactive Plot with x parameter** from the **Select Plot Type and Functions** drop-down menu.

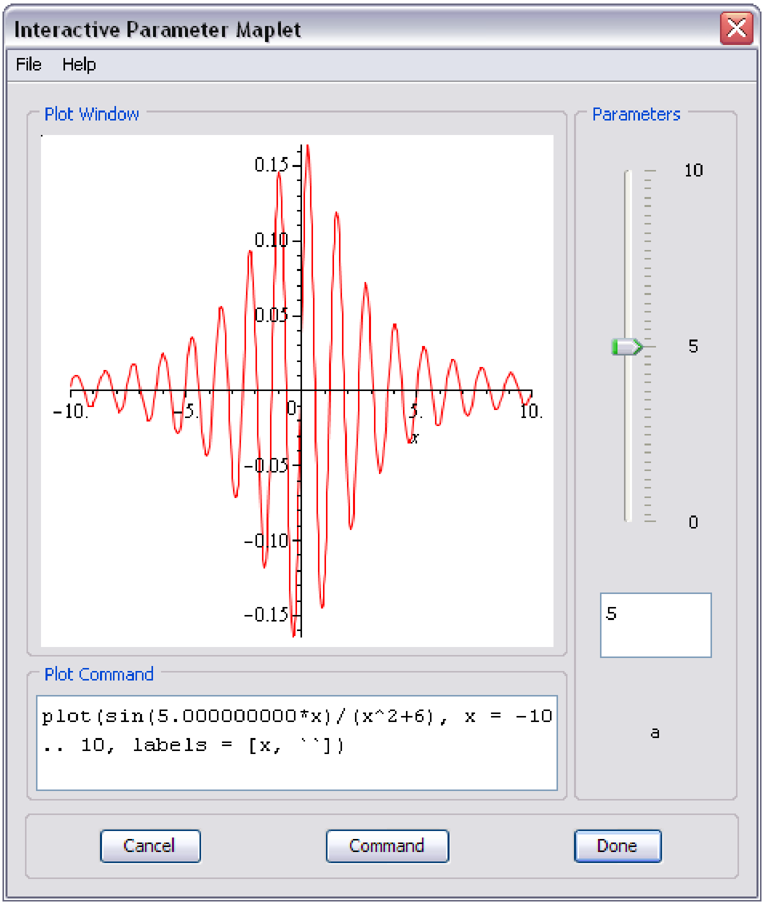


Figure 6.1: Interactive Parameter Window

Launch the Interactive Plot Builder and enter an expression:

1. Add the expression **x+3*sin(x*t)**.

In the Select Plot Type window:

2. From the **Select Plot** group box, select **Interactive Plot with 1 parameter**.

3. Change the range of the **x-axis** to **0** .. **5**.

4. Change the **t** range to **0** .. **10**.

5. Click **Plot** to open the **Interactive Parameter** window.

Note: To apply plot options before interactively adjusting the plot, click **Options** to open the **Plot Options** window. After setting the plot options, click **Plot** to display the **Interactive Parameter window**.

6. To adjust the numeric values, use the slider.

7. Click **Done** to place the plot in the Maple document.

To see the Maple syntax used to generate this plot, see *Maple commands from Creating Plots: Interactive Plot Builder (page 250)*

For information on customizing plots using the **Interactive Plot Builder**, refer to *Customizing Plots: Interactive Plot Builder Options (page 264)*.

Context Menu

A context menu in Maple displays a list of commands to manipulate, display, or calculate using a Maple expression. The commands in the menu depend on the type of the expression. To display the context menu for a Maple expression, right-click (**Control**-click for Macintosh) the expression.

For expressions, the context menu lists:

- 2-D or 3-D plot
- 2-D or 3-D implicit plot
- **Interactive Plot Builder**

based on the expression selected.

When you invoke the **Interactive Plot Builder** through the context menu, the expression automatically passes to the builder, and Maple does not display the **Specify Expression** window.

One advantage of using the context menu is the simplicity of creating an expression using menus. By using this method, you do not need any knowledge of plot command syntax.

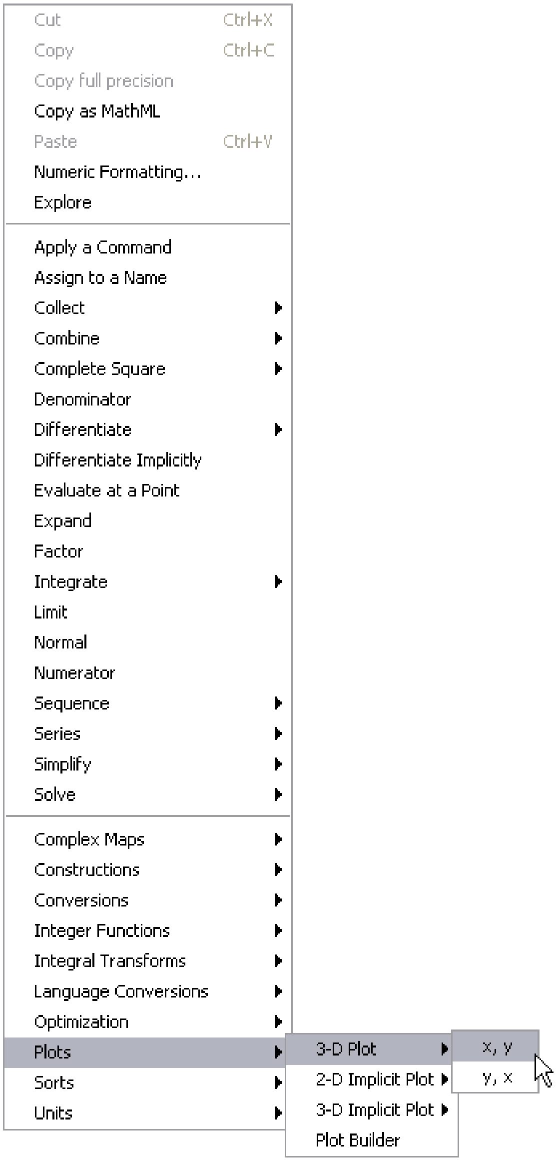
1. Enter and evaluate an expression, for example, $\frac{x\,y}{x^2+y^2}$.

2. Right-click (**Control**-click for Macintosh) the expression.

3. From the context menu, select **Plots** → **3-D Plot** → **x,y**.

> $\frac{x\,y}{x^2+y^2}$

$$\frac{x\,y}{x^2+y^2} \tag{6.1}$$



For information on customizing plots using the context menu, see *Context Menu Options (page 264)*.

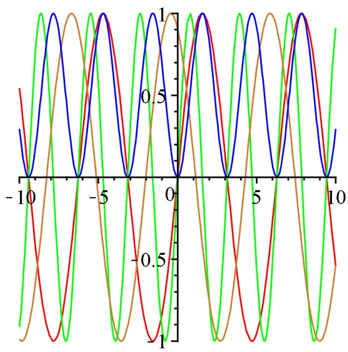
Dragging to a Plot Region

To use the drag-and-drop method, use the plot region created by one of the other methods or insert an empty plot region into the document. Empty plot regions can be two-dimensional or three-dimensional.

Advantages of the drag-and-drop method include the ease of adding and removing plots and the independence from plotting command syntax.

Example:

1. From the **Insert** menu, select **Plot** → **2-D**.
2. Enter the expression $\sin(x)$ in an input region.
3. When dragging an expression to a plot region, you can either make a copy of the expression from the input region or you can cut the expression, thereby removing it from the input region. To make a copy of the expression, select the full expression in the input region and press **Ctrl** (**Command**, Macintosh) while you drag the expression to the plot region. To cut the expression and paste it in the plot region, highlight the expression and drag it to the plot region.
4. Repeat steps 2 and 3 using the following expressions: $\sin(2x), \sin(x+2)$, and $\sin(x)^2$.
5. To remove an expression from the plot region, drag-and-drop the expression plot from the plot region to a Maple input region.



The plot and plot3d Commands

The final method for creating plots is entering plotting commands.

The main advantages of using plotting commands are the availability of all Maple plot structures and the greater control over the plot output. Plot options are discussed in *Customizing Plots (page 264)*.

Table 6.2: The plot and plot3d Commands

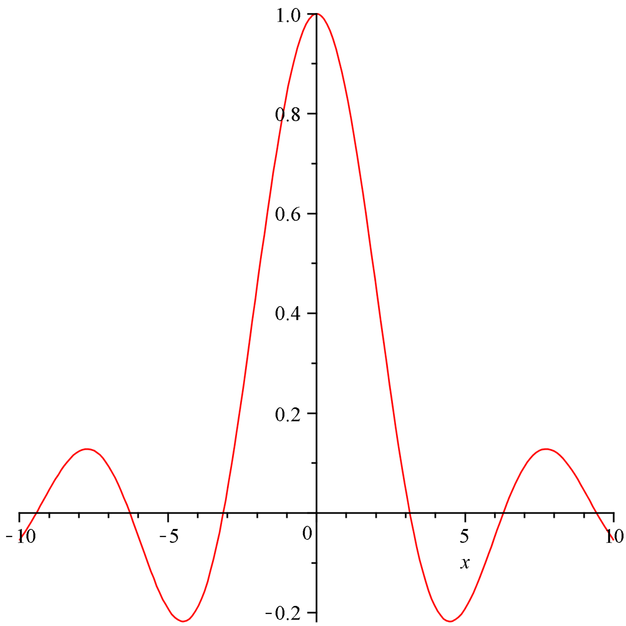
plot(plotexpression, x=a..b, ...)
plot3d(plotexpression, x=a..b, y=a..b, ...)
• **plotexpression** - expression to be plotted
• **x=a..b** - name and horizontal range
• **y=a..b** - name and vertical range

Maple commands from Creating Plots: Interactive Plot Builder

The following examples show the plotting commands returned by the examples in *Interactive Plot Builder (page 238)*.

Example 1 - Display a plot of a single variable expression

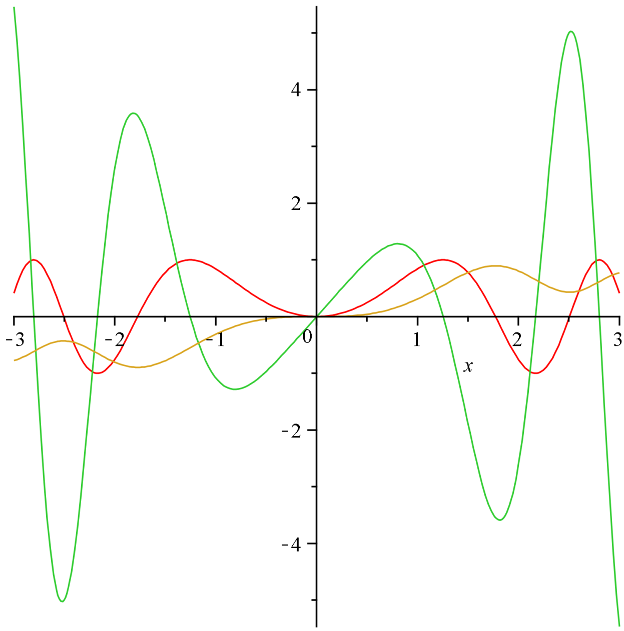
> $plot\left(\frac{\sin(x)}{x}, x = -10..10\right)$



Example 2 - Display a plot of multiple expressions in 1 variable

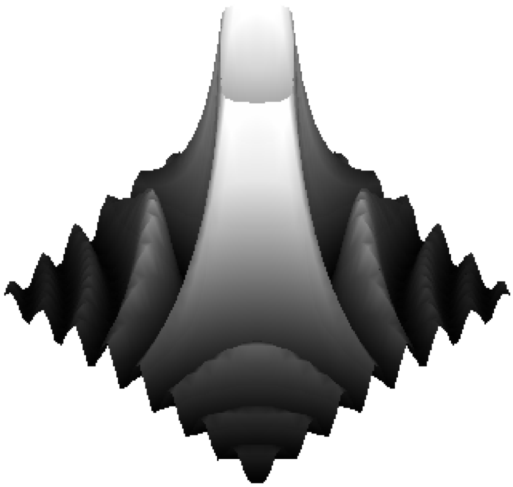
To display multiple expressions in a plot, include the expressions in a list. To enter $\frac{d}{dx}\sin(x^2)$ and $\int \sin(x^2)\,dx$, use the **Expression** palette. For more information, see *Palettes (page 22)*.

> $plot\left(\left[\sin(x^2), \frac{d}{dx}\sin(x^2), \int \sin(x^2)\,dx\right], x=-3..3\right)$



Example 3 - Display a plot of a multi-variable expression

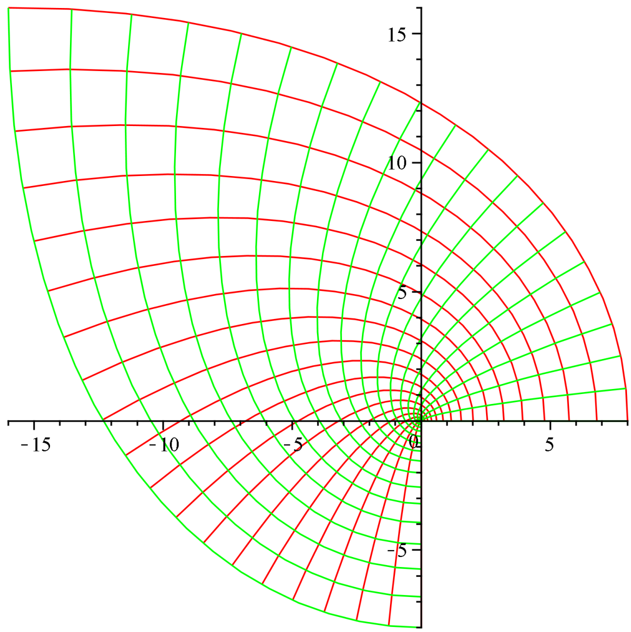
$$> plot3d\left(\frac{1+\sin(x\,y)}{x^2+y^2}, x=-5..5, y=-5..5, view=0..0.5, lightmodel=light1, shading=zgrayscale, style=patchnogrid, grid=[40, 40]\right)$$



Example 4 - Display a conformal plot

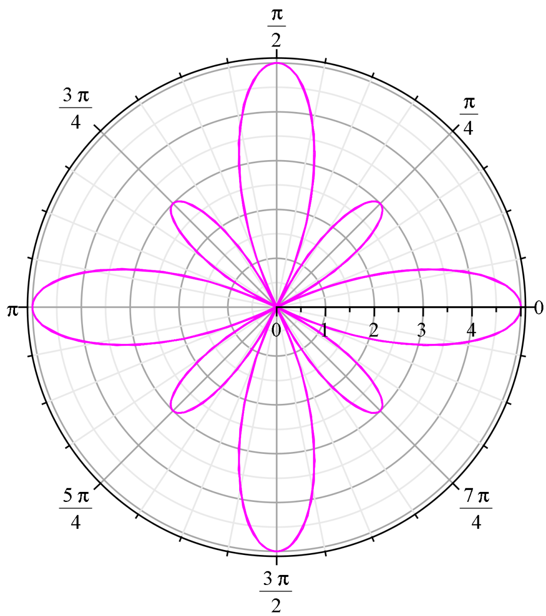
A collection of specialized plotting routines is available in the **plots** package. For access to a single command in a package, use the long form of the command.

> $plots[conformal](z^3, z = 0\,..2 + 2\,I, axes = normal, grid = [20, 20])$



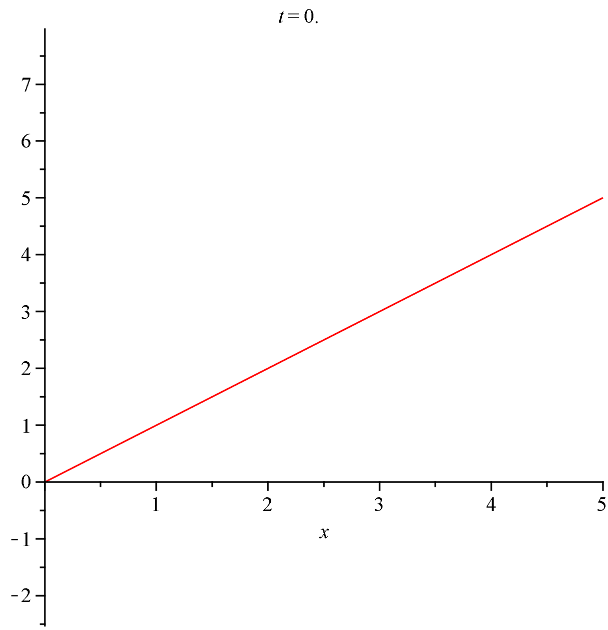
Example 5 - Display a plot in polar coordinates

> $plots[polarplot](1 + 4\cos(4\,\theta), \theta = 0..8\,\pi, color = magenta)$



Example 6 - Interactive Plotting

> $plots[animate](plot, [x+3\sin(x\,t), x=0..5], t=0..10)$

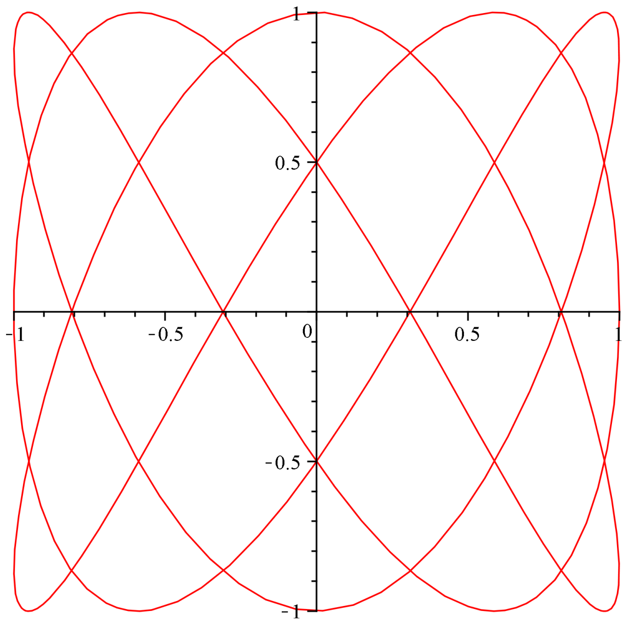


For more information on the plot options used in this section, refer to the **?plot/options** and **?plot3d/options** help pages.

Display a Parametric Plot

Some graphs cannot be specified explicitly. In other words, you cannot write the dependent variable as a function of the independent variable, $y=f(x)$. One solution is to make both the x-coordinate and the y-coordinate depend upon a parameter.

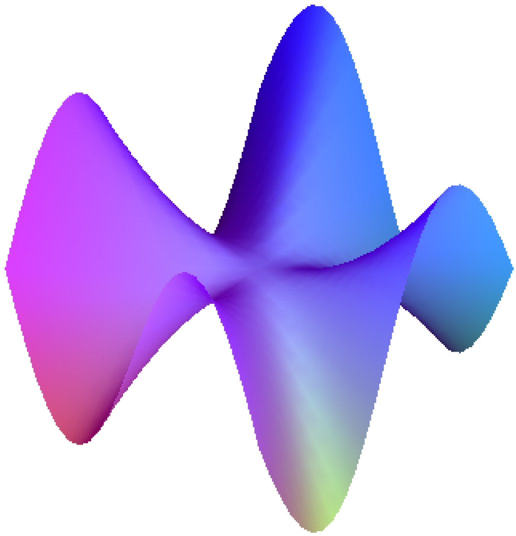
> $plot([\cos(3\,t), \sin(5\,t), t = 0\,..2\,\pi])$



Display a 3-D Plot

Maple can plot an expression of two variables as a surface in three-dimensional space. To customize the plot, include **plot3d** options in the calling sequence. For a list of plot options, see *The plot and plot3d Options (page 267)*.

> $plot3d\left(\frac{x\,y\,(x^2-y^2)}{x^2+y^2}, x=-2\,..2, y=-2\,..2, glossiness=0.5, style=patchnogrid, light=[100, 345, 0.4, 0.9, 0.7], ambientlight=[0.5, 0, 1]\right)$



The plots Package

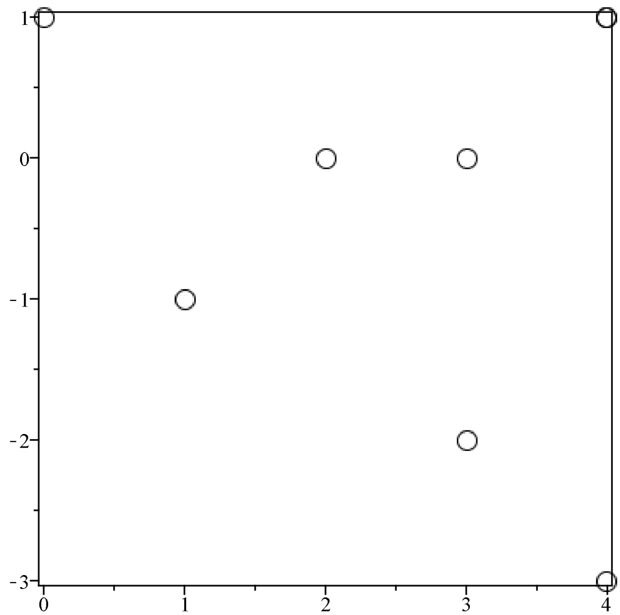
The **plots** package contains numerous plot commands for specialized plotting. This package includes: **animate**, **contourplot**, **densityplot**, **fieldplot**, **odeplot**, **matrixplot**, **spacecurve**, **textplot**, **tubeplot**, and more. For details about this package, refer to the **?plots** help page.

> $with(plots)$:

The pointplot Command

To plot numeric data, use the **pointplot** command in the **plots** package with the data organized in a list of lists structure of the form $[[x_1, y_1], [x_2, y_2], ..., [x_n, y_n]]$. By default, Maple does not connect the points. To draw a line through the points, use the **style = line** option. For further analysis of data points, use the **Curve Fitting Assistant** (**Tools→Assistants→CurveFitting**), which fits and plots a curve through the points. For more information, refer to the **?CurveFitting[Interactive]** help page.

> *pointplot*([[0, 1], [1, −1], [3, 0], [4, −3], [2, 0], [4, 1], [3, −2], [4, 1]], *axes*=*BOXED*, *symbolsize*=25, *symbol*=*circle*)



The matrixplot Command

The **matrixplot** command plots the values of a plot object of type **Matrix**. The **matrixplot** command accepts options such as **heights** and **gap** to control the appearance of the plot. For more information on Matrices, see *Linear Algebra (page 159)*.

> *with*(*LinearAlgebra*) :

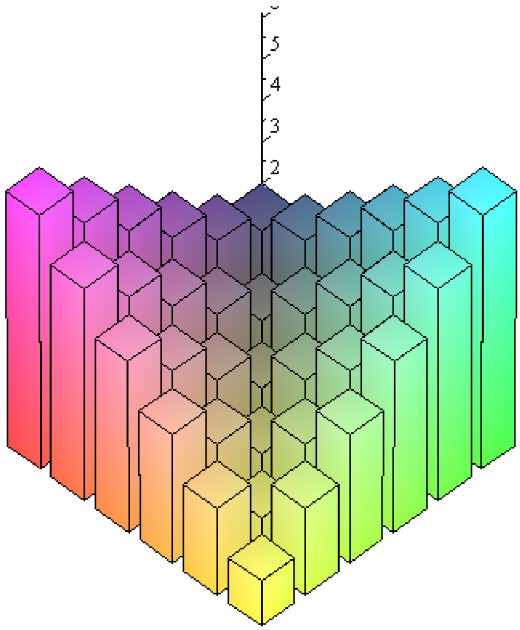
> $A := HilbertMatrix(6)$

$$A := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} \\ \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} \\ \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} \end{bmatrix}$$

> $B := ToeplitzMatrix([1, 2, 3, 4, 5, 6], symmetric)$

$$B := \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 & 2 & 3 \\ 5 & 4 & 3 & 2 & 1 & 2 \\ 6 & 5 & 4 & 3 & 2 & 1 \end{bmatrix}$$

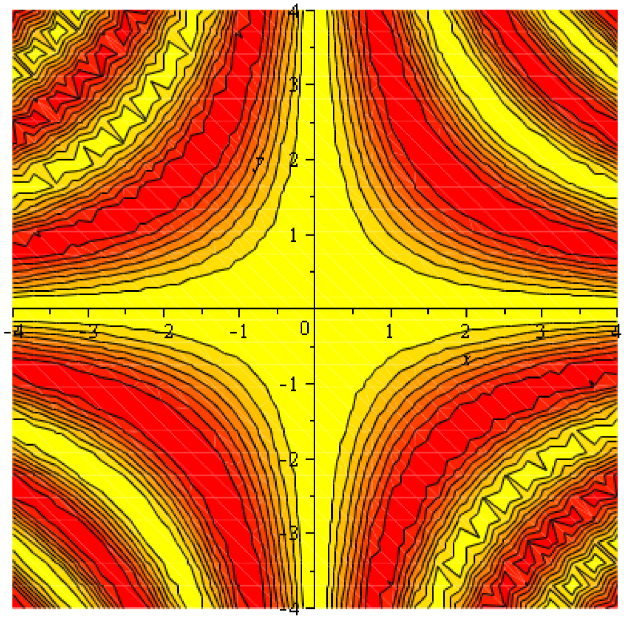
> $matrixplot(A+B, heights=histogram, axes=normal, gap=0.25, style=patch)$



The contourplot Command

The **contourplot** command generates a topographical map for an expression or function. To create a smoother and more precise plot, increase the number of points using the **numpoints** option.

> $contourplot(\cos(x\,y), x=-4..4, y=-4..4, filled=true, numpoints=750)$

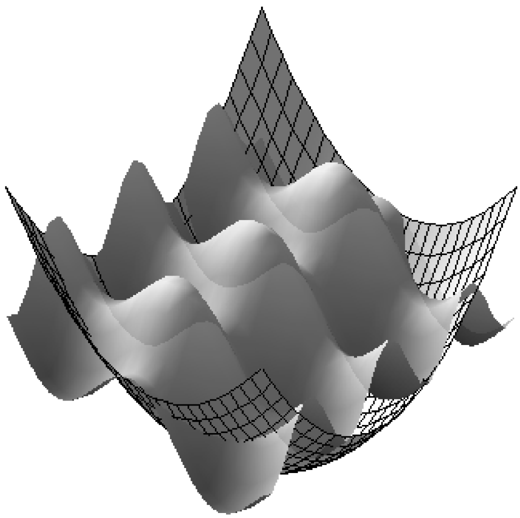


Multiple Plots in the Same Plot Region

List of Expressions

To display multiple expressions in the same plot region, enter the expressions in a **list** data structure. To distinguish the surfaces, apply different shading options, styles, or colors to each surface.

> $plot3d\left([\cos(5\,x)+\cos(5\,y), x^2+3\,y^2-4], x=-2\,..2, y=-1\,..1, shading=[zgrayscale, none], color=[default, grey], style=[patchnogrid, patch], lightmodel=light3, transparency=0.1\right)$



The display Command

To display different types of plots in the same plot region, use the **display** command in the **plots** package.

This example plots a curve over a hill with the shadow of the curve projected onto the hill.

> $z := 10\left(x^2+y^5+\frac{x}{5}\right)\mathrm{e}^{(-x^2-y^2)}:$

> $hill := plot3d(z, x=-2\,..2, y=-2.5\,..2.5, shading=zhue, style=patchnogrid, lightmodel=light3, orientation=[-125, 60]):$

> $xt := \cos(t)$:

> $yt := 2\sin(t)$:

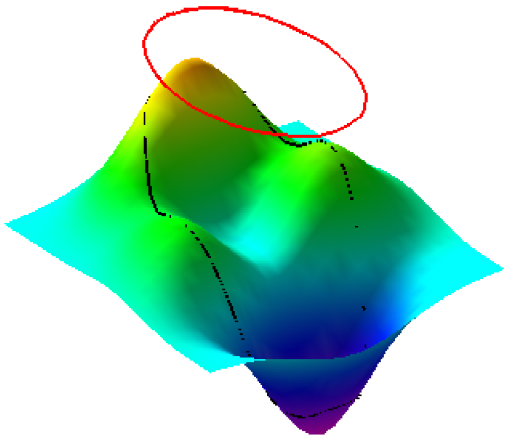
Maple can draw curves in three-dimensional space.

> $curve := spacecurve([xt, yt, 10], t = 0..10, color = red, thickness = 2)$:

> $zt := subs(\{x = xt, y = yt\}, z)$:

> $shadow := spacecurve([xt, yt, zt], t = -\pi..\pi, color = black, thickness = 2)$:

> $display(hill, curve, shadow)$



6.3 Customizing Plots

Maple provides many plot options to display the most aesthetically pleasing, illustrative results. Plot options include line styles, colors, shadings, axis styles, and titles where applicable. Plot options are applied using the **Interactive Plot Builder**, the context menus, or as options in the command syntax.

Interactive Plot Builder Options

The **Interactive Plot Builder** offers most of the plot options available in Maple in an easy-to-use interface.

Example:

Launch the Interactive Plot Builder and enter the expression:

1. Add the expression **2*x^5-10*x^3+6*x-1**. For information on interacting with the **Interactive Plot Builder**, see *Example 1 - Display a plot of a single variable expression (page 240)*.

Set the x-axis range:

2. In the **Select Plot Type** window, change the x-axis range to **-2 .. 2**.

In the Plot Options window:

3. From the **Line** group box, select **dot** from the left drop-down menu.
4. From the **Color** group box, select **Blue**.
5. From the **Axes** group box, select **frame**.
6. From the **Title** group box, enter **My Plot** in the text field.

Plot the expression:

7. Click **Plot**.

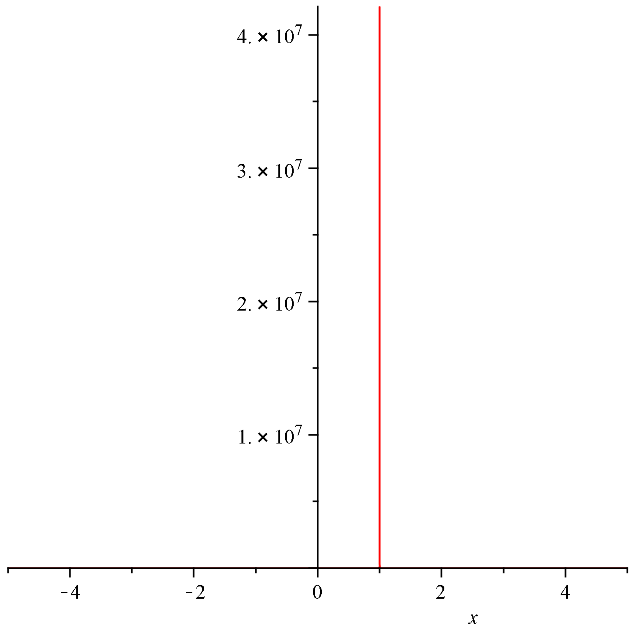
Context Menu Options

Using the context menu, you can alter a plot by right-clicking (**Control**-click for Macintosh) the plot output. You can also access a large subset of plot options using the **Plot** toolbar and **Plot** menu options. These menus display when a plot region is selected. Regardless of the method used to insert a plot into Maple, you can use the context menu to apply different plot options. For a list of options available when plotting in two and three dimensions, see *The plot and plot3d Options (page 267)*.

2-D Plot Options

Some plots do not display as you would expect using default option values. A expression with a singularity is one such example.

> $plot\left(\frac{1}{(x-1)^2}, x = -5..5\right)$



In the previous plot, all interesting details of the plot are lost because there is a singularity at x = 1. The solution is to view a narrower range, for example, from y = 0 to 7.

Alter the y-axis range:

1. Right-click the plot region. Select **Axes**, and then **Properties**.

2. In the **Axes Properties** dialog, click the **Vertical** tab.

3. De-select the **Use data extents** check box and enter **0** and **7** in the

Range min and **Range max** text regions, respectively.

4. Click **Apply** to view the changes, or **OK** to return to the document.

Change the color:

5. Place the mouse pointer on the curve and right-click (**Control**-click,

Macintosh). **Note:** The curve is selected when it becomes highlighted.

Select **Color**, and then **Green**.

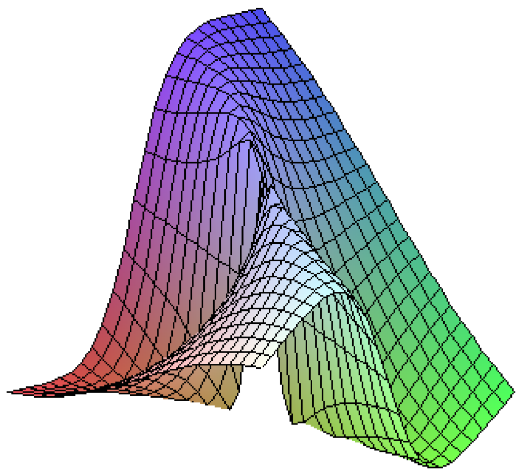
Change the line style:

6. Select **Style**, and then **Point**.

3-D Plot Options

By default, Maple displays the graph as a shaded surface with a wireframe and scales the plot to fit the window. To change these options, use the context menu.

> $plot3d\left(\frac{x\,y}{x^2+y^2}, x=-10..10, y=-5..5\right)$



Maple has many preselected light source configurations.

Change the style:

1. Right-click the plot region. Select **Style** → **Surface**.

Apply a light scheme:

2. Select **Lighting** → **Light 1**.

Change the color:

3. Select **Color** → **Z (Grayscale)**.

Change the axes style:

4. Select **Axes** → **Boxed**.

Alter the glossiness:

5. Select **Glossiness** and then select **Set...**. Using the slider, adjust the level of glossiness.

The plot and plot3d Options

If you are using commands to insert a plot, you can specify plot options as arguments at the end of the calling sequence. You can specify the options in any order. Applying plot options in the command syntax offers a few more options and greater control than what is available in the **Interactive Plot Builder** and context menus.

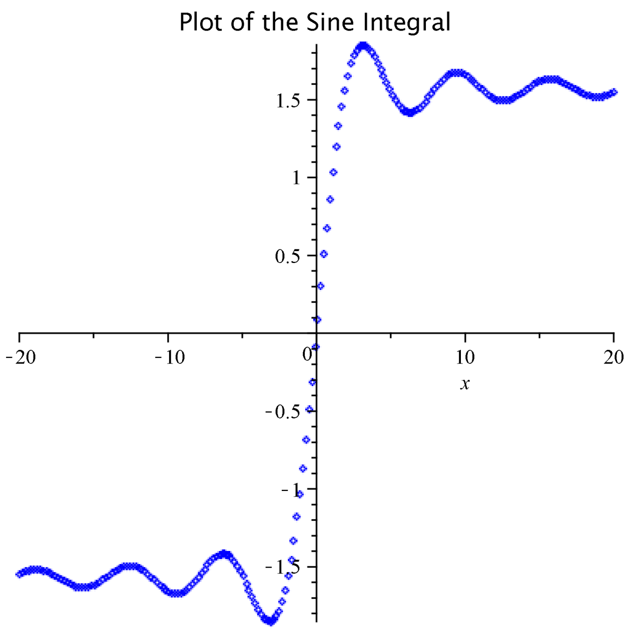
Table 6.3: Common Plot Options

Option	Description
axes	Defines the type of axes, one of: **boxed**, **frame**, **none**, or **normal**
caption	Defines the caption for the plot
color	Defines a color for the curves to be plotted
font	Defines the font for text objects in the plot
glossiness (3-D)	Controls the amount of light reflected from the surface
gridlines (2-D)	Defines gridlines in the plot
lightmodel (3-D)	Controls the light model to illuminate the plot, one of: **none**, **light1**, **light2**, **light3**, or **light4**
linestyle	Defines the dash pattern used to render lines in the plot, one of: **dot**, **dash**, **dashdot**, **longdash**, **solid**, **spacedash**, and **spacedot**
legend (2-D)	Defines a legend for the plot
numpoints	Controls the minimum total number of points generated
scaling	Controls the scaling of the graph, one of: **constrained** or **unconstrained**
shading (3-D)	Defines how the surface is colored, one of: **xyz**, **xy**, **z**, **zgrayscale**, **zhue**, or **none**
style	Defines how the surface is to be drawn, one of: **line**, **point**, **polygon**, or **polygonoutline** for 2-D plots; **contour**, **point**, **surface**, **surfacecontour**, **surfacewireframe**, **wireframe**, or **wireframeopaque** for 3-D plots
symbol	Defines the symbol for points in the plot, one of: **asterisk**, **box**, **circle**, **cross**, **diagonalcross**, **diamond**, **point**, **solidbox**, **solidcircle**, or **soliddiamond** for 2-D plots; **asterisk**, **box**, **circle**, **cross**, **diagonalcross**, **diamond**, **point**, **solidsphere**, or **sphere** for 3-D plots
title	Defines a title for the plot
thickness	Defines the thickness of lines in the plot
transparency (3-D)	Controls the transparency of the plot surface

Option	Description
view	Defines the minimum and maximum coordinate values of the axes displayed on the screen

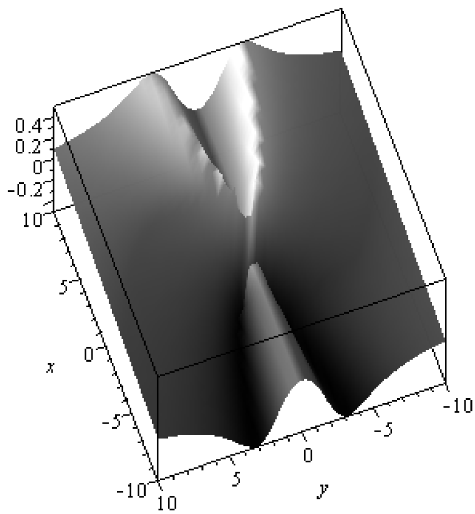
For a complete list of plot options, refer to the **?plot/options** and **?plot3d/options** help pages.

> *plot*(Si(*x*), *x* = −20 ..20, *title* = "Plot of the Sine Integral", *titlefont* = [*HELVETICA*, 12], *color* = *blue*, *style* = *point*)



To create a smoother or more precise plot, calculate more points using the **numpoints** option.

$$> plot3d\left(\frac{x\,y^2}{x^2+y^4}, x=-10..10, y=-10..10, axes=boxed, numpoints=1500, lightmodel=light3, shading=zgrayscale, orientation=[160, 20], style=patchnogrid\right)$$



6.4 Analyzing Plots

Point Probe, Rotate, Pan, and Zoom Tools

To gain further insight into a plot, Maple offers various tools to analyze plot regions. These tools are available in the **Plot menu** menu, **Context Bar**, and in the context menu under **Transform** when the plot region is selected.

Table 6.4: Plot Analysis Options

Name	Icon	Description
Point probe (2-D)		Display the coordinates corresponding to the cursor position on a two-dimensional plot in the context bar (upper left-hand corner).

Name	Icon	Description
Rotate (3-D)		Rotate a three-dimensional plot to see it from a different point of view.
Pan		Pan the plot by changing the view ranges for 2-D plots; smartplots re-sample to reflect the new view. Change the position of the plot in the plot region for 3-D plots.
Zoom		Zoom into or out of the plot by changing the view ranges for 2-D plots; smartplots re-sample to reflect the new view. Make the plot larger or smaller in the plot window for 3-D plots.
Selection Tool		Use the Selection Tool to select the information displayed in the point probe tool tooltip. You can choose to display coordinates derivated from converted pixel coordinates or data points derived from the original data points.

6.5 Creating Animations

Animations allow you to emphasize certain graphical behavior, such as the deformation of a bouncing ball, more clearly than in a static plot. A Maple animation is a number of plot frames displayed in sequence, similar to the action of movie frames. To create an animation, use the **Interactive Plot Builder** or commands.

Interactive Plot Builder

Creating Animations Using the Interactive Plot Builder:

Launch the Interactive Plot Builder and enter the expression:

1. Add the expression **sin(i*sqrt(x^2+y^2)/10)**.

For information on interacting with the **Interactive Plot Builder**, see *Example 1 - Display a plot of a single variable expression (page 240)*.

In the Select Plot Type window:

2. From the **Select Plot Type** drop-down menu, select **Animation**.

3. Change the **x Axis** range to **-6** .. **6**.

4. Change the **y Axis** range to **-6** .. **6**.

5. Change the **Animation Parameter** (**i**) range to **1** .. **30**.

In the Plot Options window:

6. From the **Style** group box, select **surface**.

7. From the **Color** group box, in the **Light Model** drop-down menu, select **red-turquoise**.

8. From the **Color** group box, in the **Shading** drop-down menu, select **z (grayscale)**.

9. In the **View** group box, select the **Constrained Scaling** check box.

Plot the expression:

10. Click **Plot**.

> *plots*[*interactive*]( );

For information on playing the animation, see *Playing Animations (page 275)*. To see the Maple syntax used to generate this plot, see *Maple Syntax for Creating Animations: Interactive Plot Builder Example (page 271)*.

The plots[animate] Command

You can also use the **animate** command, in the **plots** package, to generate animations.

Table 6.5: The animate Command

animate(plotcommand, plotarguments, t=a..b, ...)
animate(plotcommand, plotarguments, t=L, ...)
• **plotcommand** - Maple procedure that generates a 2-D or 3-D plot
• **plotarguments** - arguments to the plot command
• **t=a..b** - name and range of the animation parameter
• **t=L** - name and list of real or complex constants

To access the command, use the short form name after invoking the **with(plots)** command.

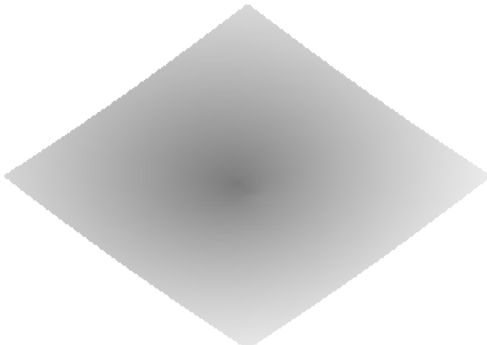
> *with*(*plots*):

Maple Syntax for Creating Animations: Interactive Plot Builder Example

The following example shows the plotting command returned by the example in *Interactive Plot Builder (page 270)*.

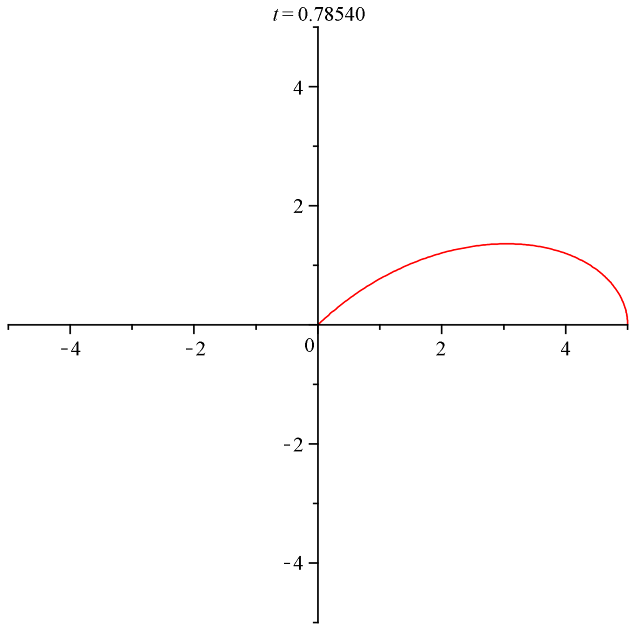
> $animate\left(plot3d,\left[\sin\left(\frac{i\sqrt{x^2+y^2}}{10}\right), x=-6..6, y=-6..6, style=patchnogrid, lightmodel=light3, shading=zgrayscale, scaling=constrained\right], i=1..30\right)$

$i = 1.$



Animate a 2-D plot

> $animate\left(plot, \left[5\cos(2\,\theta), \theta = 0\,..t, coords = \mathrm{polar}\right], t = \frac{\pi}{4}\,..2\,\pi, frames = 50\right)$



For more information on the **animate** command, refer to the **?plots[animate]** help page.

The plot3d[viewpoint] Command

You can use the **viewpoint** command to create an animation in which the position from which you view a 3-D plot moves in all directions and in various angles around the plot surface based on coordinates and parameters you specify. This type of animation creates the effect of flying through, around, beside, towards, and away from a plot surface in three-dimensional space.

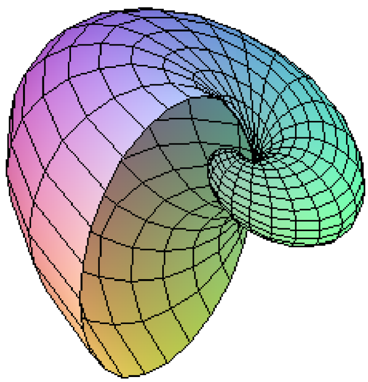
The moveable position from which you view the surface is called the *camera*. You can specify the orientation of the camera to view different sides of a surface, the path along which the camera moves throughout and around a surface, and the location of the camera in 3-D space in each animation frame. For example, you can specify coordinates to move the camera to specific points beside a surface; a pre-defined camera path to move the camera in a circle around the surface; and the range of view to move the camera close to or away from the surface. Refer to the **?viewpoint** help page for information on the available options.

To animate the following examples, click the plot object and then click the play button () in the **Animation** context bar.

Example 1: Moving the Camera Around a 3-D Plot

In the following example, a pre-defined path **circleleft** moves the camera in a counter-clockwise circle around the plot surface.

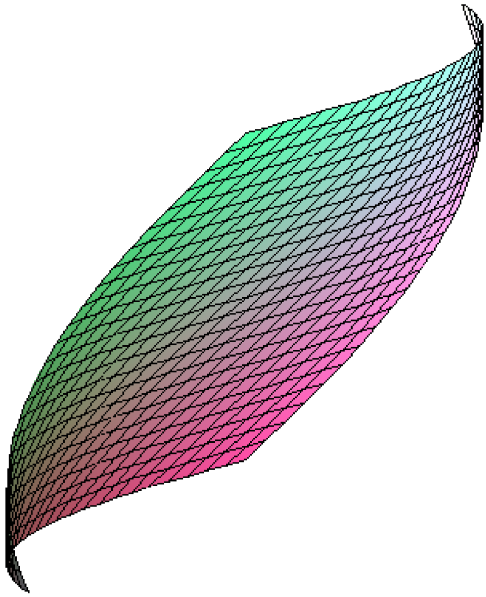
> $plot3d\left(1.3^{x}\sin(y), x=-1..2\pi, y=0..\pi, coords=spherical, style=patch, viewpoint=["circleleft"]\right)$



Example 2: Specifying a Path to Move the Camera Towards and Around a 3-D Plot

In the following example, a camera path is specified to zoom into and view different sides of the plot surface.

> $plot3d\left(\sin(x+y), x=-1..1, y=-1..1, shading=xyz, viewpoint=\left[path=\left[\left[50*x, 90*\cos(x), 100*\sin(x)\right], x=-2*\pi..\pi\right]\right]\right)$



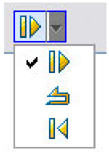
6.6 Playing Animations

Animation Context Bar

To run the animation, click the plot to display the **Animate** context bar.

Table 5.6: Animation Options

Name	Icon	Description
Previous Frame		View the previous frame in the animation.
Stop		Stop the animation.
Play		Play the selected animation.
Next Frame		View the next frame in the animation.
Current Frame		Slider control for viewing individual frames of an animated plot.

Name	Icon	Description
Forward Oscillate Backward		Forward - Play the animation forward. Oscillate - Play the animation forward and backward. Backward - Play the animation backward.
Single Continuous		Single - Run the animation in single cycle mode. The animation is displayed only once. Continuous - Run the animation in continuous mode. The animation repeats until you stop it.
Frames per second		Set the animation to play at a faster or slower speed.
Point probe		Determine the coordinates of a 2-D plot at the position of the cursor.
Zoom		Zoom into or out of the plot by changing the view ranges.
Pan		Pan the plot by changing the view ranges.
Rotate (3-D)		Rotate a three-dimensional plot to see it from a different point of view.

You can also run the animation using the context menu or the **Plot** menu.

6.7 Customizing Animations

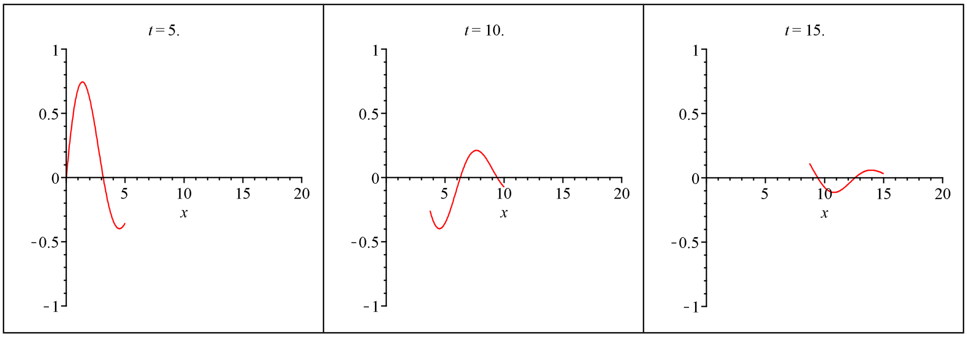
The display options that are available for static plots are also available for Maple animations.

Interactive Plot Builder Animation Options

Using the **Interactive Plot Builder**, you can apply various plot options within the **Plot Options** window. See *Interactive Plot Builder (page 270)*.

Context Menu Options

As with static plots, you can apply plot options to the animation by right-clicking (**Control**-click for Macintosh) the animation output.



Customize the animation using the context menu:

1. To change the line style, right-click the plot region. Select **Style** → **Point**.
2. To remove the axes, select **Axes** → **None**.

The animate Command Options

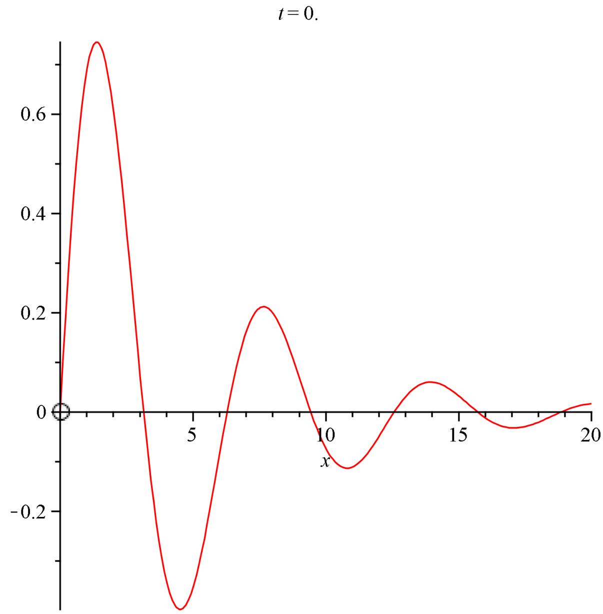
The **animate** command offers a few options that are not available for static plots. Refer to the **?animate** help page for information on these additional options. By default, a two-dimensional animation consists of sixteen plots (frames) and a three-dimensional animation consists of eight plots (frames). To create a smoother animation, increase the number of frames using the **frames** option.

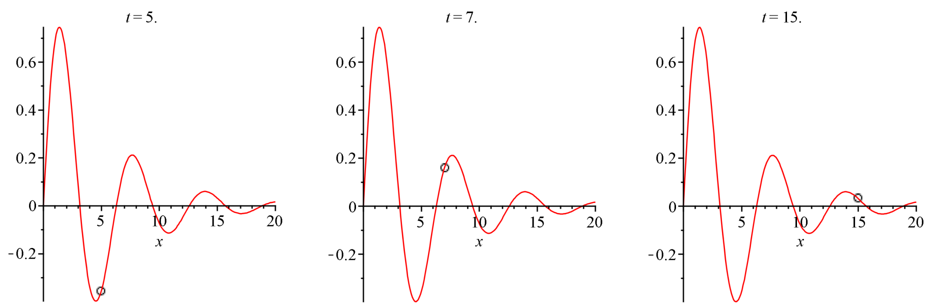
Note: Computing more frames increases time and memory requirements.

> $sinewave := plot\left(\sin(x)\,\mathrm{e}^{-\frac{x}{5}}, x = 0\,..20\right):$

> $ball := \mathbf{proc}(x, y)\ plots[pointplot]([[x, y]], symbol = circle, symbolsize = 20)\ \mathbf{end\ proc}:$

$$> \ plots[animate]\left(ball, \left[t, \sin(t)\,\mathrm{e}^{-\frac{t}{5}}\right], t=0\,..20, frames=60, background=sinewave\right)$$





6.8 Exporting

You can export a generated plot or animation to an image in various file formats, including DXF and X3D (for 3-D plots), EPS, GIF, JPEG/JPG, POV, Windows BMP, and WMF. Exporting an animation to GIF produces an animated image file. The exported images can be included in presentations, web pages, Microsoft Word, or other software.

To export an image:

1. Right-click the plot region (**Control**-click for Macintosh).

2. Select **Export** and the file format.

Alternatively:

1. Click the plot.

2. From the **Plot** menu, select **Export**, and then the file format.

Maple has various plot drivers. By setting the **plotdevice**, a file can be automatically created without returning the image to the document. For more information, refer to the **?plot,device** help page.

6.9 Code for Color Plates

Generating impressive graphics in Maple can require only a few lines of code, as shown by the examples in this chapter. However, other graphics require many lines of code. Code for the color plates is available at the Maple Application Center.

From the **Help** menu, select **On the Web**, **User Resources**, and then **Application Center**.

To access the color plate code:

1. Go to the Maple Application Center.

2. In the **Keyword or phrase** region, enter **Color Plate**.

7 Creating Mathematical Documents

Maple allows you to create powerful documents as business and education tools, technical reports, presentations, assignments, and handouts.

You can:

- Copy, cut, and paste information
- Format text for reports or course material
- Add headers and footers
- Insert images, tables, and symbols
- Generate two- and three-dimensional plots and animations
- Sketch in the document or on a plot
- Insert hyperlinks to other Maple files, web sites, or email addresses
- Place instructions and equations side by side
- Bookmark specific areas
- Easily update, revise, and distribute your documents

In this chapter, we will create a document that demonstrates many of Maple's documentation features. For further examples, note that this guide was written using Maple.

7.1 In This Chapter

Section	Topics
Tables (page 306) - Create tables and modify their attributes	• Creating a table • Cell contents • Navigating table cells • Modifying Structural Layout • Modifying Physical Dimensions • Modifying Appearance • Printing Options • Execution Order • Tables in the Classic Worksheet
Canvas (page 318) - Sketch an idea in the document by inserting a canvas	• Insert a Canvas • Drawing • Canvas Style • Inserting Images
Hyperlinks (page 323) and Bookmarks - Add hyperlinks to various sources	• Inserting a Hyperlink in the Document • Linking to an Email Address, Dictionary Topic, Help Page, Maplet Application, Web Page, or Document • Bookmarks
Embedded Components (page 329) - Insert buttons, sliders, and more in your document	• Overview of available components • Example using a task template
Spell Checking (page 332) - Verify text with the Maple spell checking utility	• How to Use the **Spellcheck** Utility • Selecting a Suggestion • User Dictionary
Creating Graded Assignments (page 334) - Create documents for automated testing and assessment	• Creating a Question • Viewing Questions in Maple • Saving Test Content
Worksheet Compatibility (page 335) - Compatibility Issues	• Classic Worksheet interface does not support all Standard Worksheet interface features

7.2 Document Formatting

To begin, create a new Maple document. From the **File** menu, select **New** → **Document Mode**. For this example, you can copy and paste text from any file. The example text below is from a Maple help page, **?plot**, but the formatting has been removed for demonstration purposes.

Copy and Paste

You can cut, copy, and paste content within Maple documents, and from other sources.

To copy an expression, or part of an expression, to another location on the document:

1. Select the expression, or part of the expression, to copy.

2. From the **Edit** menu, select **Copy**.

3. Place the cursor at the insertion point.

4. From the **Edit** menu, select **Paste**.

Result:

plot - create a two-dimensional plot

Calling Sequence

plot(f, x)

plot(f, x=x0..x1)

plot(v1, v2)

Parameters

f - expression in independent variable x

x - independent variable

x0, x1 - left and right endpoints of horizontal range

v1, v2 - x-coordinates and y-coordinates

If you paste into a math input region, Maple interprets all the pasted content as input. If you paste into a text region, Maple interprets all the pasted content as text. However, note that 2-D Math retains its format in both input and text regions.

When you copy and paste to another application, in general, Maple retains the original structure.

Quick Character Formatting

The **Format→Character** menu provides access to the following quick formatting features: **Bold, Italic**, **Underline**, **Superscript**, **Subscript**, **font Color**, and **Highlight Color**.

To modify text:

1. In the document, select the text to modify.

2. From the **Format** menu, select **Character**, and then the appropriate feature.

For example, in the pasted text, select "Calling Sequences" and apply **Bold** character formatting.

Alternatively, use the context bar icons. For example, to apply a color to the parameters "f, x=x0..x1":

- Font Color Context Bar Icon 
- Highlight Color Context Icon 

For font and highlight colors, you can select from Swatches, a color wheel, RGB values, or choose a color using the eye dropper tool. See Figure 7.1.

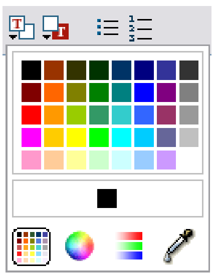


Figure 7.1: Select Color Dialog

In this example, choose a dark purple color, as in the help pages.

To format this text as bold, click the **Bold** toolbar icon, **B**. Also, select the text "Calling Sequence" and format as bold.

Result:

plot - create a two-dimensional plot

Calling Sequence

plot(f, x)

plot(**f, x=x0..x1**)

plot(v1, v2)

Parameters

f - expression in independent variable x

x - independent variable

x0, x1 - left and right endpoints of horizontal range

v1, v2 - x-coordinates and y-coordinates

Attributes Submenu: Setting Fonts, Character Size, and Attributes

You can also change various character attributes such as font, character size, style, and color in one dialog.

To modify text:

1. In the document, select text to modify.

2. From the **Format** menu, select **Character**, and then **Attributes**. The **Character Style** dialog opens. See Figure 7.2.

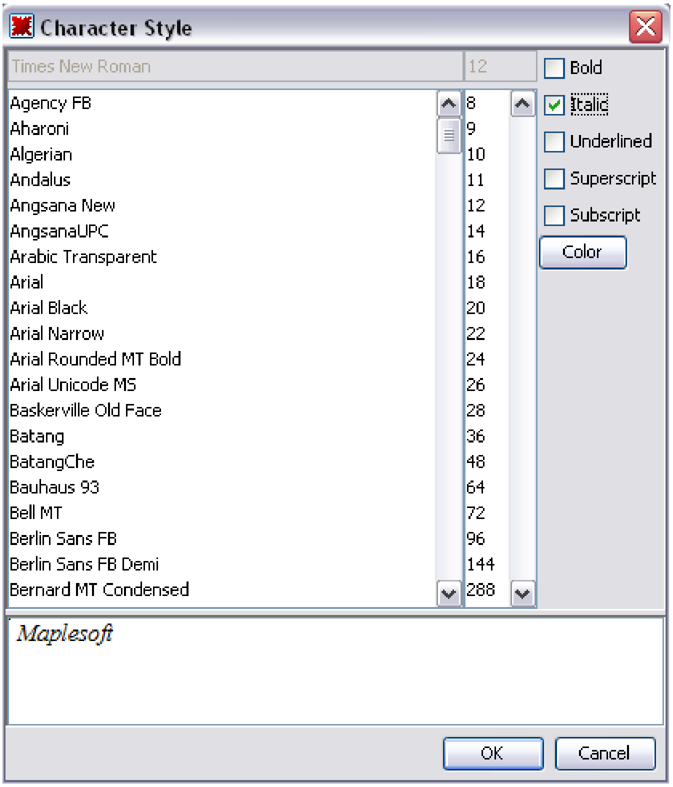


Figure 7.2: Character Style Dialog

Quick Paragraph Formatting

The **Format→Paragraph** menu provides access to the following quick alignment features: **Align Left**, **Center**, **Align Right**, and **Justify**.

To modify a paragraph:

1. In the document, select the paragraph to modify.

2. From the **Format** menu, select **Paragraph**, and then the appropriate feature.

Attributes Submenu: Spacing, Indent, Alignment, Bullets, Line Break, and Page Break

You can change various paragraph attributes in one dialog.

- From the **Format** menu, select **Paragraph**, and then **Attributes**. The **Paragraph Style** dialog opens. See Figure 7.3.

- When changing spacing, you must indicate units (inches, centimeters, or points) in the **Units** drop-down list.

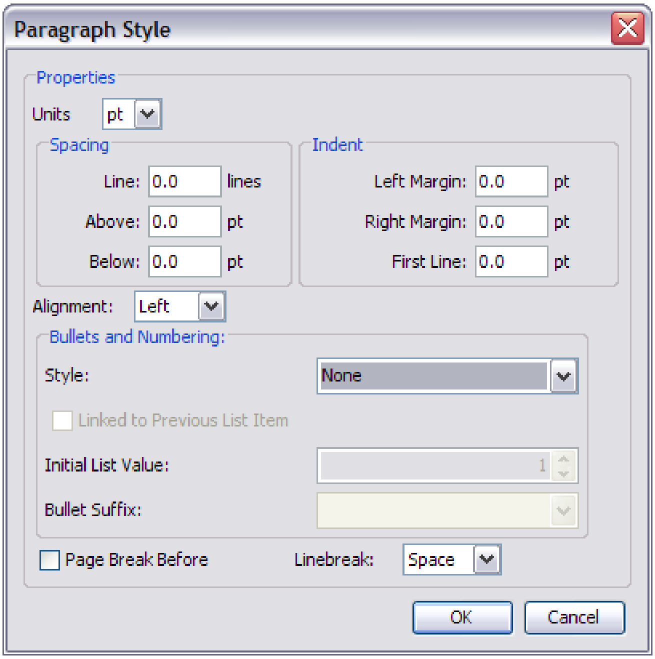


Figure 7.3: Paragraph Style Dialog

For example, in the pasted text, select all of the items under "Parameters", then open the **Paragraph Style** dialog. Notice that the spacing has already been set.

In the **Indent** section, change the **Left Margin** indent to 10.0 pt.

In the **Bullets and Numbering** section, click the **Style** drop-down and select **Dash**. Click **OK** to close the dialog and apply the styles.

Result:

plot - create a two-dimensional plot

Calling Sequence

plot(f, x)

plot(**f, x=x0..x1**)

plot(v1, v2)

Parameters

– f - expression in independent variable x

– x - independent variable

– x0, x1 - left and right endpoints of horizontal range

– v1, v2 - x-coordinates and y-coordinates

For more information, refer to the **?paragraphmenu** help page.

Character and Paragraph Styles

Maple has predefined styles for characters and paragraphs. A style is a set of formatting characteristics that you can apply to text in your document to change the appearance of that text. When you apply a style, you apply a group of formats in one action.

- A **character style** controls text font, size, color, and attributes such as bold and italic. To override the character style within a paragraph style, you must apply a character style or character formatting.
- A **paragraph style** controls all aspects of a paragraph's appearance, such as text alignment, line spacing, and indentation. In Maple, each paragraph style includes a character style.

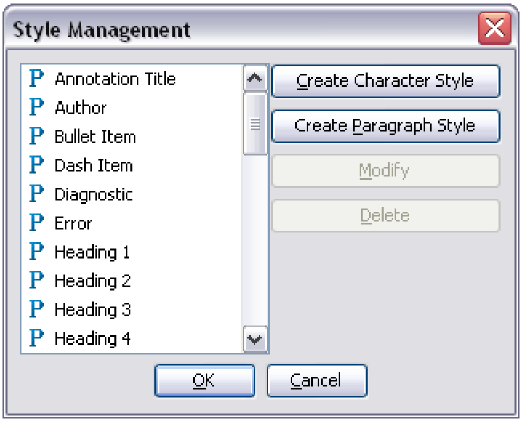


Figure 7.4: Style Management Dialog

Applying Character Styles

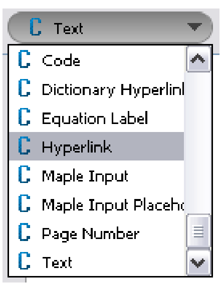
By using the drop-down list in the document context bar, you can apply:

- Existing Maple character styles.
- New styles that you have created through the **Style Management** (Figure 7.4) and **Character Style** (Figure 7.5) dialogs.

To apply a character style to text in your document:

1. Select the text to modify.

2. In the styles drop-down list in the context bar of your document, select an appropriate character style. All character styles are preceded by the letter **C**. The selected text now reflects the attributes of the character style you have chosen.



3. (Optional) If necessary, you can remove this style. From the **Edit** menu, select **Undo**.

Creating and Modifying Character Styles

You can create custom character styles to apply to text, or change existing character styles. New styles are automatically added to the styles drop-down list in the context bar of your document.

1. From the **Format** menu, select **Styles**. The **Style Management** dialog opens. See Figure 7.4.

To create a character style:

- Click **Create Character Style**. The **Character Style** dialog opens. See Figure 7.5.
- In the first row of the dialog, enter a style name in the blank text region.

To modify a character style:

- From the style list, select the character style to modify. Recall that all character styles are preceded by the letter **C**, while paragraph styles are preceded by the letter **P**.
- Click **Modify**. The **Character Style** dialog opens with the current attributes displayed. See Figure 7.5.

For either action, continue:

2. Select the properties for the new character style, such as font, size, attributes, and color. In the font attributes, the **Superscript** and **Subscript** check boxes are mutually exclusive. When you select one of the two check boxes, the other is disabled. You must clear one before selecting the other.

Note: A preview of the style is displayed in the last row of the **Character Style** dialog.

3. To save the style, click **OK** or to abandon, click **Cancel**. If you have modified a style, all text in your document that uses the altered style is updated to reflect the changes.

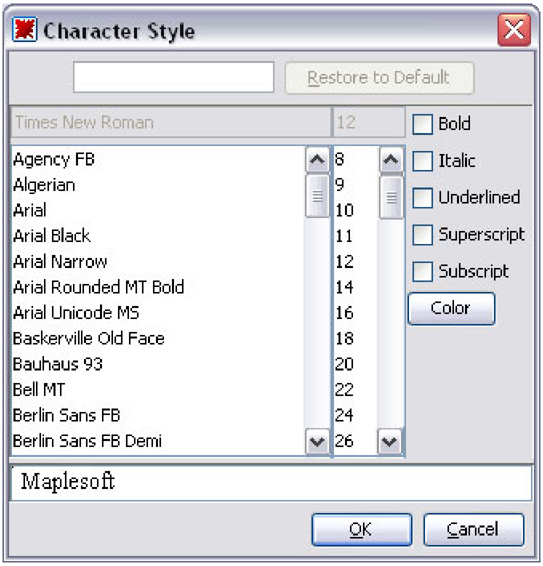
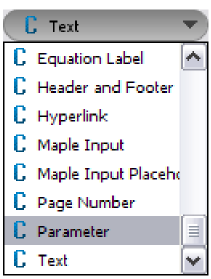


Figure 7.5: Character Style Dialog

For example, in the pasted text, suppose we want to create a character style for the bold, purple parameter.

- From the **Format** menu, select **Styles**, then click **Create Character Style**.
- Enter the style name, "Placeholder", and then select the character attributes. In this case, click the **Bold** check box. Then click the **Color** button and choose a dark purple. Click **OK** to create the character style.

Now you can apply the style to any text. Under **Calling Sequences**, select each list of parameters inside the command. To apply the style, from the **Styles** drop-down menu in the toolbar, select **Parameter**.



Result:

plot - create a two-dimensional plot

Calling Sequence

plot(**f, x**)

plot(**f, x=x0..x1**)

plot(**v1, v2**)

Parameters

– f - expression in independent variable x

– x - independent variable

– x0, x1 - left and right endpoints of horizontal range

– v1, v2 - x-coordinates and y-coordinates

Applying Paragraph Styles

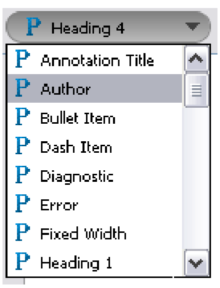
By using the drop-down list in the document context bar, you can apply:

- Existing Maple paragraph styles.
- New styles that you have created through the **Style Management** (Figure 7.4) and **Paragraph Style** (Figure 7.6) dialogs.

To apply a Maple paragraph style to text in your document:

1. Select the text to modify.

2. In the styles drop-down list in the context bar of your document, select an appropriate paragraph style. All Maple paragraph styles are preceded by the letter **P**. The selected text now reflects the attributes of the paragraph style you have chosen.



For example, to format the title of the pasted text as a title, first select the line: "plot - create a two-dimensional plot". In the Styles drop-down, select **Title**.

Result:

plot - create a two-dimensional plot

Calling Sequence

plot(**f**, **x**)

plot(**f**, **x=x0..x1**)

plot(**v1**, **v2**)

Parameters

– f - expression in independent variable x

– x - independent variable

– x0, x1 - left and right endpoints of horizontal range

– v1, v2 - x-coordinates and y-coordinates

3. (Optional) If necessary, you can remove this style. From the **Edit** menu, select **Undo**.

Creating and Modifying Paragraph Styles

You can create custom paragraph styles to apply to text, or change existing paragraph styles. New styles are automatically added to the styles drop-down list in the context bar of your document.

1. From the **Format** menu, select **Styles**. The **Style Management** dialog opens. See Figure 7.4.

To create a paragraph style:

- Click **Create Paragraph Style**. The **Paragraph Style** dialog opens. See Figure 7.6.
- In the first row of the dialog, enter a style name in the blank text field.

To modify a paragraph style:

- Select a paragraph style to modify. Recall that all paragraph styles are preceded by the letter **P**.
- Click **Modify**. The **Paragraph Style** dialog opens with the current attributes displayed.

For either action, continue:

4. In the **Units** drop-down menu, select the units used to determine spacing and indentation. Select from inches (**in**), centimeters (**cm**), or points (**pt**).

5. Select the properties to use for this paragraph style, such as **Spacing**, **Indent**, **Alignment**, **Bullets and Numbering**, **Page Break Before**, and **Linebreak**.

6. To add or modify a font style, click **Font**. The **Character Style** dialog opens. For detailed instructions, see *Creating and Modifying Character Styles (page 290)*.

7. To save the style, click **OK**, or to abandon, click **Cancel**. If you are modifying an existing style, all text in your document that uses the altered style is updated to reflect the changes.

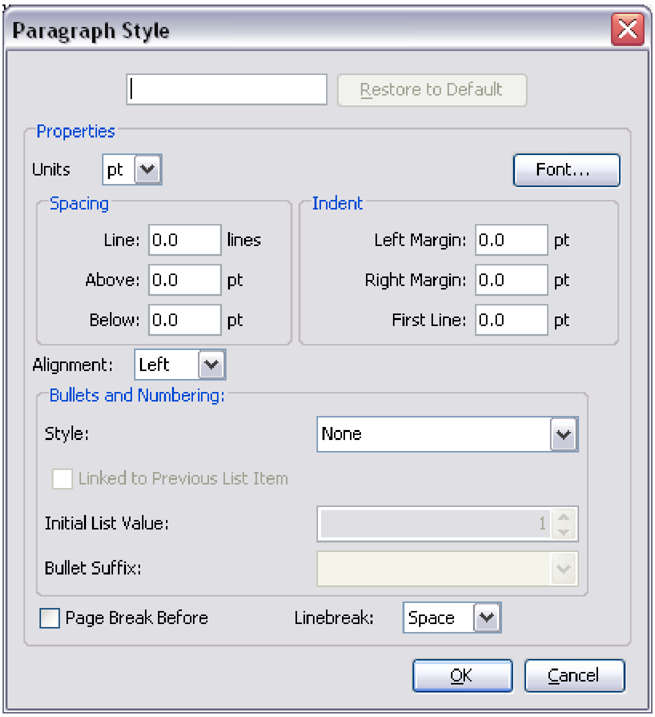


Figure 7.6: Paragraph Style Dialog

Style Set Management: Saving Styles for Future Use

You can use the style set of a particular document as the default style for all documents.

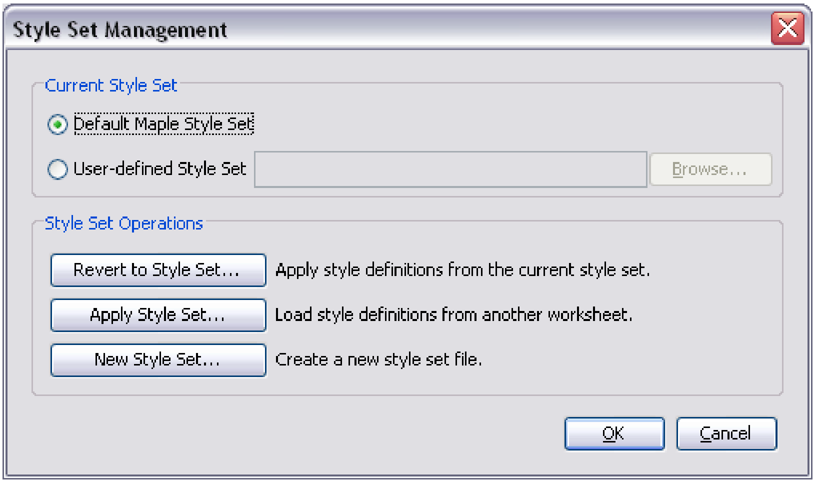
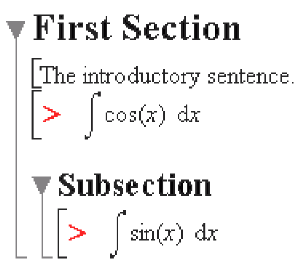


Figure 7.7: Style Set Management Dialog

For information on creating and managing style sets, see the **?worksheet/documenting/styles** help page.

Sections

You can organize your document into sections, either before or after the text has been entered.



Using the Insert Menu to Add Sections

1. Place the cursor in the paragraph or execution group above the location at which you want to insert a new section.

- If the cursor is inside a section, Maple inserts the new section after the current section.
- If the cursor is in an execution group, Maple inserts the new section after the execution group.

2. From the **Insert** menu, select **Section**. An arrow marks the start of the section.

3. Enter the section heading.

4. Press the **Enter** key.

5. Enter the body of the section.

Tips for Adding Subsections

The insert location of subsections is the same as for sections, with a few exceptions.

- Subsections are inserted at the current cursor location when in a subsection.
- To insert a subsection immediately after the current subsection, collapse the subsection and place the cursor in the subsection title.

Using the Indent and Outdent Toolbar Icons

You can shift sections to create or remove subsections.

	Enclose the selection in a section or subsection
	Outdent the selection to the next section level, if possible.

For example, to create two sections containing the two categories of information in the pasted text:

1. Select "Parameters" and all of the items under it.

2. Click the Indent toolbar item.

3. Cut and paste "Parameters" from inside the section to its title.

4. Similarly, create a section with the title "Calling Sequence", containing the items under that heading.

Result:

plot - create a two-dimensional plot

Calling Sequence

plot(**f, x**)
plot(**f, x=x0..x1**)
plot(**v1, v2**)

Parameters

– f - expression in independent variable x
– x - independent variable
– x0, x1 - left and right endpoints of horizontal range
– v1, v2 - x-coordinates and y-coordinates

Note: the section titles are automatically formatted as section titles, but you can change the formatting through the **Paragraph Style** dialog.

Headers and Footers

You can add headers and footers to your document that will appear at the top and bottom of each page when you print the document.

To add or edit headers and footers:

From the **View** menu, select **Header Footer**. The **Header Footer** dialog appears. See Figure 7.8.

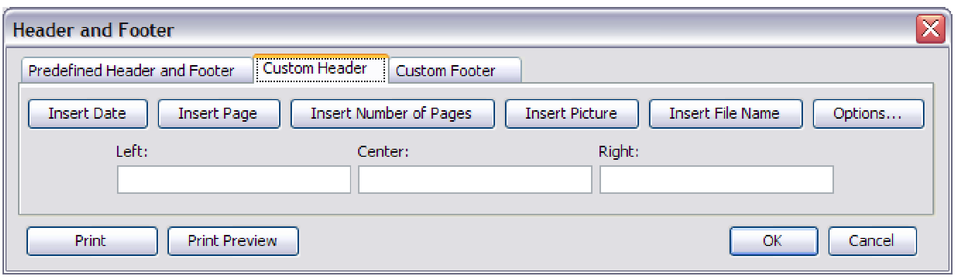


Figure 7.8: Header and Footer Dialog - Custom Header

The available elements include the current date, page number, number of pages, an image, the filename, or any plain text. These elements can be placed in the left or right corner or the center of the page.

You can choose one of the predefined header or footer styles in the **Predefined Header and Footer** tab, or create your own by clicking the **Custom Header** or **Custom Footer** tab.

For more information on header and footer options, refer to the **?headerfooter** help page.

Show or Hide Worksheet Content

You can hide document elements of a specific type so that they are not visible. This does not delete them, but hides them from view. Hidden elements are not printed or exported, but they can be copied and pasted.

In a document, use the **Show Contents** dialog to hide all spreadsheets, input, output, or graphics, plus markers for section boundaries, execution group boundaries, hidden table borders on mouse pointer roll over, and annotations. The dialog is accessed from the **View→Show/Hide Contents** menu.

Using the Show Contents Dialog

A check mark beside the item indicates that all document elements of that type are displayed for the current document. See Figure 7.9.

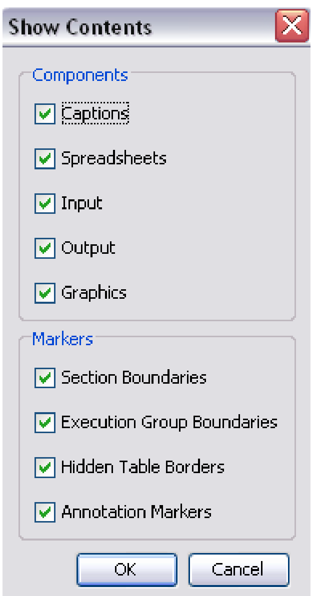


Figure 7.9: Show Contents Dialog

1. From the **View** menu, select **Show/Hide Contents**. The **Show Contents** dialog opens with all items selected for display.

2. Clear the check box associated with the document components or markers to hide them.

Note: By clearing the **Input** check box, only Maple Input and 2-D Math input, that is, 2-D Math content that has been evaluated, are hidden. Clearing the **Graphics** check box ensures that a plot, an image, or the **Canvas** inserted in the document by using the **Insert** menu option is also hidden.

Command Output Versus Inserted Content

Output is considered an element that results from executing a command. Inserted components are not considered output.

Consider the following examples.

The plot resulting from executing the **plot(sin)** command is considered output.

- To show a plot from the **plot(sin)** command, select both the **Output** and **Graphics** check boxes in the **Show Contents** dialog.

If you insert a plot by using the **Insert** menu option, that plot is not considered output. Therefore, if you clear the **Output** check box in the **Show Contents** dialog, that plot will be visible in the document.

- To hide an inserted plot, clear the **Graphics** check box in the **Show Contents** dialog.

Inserted images and the **Canvas** are not considered output. As such, they are not hidden if you clear the **Output** check box.

- To hide an inserted image or canvas, clear the **Graphics** check box in the **Show Contents** dialog.

Indentation and the Tab Key

The Tab icon allows you to set the **Tab** key either to move between placeholders or to indent. For example, with the Tab icon off, click the exponent button in the **Expression** palette. The expression is inserted with the first placeholder highlighted. To move to the next placeholder, use the **Tab** key.

	Tab icon **off**. Allows you to move between placeholders using the **Tab** key.
	The Tab icon is disabled when using 2-D Math (**Math** mode), and as such, the **Tab** key allows you to move between placeholders.
	Tab icon **on**. Allows you to indent in the document using the **Tab** key.

7.3 Commands in Documents

Document Blocks

With document blocks, you can create documents that present text and math in formats similar to those found in business and education documents.

In a document block, an input prompt or execution group is not displayed.

By hiding Maple input such that only text and results are visible, you create a document with better presentation flow. Before using document blocks, it is recommended that you display **Markers**. A vertical bar is displayed along the left pane of the document. Icons representing document blocks are displayed in this vertical bar next to associated content.

To activate Markers:

- From the **View** menu, select **Markers**.

For further details on document blocks, see *Document Blocks (page 51)* in Chapter 1.

Working with Document Blocks

In document mode, each time you press **Enter**, a new document block appears. Documents consist of a series of document blocks.

1. Create a new document block after the last section of the pasted example, either by pressing **Enter**, or by selecting, from the **Format** menu, **Create Document Block**.

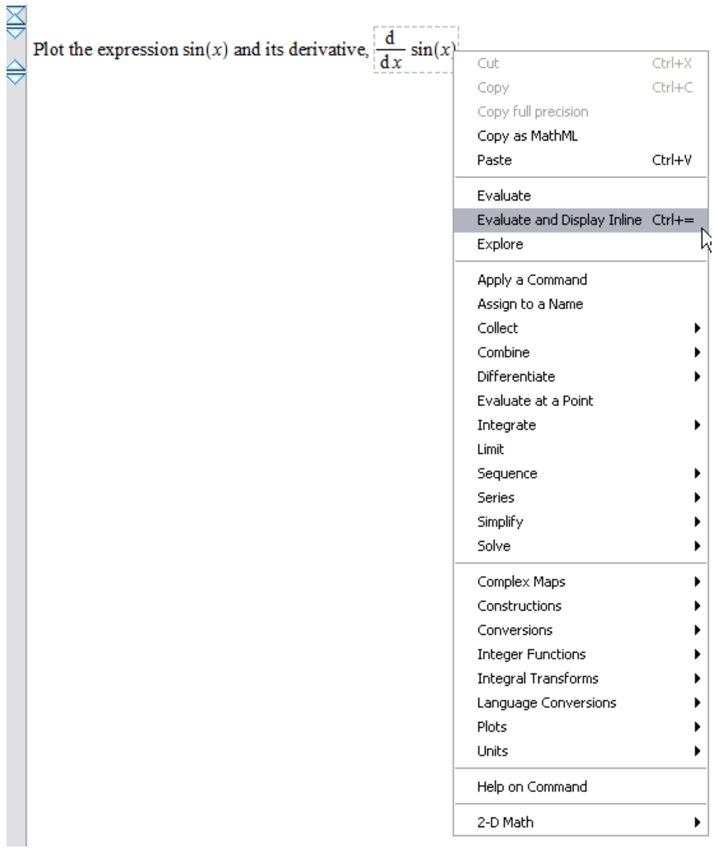
2. Enter text and an expression to evaluate. For example, enter "Plot the expression $\sin(x)$ and its derivative, $\frac{d}{dx}\sin(x)$ ". For detailed instructions on entering this phrase, see *Example 6 - Enter Text and 2-D Math in the Same Line (page 31)* in Chapter 1.

3. Select the expression **Control**-click, for Macintosh) to display the context menu.

4. Click the **Evaluate and Display Inline** menu item. The expression is evaluated.

5. Check that the input mode is **Text**, then enter the rest of the sentence: ", in the same plot." See Figure 7.10.

Before



After

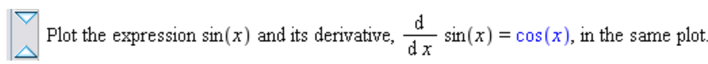


Figure 7.10: Working with Document Blocks

Result:

plot - create a two-dimensional plot

Calling Sequence

plot(**f, x**)
plot(**f, x=x0..x1**)
plot(**v1, v2**)

Parameters

– f - expression in independent variable x
– x - independent variable
– x0, x1 - left and right endpoints of horizontal range
– v1, v2 - x-coordinates and y-coordinates

Plot the expression $\sin(x)$ and its derivative, $\frac{d}{dx}\sin(x) = \cos(x)$, in the same plot.

Inline Document Output

Document blocks can display content inline, that is, text, input, and output in one line as presented in business and education documents. In document mode, content is displayed inline by default.

To display content inline:

1. Place the cursor in the document block.

2. From the **View** menu, select **Inline Document Output**.

View Document Code

To view the contents, that is, all code and expanded execution groups within a document block, you must expand the document block.

1. Place the cursor in the document block region.

2. From the **View** menu, select **Expand Document Block**.

Plot the expression $\sin(x)$ and its integral,

> $\int \sin(x)\, dx$

output redirected...

=

```
> print((1)); # input placeholder
```

$-\cos(x)$

, in the same plot.

3. To hide code again, select **View** and then **Collapse Document Block**.

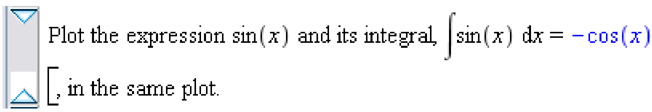
Expand an Execution Group within a Document Block

An execution group is a grouping of Maple input with its corresponding Maple output. It is distinguished by a large square bracket at the left called a group boundary.

As document blocks can contain many execution groups, you can select to expand an execution group within a document block.

1. Place the cursor near the end of the document block region.

2. From the **View** menu, select **Expand Execution Group**.



3. To hide the group, select **View** and then **Collapse Execution Group**.

Switch between Input and Output

1. Place the cursor in the document block region.

2. From the **View** menu, select **Toggle Input-Output Display**.

Input from any executable math or commands is displayed in one instance, or only output is displayed.

Typesetting

You can control typesetting and 2-D Math equation parsing options in the Standard Worksheet interface. Extended typesetting uses a customizable set of rules for displaying expressions.

The rule-based typesetting functionality is available when the **Typesetting level** is set to **Extended** (**Tools→Options→Display** tab). This parsing functionality applies to 2-D Math editing (**Math** mode) only.

For example, you can change the display of derivatives to suit the content and audience of your document.

> $\frac{d}{dx} f(x)$

$$\frac{d}{dx} f(x)$$

Tools→Options→Display tab: Typesetting level = Maple Standard.

> $\frac{d}{dx} f(x)$

$$f'(x)$$

Tools→Options→Display tab: Typesetting level = Extended.

To specify rules, use the **Typesetting Rule Assistant**.

- From the **View** menu, select **Typesetting Rules**. The **Typesetting Rule Assistant** dialog opens.

For more information, see the **?Typesetting**, **?TypesettingRuleAssist**, and **?OptionsDialogDisplay** help pages.

Auto-Execute

The **Autoexecute** feature allows you to designate regions of a document for automatic execution. These regions are executed when the document opens or when the command is executed. This is useful when sharing documents. Important commands can be executed as soon as the user opens your document. The user is not required to execute all commands.

Setting the Auto-Execute Feature

1. Select the region to be automatically executed when the document opens.

2. From the **Format** menu, select **Autoexecute**, and then **Set**.

Regions set to **Autoexecute** are denoted by exclamation mark symbols in the Markers region (**View → Markers**), .

For example, to display a plot in your document without saving the plot, making your document use less memory, you can set a plot command to autoexecute.

1. After the plot instruction, enter a Maple prompt (**Insert** → **Execution Group** → **After Cursor**).

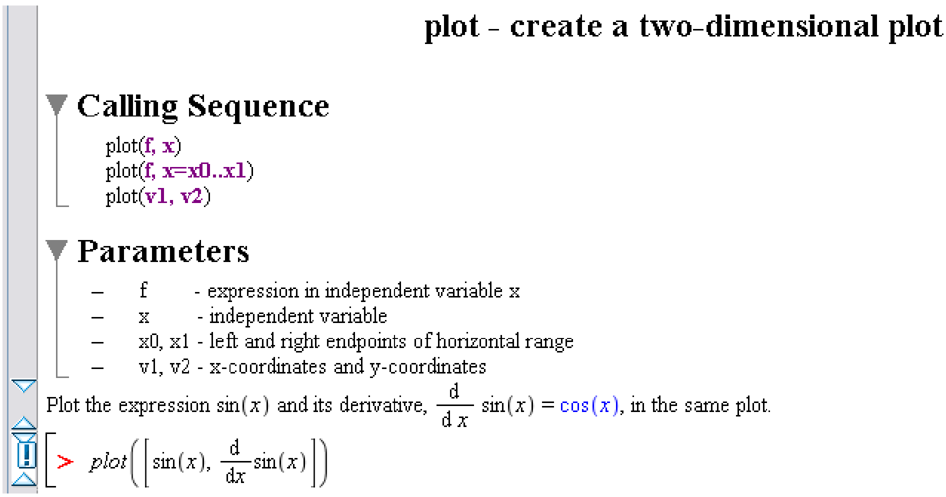
2. Enter the plot command: $plot\left(\left[\sin(x), \int \sin(x)\, dx\right]\right)$ and press **Enter** to execute.

3. Select the plot, then select **Edit** → **Remove Output** → **From Selection**.

4. Place the cursor in the plot command, the select **Format** → **Autoexecute** → **Set**.

5. Save and close the document; on reopening, the command is re-executed.

Result:



Removing the Auto-Execute Setting

To remove the setting in a region:

1. Select the region.

2. From the **Format** menu, select **Autoexecute**, and then **Clear**.

To remove all autoexecuted regions from a document:

- From the **Format** menu, select **Autoexecute**, and then **Clear All**.

Repeating Auto-Execution

To execute all marked groups:

- From the **Edit** menu, select **Execute**, and then **Repeat Autoexecution**.

Security Levels

By default, Maple prompts the user before automatically executing the document.

To set security levels for the autoexecute feature, use the **Security** tab in the **Options** dialog. For details, refer to the **?OptionsDialogSecurity** help page.

7.4 Tables

Tables allow you to organize content in a document.

Creating a Table

To create a table:

1. From the **Insert menu**, select **Table**.

2. Specify the number of rows and columns in the table creation dialog.

3. Click **OK**.

The default properties for the table include visible borders and auto-adjustment to 100% of the document width. These options, as well as the table dimensions, can be modified after table creation.

Create a table with 4 rows and 2 columns at the end of your document. In document mode, the input mode is set to **Math** by default; in worksheet mode, the default is **Text** mode.

Cell Contents

Any content that can be placed into a document can also be placed into a table cell, including other sections and tables. Table cells can contain a mix of:

- Input commands
- 2-D Math
- Embedded components: buttons, sliders, check boxes, and more
- Plots
- Images

Enter a heading in both columns of the first row, in 2-D Math. You can use any text formatting features within each cell; for example, bold and center the headings.

$\boldsymbol{f(x)}$	$\boldsymbol{\frac{d}{dx} f(x)}$

Navigating Table Cells

Use the **Tab** key to move to the next cell. Ensure that the Tab toolbar icon is **off**.

	Tab icon **off**. Allows you to move between cells using the **Tab** key.
	Tab icon **on**. Allows you to indent in the table using the **Tab** key.

Tab between the cells of the table and enter the following expressions in the first column. For each function, from the context menu, select **Differentiate** → **x**. Cut and paste the resulting expression into the second column.

$\boldsymbol{f(x)}$	$\boldsymbol{\frac{d}{dx} f(x)}$
$\frac{1}{1+\frac{1}{1+\frac{1}{x}}}$	$-\frac{1}{\left(1+\frac{1}{1+\frac{1}{x}}\right)^2 \left(1+\frac{1}{x}\right)^2 x^2}$
$\sin(\omega x)\, e^{(-5\, x)}$	$\cos(\omega x)\, \omega\, e^{-5\, x} - 5 \sin(\omega x)\, e^{-5\, x}$
$\frac{d^2}{d x^2} \sin^2(x)$	$-8 \sin(x) \cos(x)$

Modifying the Structural Layout of a Table

The number of rows and columns in a table are modified using the **Insert** and **Delete** submenus in the **Table** menu or by using the **Cut** and **Copy/Paste** tools.

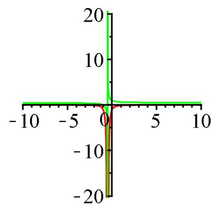
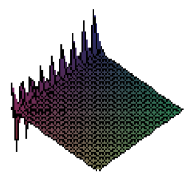
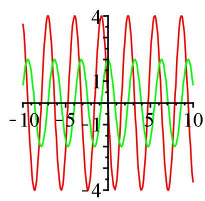
Inserting Rows and Columns

Row and column insertion is relative to the table cell that currently contains the cursor. If the document has an active selection, insertion is relative to the selection boundaries.

- Column insertion can be to the left or right of the document position marker or selection.
- Row insertion can be above or below the marker or selection.

In your table, add a third column on the right to display the plots of these expressions. Add the heading, and insert a blank plot region in each cell below it, by selecting **Insert** → **Plot** → **2-D** (or **3-D** for the second expression). Then **Ctrl**-drag (**Control**-drag for Macintosh) each expression in the row into its plot region to display it. For details on this procedure, see *Plots and Animations (page 237)*.

Resize the plots and table as desired.

$f(x)$	$\frac{d}{dx} f(x)$	**Plot of** $f(x)$ **and** $\frac{d}{dx} f(x)$
$\frac{1}{1+\frac{1}{1+\frac{1}{x}}}$	$-\frac{1}{\left(1+\frac{1}{1+\frac{1}{x}}\right)^2 \left(1+\frac{1}{x}\right)^2 x^2}$	
$\sin(\omega x)\, e^{(-5x)}$	$\cos(\omega x)\, \omega\, e^{-5x} - 5 \sin(\omega x)\, e^{-5x}$	
$\frac{d^2}{dx^2} \sin^2(x)$	$-8 \sin(x) \cos(x)$	

Deleting Rows and Columns

With deleting operations using the **Delete** key, the **Delete Table Contents** dialog opens allowing you to specify the desired behavior. For example, you can delete the selected rows, or delete the contents of the selected cells. See Figure 7.11.

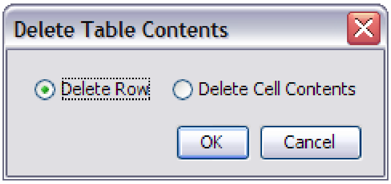


Figure 7.11: Delete Table Contents Verification Dialog

Pasting

Pasting a table subselection into a table may result in the creation of additional rows or columns, overwriting existing cell content, or the insertion of a subtable within the active table cell. When there is a choice, the **Table Paste Mode** dialog opens, allowing you to choose. See Figure 7.12.

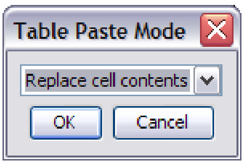


Figure 7.12: Table Paste Mode Selection Dialog

Merging Cells

To merge adjacent cells in a table, select the cells you would like to merge. From the **Table** menu, select **Merge Cells**. You can merge cells across row or column borders. See Figure 7.13. The resulting cell must be rectangular. The contents of the individual cells in the merge operation are concatenated in execution order. See Figure 7.14. For details on cell execution order, see *Execution Order Dependency (page 315)*.

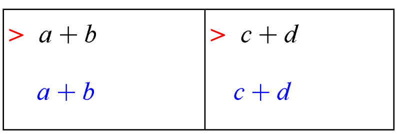


Figure 7.13: Two cells

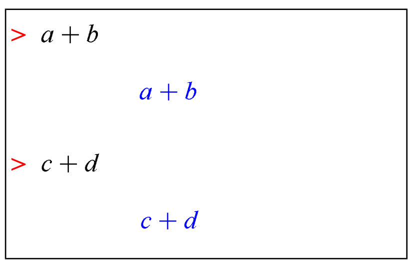


Figure 7.14: Merged Cells

Modifying the Physical Dimensions of a Table

The overall width of the table can be controlled in several ways.

The most direct way is to press the left mouse button (press mouse button, for Macintosh) while hovering over the left or right table boundary and dragging the mouse left or right. Upon release of the mouse button, the table boundary is updated. This approach can also be used to resize the relative width of table columns.

Alternatively, the size of the table can be controlled from the **Table Properties** dialog. Select the **Table** menu and then **Properties**. Two sizing modes are supported.

(1) **Fixed percentage of page width**. Using this option, the table width adjusts whenever the width of the document changes. This option is useful for ensuring that the entire content of the table fits in the screen or printed page.

(2) **Scale with zoom factor**. This option is used to preserve the size and layout of the table regardless of the size of the document window or the zoom factor. If the table exceeds the width of the document window, the horizontal scroll bar can be used to view the rightmost columns. **Note**: Using this option, tables may be incomplete when printed.

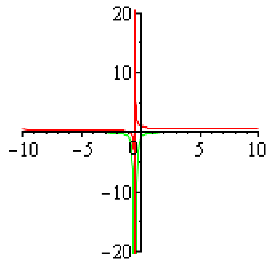
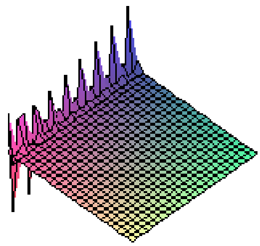
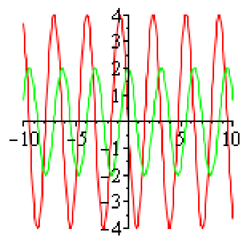
Modifying the Appearance of a Table

Table Borders

The style of exterior and interior borders is set using the **Table Properties** dialog. From the **Table** menu, select **Properties**.

- You can set all, none, or only some of the borders to be visible in a table. Exterior borders are controlled separately.
- You can control the visibility of interior borders by using the Group submenu of the Table menu; grouping rows or columns suppresses interior borders, provided that the interior border style is set by row and column group.

For example, group the columns together, and group rows 2 to 4 together. Then in the **Table Properties** dialog, select **Exterior Borders**: **Top and bottom**, and **Interior Borders**: **By row and column group**.

$f(x)$	$\frac{d}{dx} f(x)$	Plot of $f(x)$ and $\frac{d}{dx} f(x)$
$\frac{1}{1+\frac{1}{1+\frac{1}{x}}}$	$-\frac{1}{\left(1+\frac{1}{1+\frac{1}{x}}\right)^2 \left(1+\frac{1}{x}\right)^2 x^2}$	
$\sin(\omega x)\, e^{(-5x)}$	$\cos(\omega x)\, \omega\, e^{-5x} - 5 \sin(\omega x)\, e^{-5x}$	
$\frac{d^2}{dx^2} \sin^2(x)$	$-8 \sin(x) \cos(x)$	

- Hidden borders are visible when the mouse hovers over a table. **Note**: You can hide the visibility of lines on mouse pointer roll over by using the **View→Show/Hide Contents** dialog, and clearing the **Hidden Table Borders** check box.

Alignment Options

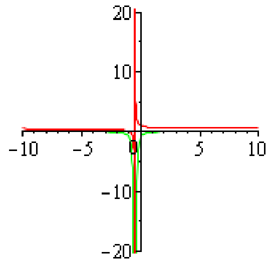
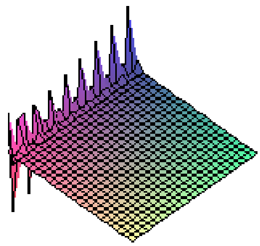
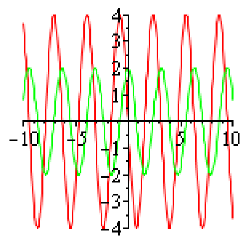
The table alignment tools control the horizontal alignment of columns and vertical alignment of rows.

For column alignment, the current selection is expanded to encompass all rows in the selected columns. The alignment choice applies to all cells within the expanded selection. If the document does not contain a selection, the cursor position is used to identify the column.

Similarly, the selection is expanded to include all columns in the selected rows for vertical alignment options. The following table illustrates the vertical alignment options. The baseline option is useful for aligning equations across multiple cells within a row of a table.

Top	$x^{\left(\frac{1}{y}\right)}$	$\frac{1}{x^{\left(\frac{1}{y}\right)}}$
Center	$x^{\left(\frac{1}{y}\right)}$	$\frac{1}{x^{\left(\frac{1}{y}\right)}}$
Bottom	$x^{\left(\frac{1}{y}\right)}$	$\frac{1}{x^{\left(\frac{1}{y}\right)}}$
Baseline	$x^{\left(\frac{1}{y}\right)}$	$\frac{1}{x^{\left(\frac{1}{y}\right)}}$

For example, set the **Row** alignment to **Baseline** for all rows, and set the **Column** alignment to **Center** for all columns.

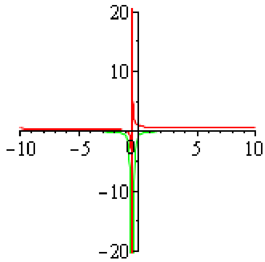
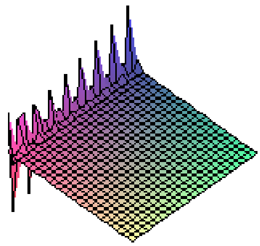
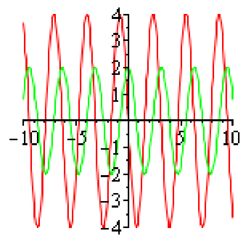
f(x)	$\frac{d}{dx}f(x)$	**Plot of** $f(x)$ **and** $\frac{d}{dx}f(x)$
$\frac{1}{1+\frac{1}{1+\frac{1}{x}}}$	$-\frac{1}{\left(1+\frac{1}{1+\frac{1}{x}}\right)^2\left(1+\frac{1}{x}\right)^2 x^2}$	
$\sin(\omega x)\, e^{(-5x)}$	$\cos(\omega x)\,\omega\, e^{-5x} - 5\sin(\omega x)\, e^{-5x}$	
$\frac{d^2}{dx^2}\sin^2(x)$	$-8\sin(x)\cos(x)$	

Cell Color

You can set the background color of any cell or collection of cells to be any color. This coloring is independent of any highlighting or text color that may also be applied.

To change the color of a cell, place the cursor in the cell, then from the **Table** menu, select **Cell Color...**. In the **Select A Color** dialog, choose a color from the swatches, the color wheel, or RGB. See the **?DrawingTools** help page for details on color selection.

For example, select the first row of the table and apply a light blue color. This sets the header off from the content below.

$f(x)$	$\frac{d}{dx} f(x)$	Plot of $f(x)$ and $\frac{d}{dx} f(x)$
$\frac{1}{1+\frac{1}{1+\frac{1}{x}}}$	$-\frac{1}{\left(1+\frac{1}{1+\frac{1}{x}}\right)^2 \left(1+\frac{1}{x}\right)^2 x^2}$	
$\sin(\omega x)\, e^{(-5x)}$	$\cos(\omega x)\, \omega\, e^{-5x} - 5 \sin(\omega x)\, e^{-5x}$	
$\frac{d^2}{dx^2} \sin^2(x)$	$-8 \sin(x) \cos(x)$	

Controlling the Visibility of Cell Content

The **Table Properties** dialog includes two options to control the visibility of cell content. These options allow control over the visibility of Maple input and execution group boundaries. Thus, these elements can be hidden in a table even if they are set to visible for the document in the **View→Show/Hide Contents** dialog.

Printing Options

The **Table Properties** dialog contains options to control the placement of page breaks when printing. You can fit a table on a single page, allow page breaks between rows, or allow page breaks within a row.

Execution Order Dependency

The order in which cells are executed is set in the **Table Properties** dialog. The following tables illustrate the effect of execution order.

Row-wise execution order	
`> x:=1;` $x := 1$ (7.1)	`> x:=x+1;` $x := 2$ (7.2)
`> x:=x+1;` $x := 3$ (7.3)	`> x:=x+1;` $x := 4$ (7.4)

Column-wise execution order	
`> x:=1;` $x := 1$ (7.5)	`> x:=x+1;` $x := 3$ (7.6)
`> x:=x+1;` $x := 2$ (7.7)	`> x:=x+1;` $x := 4$ (7.8)

Tables and the Classic Worksheet

Tables are flattened on export to the Classic Worksheet interface. For example, the following table in the Standard Worksheet appears as one column in the Classic Worksheet interface.

<table>
<tr><th>Table in Standard Worksheet</th><th>Table in Classic Worksheet</th></tr>
<tr><td><table><tr><td>aaa</td><td>ddd</td></tr><tr><td>bbb</td><td>eee</td></tr><tr><td>ccc</td><td>fff</td></tr></table></td><td>aaa
bbb
ccc
ddd
eee
fff</td></tr>
</table>

Additional Examples

For more practice creating and manipulating tables, try creating the following tables at the end of your document.

Table of Values

This example illustrates how to set the visibility options for cell contents to display a table of values.

$$> \; y := t \rightarrow \frac{1}{2} t^2 :$$

Create a table with 2 rows and 7 columns. Enter the values as below, and then select all table cells. In the **Table → Alignment** menu, select **Columns**, and then **Center**.

t ⟦s⟧	0	1	2	3	4	5	6
$y(t)$ ⟦m⟧	> $y(0)$ 0	> $y(1)$ $\frac{1}{2}$	> $y(2)$ 2	> $y(3)$ $\frac{9}{2}$	> $y(4)$ 8	> $y(5)$ $\frac{25}{2}$	> $y(6)$ 18

Table settings:

In the **Properties** dialog (**Table → Properties** menu):

1. Set **Table Size Mode** to **Scale with zoom factor**.

2. Hide Maple input and execution group boundaries: Clear the **Show input** and **Show execution group boundaries** check boxes.

t ⟦s⟧	0	1	2	3	4	5	6
$y(t)$ ⟦m⟧	0	$\frac{1}{2}$	2	$\frac{9}{2}$	8	$\frac{25}{2}$	18

Formatting Table Headers

The following table uses cell merging for formatting row and column headers, and row and column grouping to control the visibility of cell boundaries.

By default, invisible cell boundaries are visible on mouse pointer roll over. You can hide the visibility of lines on mouse pointer roll over by using the **View→Show/Hide Contents** dialog, and clearing the **Hidden Table Borders** check box.

		Parameter 2	
		Low	High
Parameter 1	Low	13	24
	High	18	29

Table settings:

1. Insert a table with 4 rows and 4 columns and enter the information shown above.

Using the **Table** menu:

2. **Merge** the following sets of (**R**ow,**C**olumn) cells: (R1,C1) to (R2,C2), (R1,C3) to (R1,C4), and (R3,C1) to (R4,C1).

3. **Group** columns 1 and 2, and columns 3 and 4.

4. **Group** rows 1 and 2, and rows 3 and 4.

In the **Properties** dialog (**Table→Properties** menu):

5. Set **Exterior Borders** to **None**.

6. (Optional) Change **Table Size Mode** size option to **Scale with zoom factor**.

Using the **Table** menu:

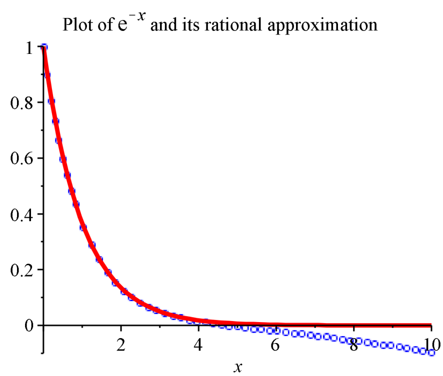
7. Set **Alignment** of columns 3 and 4 to **Center**.

2-D Math and Plots

The following example illustrates the use of tables to display 2-D Math and plots side by side.

Approximating exp(-x) as a rational polynomial using a 3^{rd} order Padé approximation.

$$e^{-x} \approx \frac{1 - \frac{1}{2}x + \frac{1}{10}x^2 - \frac{1}{120}x^3}{1 + \frac{1}{2}x + \frac{1}{10}x^2 + \frac{1}{120}x^3}$$



Insert a table with 1 row and 2 columns. Enter the information in text and executable 2-D Math to create the calculation and plot, as shown.

Table Settings:

In the **Properties** dialog (**Table→Properties** menu):

1. Set **Exterior** and **Interior Borders** to **None**.

2. Hide Maple input and execution group boundaries: Clear the **Show input** and **Show execution group boundaries** check boxes.

Using the **Table** menu:

3. Change row **Alignment** to **Center**.

7.5 Canvas

Using the drawing tools, you can sketch an idea in a canvas, draw on plots, and draw on images. See Figure 7.15. For details about the drawing feature, refer to the **?DrawingTools** help page.

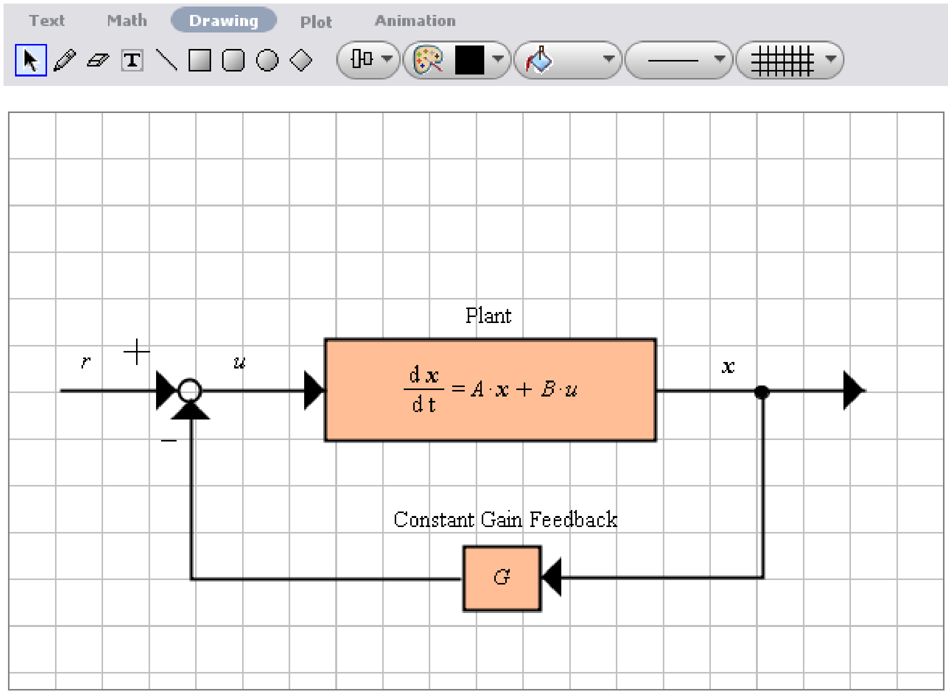


Figure 7.15: Drawing Tools and Canvas

Insert a Canvas

To insert a canvas:

1. Place the cursor where the canvas is to be inserted.

2. From the **Insert** menu, select **Canvas**. A canvas with grid lines appears in the document at the insertion point. The **Drawing** icon is available and associated context bar icons are displayed.

The tools include the following: selection tool, pencil (free style drawing), eraser, text insert, straight line, rectangle, rounded rectangle, oval, diamond, alignment, drawing outline, drawing fill, drawing linestyle, and drawing canvas properties.

Drawing

To draw with the pencil tool in the canvas:

1. From the **Drawing** icons, select the pencil icon.

2. Click and drag your mouse in the canvas to draw lines. Release the mouse to complete the drawing.

To adjust the color of drawing tools:

1. From the **Drawing** icons, select the **Drawing Outline** icon. See Figure 7.16.

2. Select one of the color swatches available or select the color wheel, RGB ranges, or eye dropper icon at the bottom of the dialog and customize the color to your preference.

3. After selecting a new color, draw on the canvas using the pencil icon and notice the new color.

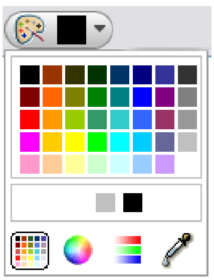
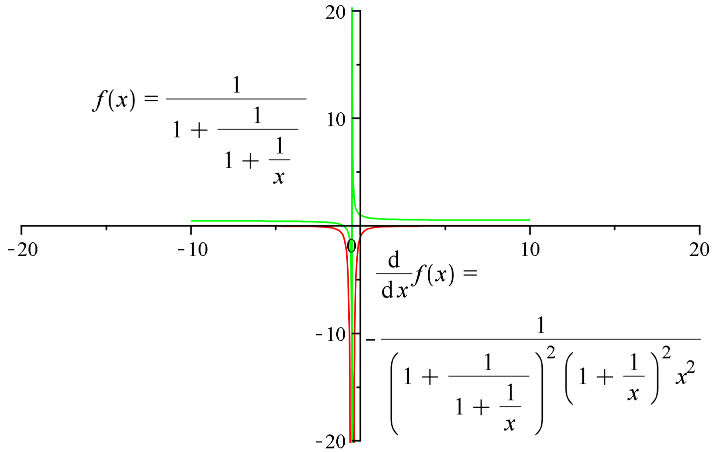


Figure 7.16: Drawing Outline Color Icon

In your document, there are three plots, two of which are 2-D plots that can be drawn on. All of the information in the table you made in the previous section could be drawn onto the plot, putting the information in a more concise layout.

Consider one of the plots from the table:



Click on the plot, and notice that the **Plot** toolbar is open. However, the **Drawing** toolbar is also available. Click on **Drawing** to see the toolbar.

Select the **Text** icon, T, and click on the plot. Enter the expression $f(x)$ in one text area, and its derivative in another, as shown. You can move the text areas around on the plot so that they indicate the correct lines.

For details on the rest of the drawing features, refer to the **?DrawingTools** help page.

Canvas Style

You can alter the **Canvas** in the following ways:

- Add a grid of horizontal and/or vertical lines. By default, the canvas opens with a grid of horizontal and vertical lines.
- Change the grid line color.
- Change the spacing between grid lines.
- Change the background color.

These options can be changed in the **Drawing Properties Canvas Icon**. See Figure 7.17.

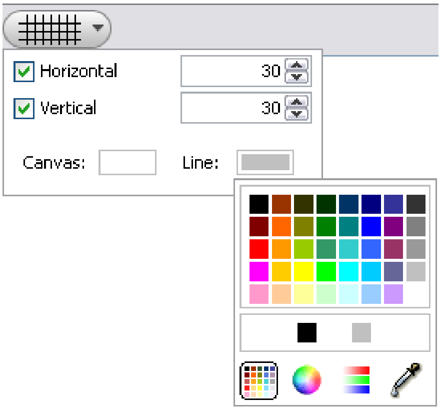


Figure 7.17: Drawing Properties Canvas Icon - Change the Gridline Color

Inserting Images

You can insert images in these file formats into your document. • Graphics Interchange Format - gif • Joint Photographic Experts Group - jpe, jpeg, jpg • Portable Network Graphics - png • Bitmap Graphics - bmp • Tagged Image File Format - tif, tiff, jfx • Portable aNyMap - pnm • Kodak FlashPix - fpx	

To insert an image into the document at the cursor location:

1. From the **Insert** menu, select **Image**. The **Load Image** dialog opens.

2. Specify a path or folder name.

3. Select a filename.

4. Click **Open**. The image is displayed in the document.

If the source file is altered, the embedded image does not change because the original object is pasted into the document.

To resize an inserted image:

1. Click the image. Resizing anchors appear at the sides and corners of the image.

2. Move the mouse over the resize anchor. Resizing arrows appear.

3. Click and drag the image to the desired size.

Note: To constrain the proportions of the image as it is resized, press and hold the **Shift** key as you drag.

You can also draw on images in the same way as the drawing canvas For more information, refer to the **?worksheet/documenting/drawingtools** help page..

ImageTools Package

You can manipulate image data using the **ImageTools** package. This package is a collection of utilities for reading and writing common image file formats, and for performing basic image processing operations within Maple.

Within Maple, images are represented as dense, rectangular Arrays of 64-bit hardware floating-point numbers. Grayscale images are 2-D, whereas color images are 3-D (the third dimension representing the color channels).

In addition to the commands in the **ImageTools** package, many ordinary Array and Matrix operations are useful for image processing.

For details about this feature, refer to the **?ImageTools** help page.

7.6 Hyperlinks

Use a hyperlink in your document to access any of the following.

- Web Page (URL)
- Email
- Worksheet
- Help Topic
- Task
- Dictionary Topic
- Maplet

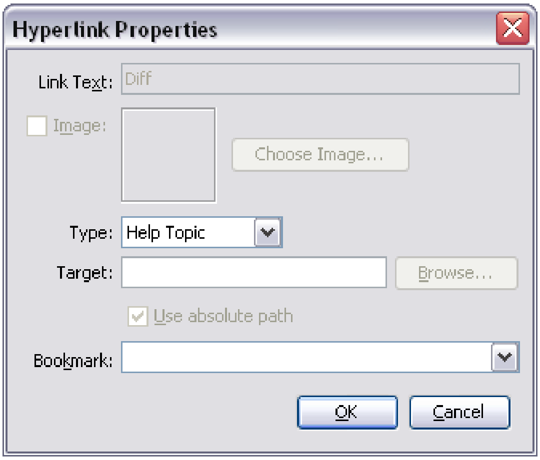


Figure 7.18: Hyperlink Properties Dialog

Inserting a Hyperlink in a Document

To create a hyperlink from existing text in the document:

1. Highlight the text that you want to make a hyperlink.

2. From the **Format** menu, select **Convert To** and then **Hyperlink**.

3. In the **Hyperlink Properties** dialog box, the **Link Text** field is grayed out since the text region you highlighted is used as the link text. This is demonstrated in Figure 7.18. The highlighted text region, Diff is grayed out.

4. Specify the hyperlink **Type** and **Target** as described in the appropriate following section.

To insert a text or image hyperlink into the document:

1. From the **Insert** menu, select **Hyperlink**.

2. In the **Hyperlink Properties** dialog box, enter the **Link Text.**

Optionally, use an image as the link. Select the **Image** check box and click **Choose Image** for the file. In **.mw** files, the image appears as the link. You can resize the image as necessary. Click and drag from the corners of the image to resize.

3. Specify the hyperlink **Type** and **Target** as described in the appropriate following section.

Linking to a Web Page

To link to a Web page:

1. In the **Type** drop-down list, select **URL**.

2. In the **Target** field, enter the full URL, for example, **http://www.maplesoft.com**.

3. Click **OK**.

Linking to an Email Address

To link to an email address:

1. In the **Type** drop-down list, select **Email**.

2. In the **Target** field, enter the email address.

3. Click **OK**.

Note: For information about email hyperlinks in the Classic Worksheet interface, see *Worksheet Compatibility (page 335)*.

Linking to a Worksheet

To link to a Maple worksheet or document:

1. In the **Type** drop-down list, select **Worksheet**.

2. In the **Target** field, enter the path and filename of the document or click **Browse** to locate the file. (Optional) In the **Bookmark** drop-down list, enter or select a bookmark.

Note: To link within a single Maple document, leave the **Target** field blank and choose the bookmark from the **Bookmark** drop-down list.

Note: When linking to a custom document, the path is absolute. When sharing documents that contain hyperlinks, ensure that target documents are in the same directory.

3. Click **OK**.

Linking to a Help Page

To link to a help page:

1. In the **Type** drop-down list, select **Help Topic**.

2. In the **Target** field, enter the topic of the help page. (Optional) In the **Bookmark** drop-down list, enter or select a bookmark.

3. Click **OK**.

Linking to a Task

To link to a task:

1. In the **Type** drop-down list, select **Task**.
2. In the **Target** field, enter the topic name of the task template (see the status bar at the bottom of the Task Browser window).
3. Click OK.

Linking to a Dictionary Topic

To link to a Dictionary topic:

1. In the **Type** drop-down list, select **Dictionary Topic**.

2. In the **Target** field, enter a topic name. Dictionary topics begin with the prefix **Definition/**, for example, **Definition/dimension**.

3. Click **OK**.

Linking to a Maplet Application

To link to a Maplet application:

1. In the **Type** drop-down list, select **Maplet**.

2. In the **Target** field, enter the local path to a file with the **.maplet** extension. Optionally, click **Browse** to locate the file.

If the Maplet application exists, clicking the link launches the Maplet application. If the Maplet application contains syntax errors, then error messages are displayed in a pop-up window.

When linking to a custom Maplet application, the path is absolute. When sharing documents that contain links to Maplet applications, ensure that target Maplet applications are in the same directory.

3. Click **OK**.

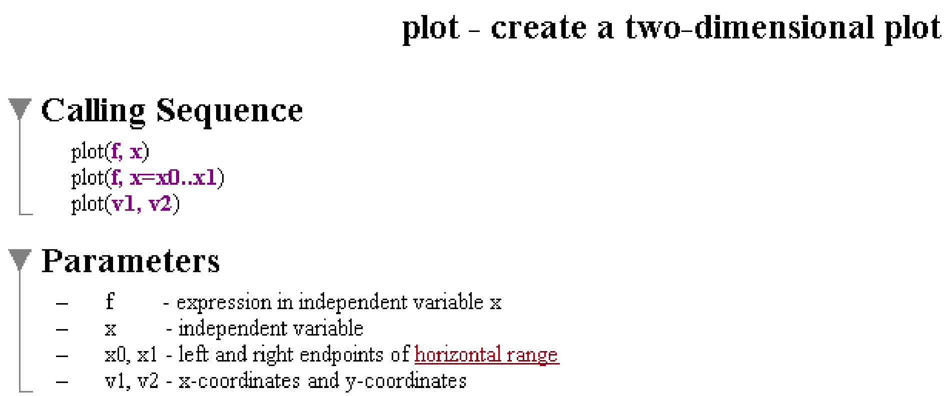
Note: To link to a Maplet application available on a MapleNet Web page, use the URL hyperlink type to link to the Web page. For information on MapleNet, see *Embedded Components and Maplets (page 389)*.

Example

For this example, link the text "horizontal range" to the dictionary page for domain. As indicated in the section for Linking to a Dictionary Topic, select **Dictionary Topic** in the **Type** drop-down list, and then enter **Definition/domain** in the **Target** field.

Links to dictionary topics appear underlined and in red.

Result:



Bookmarks

Use a bookmark to designate a location in an active document. This bookmark can then be accessed from other regions in your document or by using hyperlinks in other documents.

To display bookmark formatting icons, activate the **Marker** feature.

- From the **View** menu, select **Markers**.

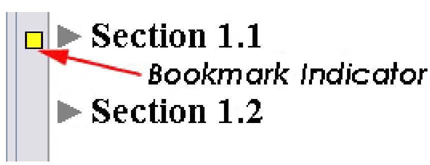


Figure 7.19: Bookmark Indicator

Note: You can display bookmark properties by holding the pointer over a bookmark indicator. See Figure 7.19.

Inserting, Renaming, and Deleting a Bookmark

To insert a bookmark:

1. Place the cursor at the location at which to place the bookmark. For example, place the cursor in the **Parameters** section title.

2. From the **Format** menu, select **Bookmarks**. The **Bookmark** dialog opens, listing existing bookmarks in the document.

3. Click **New**. The **Create Bookmark** dialog opens. See Figure 7.20. Enter a bookmark name, "parameters", and click **Create**.

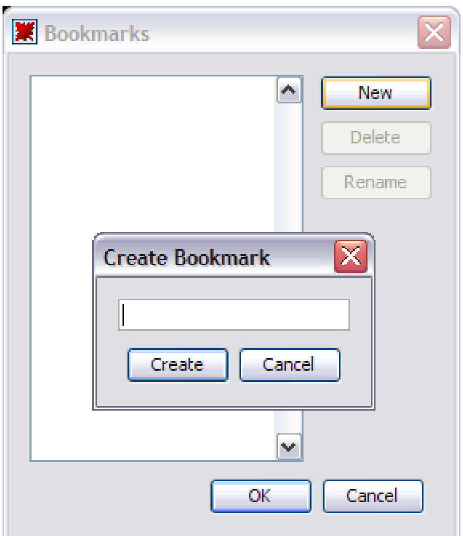
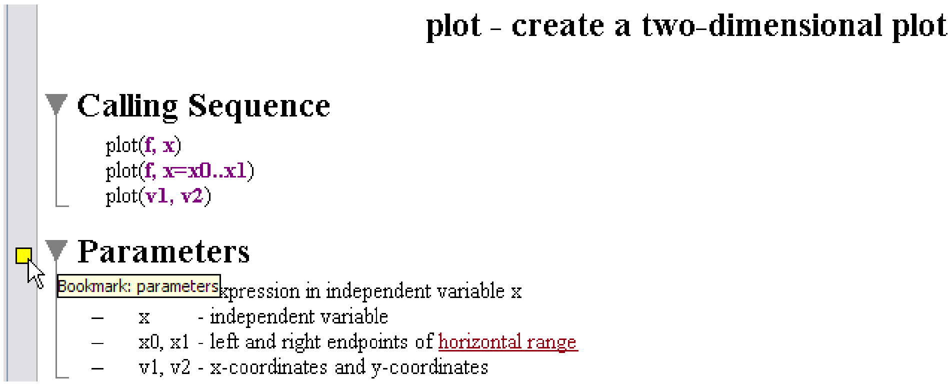


Figure 7.20: Create Bookmark Dialog

4. The new bookmark appears in the **Bookmark** dialog list. Click **OK**.

Note: You can also rename and delete bookmarks using the **Bookmark** dialog.

Result:



Go to a Bookmark

You can automatically move the cursor to the location of the bookmark in the active document.

1. From the **Edit** menu, select **Go To Bookmark**. The **Go To Bookmark** dialog opens with the current bookmarks listed.

2. Select the bookmark "parameters" and click **OK**. The cursor moves to the bookmark, at the beginning of the **Parameters** section.

For more information, refer to the **?bookmarks** help page.

7.7 Embedded Components

You can embed simple graphical interface components, such as a button, in your document. These components can then be associated with actions that are to be executed. For example, the value of a slider component can be assigned to a document variable, or a text field can be used to input an equation.

Adding Graphical Interface Components

The graphical interface components can be inserted by using the **Components** palette (Figure 7.21) or by cutting/copying and pasting existing components to another area of the document. Although copied components have most of the same characteristics, they are distinct.

By default, palettes are displayed when you launch Maple. If palettes are not visible, use the following procedure.

1. From the **View** menu, select **Palettes**.

2. Select **Expand Docks**.

3. If the **Components** palette is not displayed, right-click (**Control**-click, for Macintosh) the palette dock. From the context menu, select **Show Palette**, and then **Components**.

For more information, see *Palettes (page 22)*.

You can embed the following items.

- Button, Toggle Button
- Combo Box, Check Box, List Box, Radio Button
- Text Area, Label
- Slider, Plot, Mathematical Expression
- Dial, Meter, Rotary Gauge, Volume Gauge

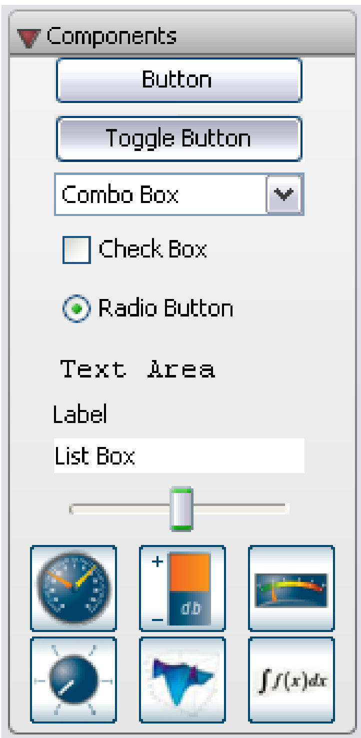


Figure 7.21: Components Palette

Task Template with Embedded Components

In your document, you can add components that have already been configured to work together, by using a task template. Here, we use the template. For details on how to create and modify components, see *Creating Embedded Components (page 393)*.

To insert the task template, from the **Tools** menu, select **Tasks** → **Browse**. In the table of contents, expand **Document Templates**, and select **Interactive Application**. Click **Insert Minimal Content**. The following is inserted into your document.

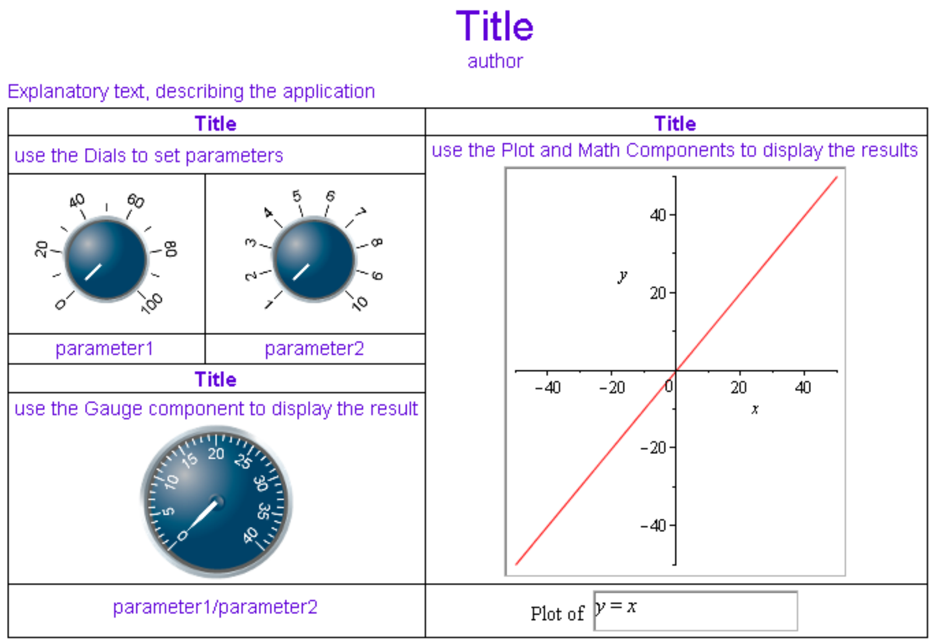


Figure 7.22: Interactive Application Task Template

This configuration of components plots a linear function with slope and y-intercept given respectively by the two dials *parameter2* and *parameter1* , and displays the function $\frac{parameter2}{parameter1}$ on a gauge. For details on how these components work together, see *Embedded Components and Maplets (page 389)*.

7.8 Spell Checking

The **Spellcheck** utility examines all designated text regions of your document for potential spelling mistakes, including regions that are in collapsed sections. It does not check input, output, text in execution groups, or math in text regions. See Figure 7.23.

Note: The **Spellcheck** utility uses American spelling.

The CodeGeneration package is a collection of comands and subpackages
that enable the translation of Maple code to other programming languages.

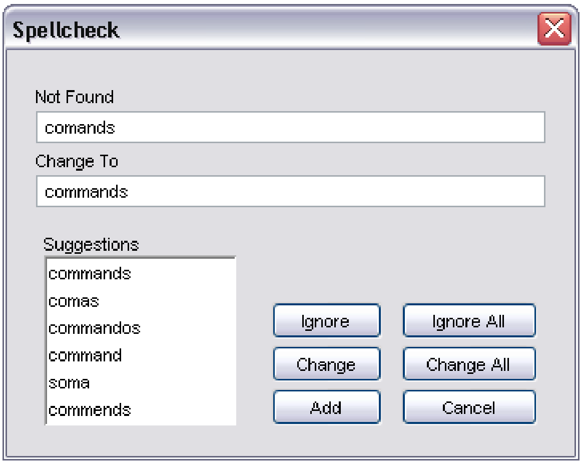


Figure 7.23: Spellcheck Dialog

How to Use the Spellcheck Utility

1. From the **Tools** menu, select **Spellcheck**. Alternatively, press **F7**. The **Spellcheck** dialog appears. It automatically begins checking the document for potential spelling mistakes.

2. If the **Spellcheck** utility finds a word that it does not recognize, that word is displayed in the **Not Found** text box.

You have six choices:

- To ignore the word, click **Ignore**.
- To ignore all instances of the word, click **Ignore All**.
- To change the word, that is, accept the suggested spelling that is in the **Change To** text box, click **Change**.
- To change all instances of the word, that is, accept the suggested spelling to replace all instances of the word, click **Change All**.

- To add the word to your dictionary, click **Add**. For details, see the following **User Dictionary** section.
- To close the **Spellcheck** dialog and stop the spelling check, click **Cancel**.

3. When the **Spellcheck** is complete, a dialog containing the message "The spelling check is complete" appears. Click **OK** to close this dialog.

Note: when using the **Spellcheck** utility, you can fix spelling errors in the dialog, but you cannot change the text in document. The **Spellcheck** utility does not check grammar.

Selecting a Suggestion

To select one of the suggestions as the correct spelling, click the appropriate word from the list in the **Suggestions** text box.

If none of the suggestions are correct, highlight the word in the **Change To** text box and enter the correct spelling. Click **Change** to accept this new spelling.

User Dictionary

You can create and maintain a custom dictionary that works with the Maple **Spellcheck** utility.

Properties of the Custom Dictionary File

- It must be a text file, that is, have the file extension **.txt**. For example, **mydictionary.txt.**
- It is a list of words, one word per line.
- It is case sensitive. This means that integer and Integer require individual entries in the dictionary file.
- It does not require manual maintenance. You build your dictionary file by using the **Add** functionality of the **Spellcheck**. However, you can manually edit the file.

To specify a custom dictionary to be used with the Maple Spellcheck utility:

1. Create a **.txt** file in a directory/folder of your choice.

2. In Maple, open the **Options** dialog, **Tools → Options**, and select the **General** tab.

3. In the **User Dictionary** field, enter the path and name of the **.txt** file you created, or click **Browse** to select the location and filename.

4. To ignore Maple words that are command and function names, clear the **Use Maple Words in spellchecker** check box.

5. Click **Apply to Session** or **Apply Globally** to save the settings, or **Cancel** to discard.

Adding a Word to Your Dictionary

When running the spellcheck, if the word in the **Not Found** text box is correct, you can add the word to your dictionary.

1. Click the **Add** button. If this is the first time you are adding a word, the **Select User Dictionary** dialog opens.

2. Enter or select the custom dictionary (**.txt** file) you created. See *User Dictionary (page 333)*.

3. Click **Select**. The word is automatically added to your custom dictionary file.

Note: Specifications in the **Options** dialog determine whether this word is recognized in your next Maple session. If you set your custom dictionary and clicked **Apply to Session**, then this word will *not* be recognized in a new Maple session. If you set your custom dictionary and clicked **Apply Globally**, then this new word will be recognized.

7.9 Creating Graded Assignments

You can use Maple to create graded assignments. Question types include multiple choice, essay, true-or-false, fill-in-the-blanks, and Maple-graded.

Note: This feature can be used to create questions for Maple T.A.—an online automated testing and assessment system. For details about Maple T.A., see *Maple T.A. (page 419)*.

Creating a Question

To create a question:

1. Open the Task browser (**Tools→Tasks→Browser**).
2. From the **Maple T.A.** folder, select the appropriate question type.
3. Insert the question template into a document.
4. Enter the question content as described in the template.
5. Repeat steps 1 to 4 for each question to add to the document.

Viewing Questions in Maple

To view and test your questions in Maple:

- From the **View** menu, select **Assignment**. This view displays all of the questions in your assignment with access to hints, plotting, and grading.

After answering your questions, you can test the grading function by clicking the **Grade** button. A Maplet dialog is displayed indicating if the question was answered correctly. If hints were provided in the question, these are also displayed.

Saving Test Content

When you save a document with test content, the authoring and assignment modes determine what the user sees when opening your document.

- If you save the document in authoring mode (task template contents visible), the user sees this content when opening the document.
- If you save the document in assignment mode, the user sees only the assignment layout.

In both cases the **View→Assignment** menu is accessible. As such, users (students) can switch between the original document contents and the displayed assignment.

7.10 Worksheet Compatibility

Maple provides users with two worksheet interfaces: the Standard Worksheet and the Classic Worksheet. Both have access to the full mathematical engine of Maple and take advantage of the new functionality in Maple. The Classic Worksheet has the traditional Maple worksheet look and uses less memory.

If you create a document in the Standard Worksheet interface of Maple and then open it in the Classic Worksheet interface, you should note possible changes to your file. For example, a bulleted list in the Standard Worksheet will not be displayed with bullets in the Classic Worksheet. Many of the graphical features in this manual, especially those in this chapter, are not available in the Classic Worksheet interface.

If you are creating documents for distribution, refer to the **?Compatibility** help page.

8 Maple Expressions

This chapter provides basic information on using Maple expressions, including an overview of the basic data structures. Many of the commands described in this chapter are useful for programming. For information on additional Maple programming concepts, such as looping, conditional execution, and procedures, see *Basic Programming (page 369)*.

8.1 In This Chapter

Section	Topics
Creating and Using Data Structures (page 337) - How to define and use basic data structures	• Expression Sequences • Sets • Lists • Tables • Arrays • Matrices and Vectors • Functional Operators • Strings
Working with Maple Expressions (page 348)- Tools for manipulating and controlling the evaluation of expressions	• Low-Level Operations • Manipulating Expressions • Evaluating Expressions

8.2 Creating and Using Data Structures

Constants, data structures, mathematical expressions, and other objects are Maple expressions. For more information on expressions, refer to the Maple Help System.

This section describes the key data structures:

- Expression sequences
- Sets
- Lists
- Tables
- Arrays
- Matrices and Vectors
- Functional operators
- Strings

Expression Sequences

The fundamental Maple data structure is the *expression sequence*. It is a group of expressions separated by commas.

> $S := 2, y, \sin(x^2), I:$

Accessing Elements

To access one of the expressions:

- Enter the sequence name followed by the position of the expression enclosed in brackets(**[ ]**).

For example:

> $S[2]$

$$y$$

Using negative integers, you can select an expression from the end of a sequence.

> $S[-2]$

$$\sin(x^2)$$

You can select multiple expressions by specifying a range using the range operator (**..**).

> $S[2..-2]$

$$y, \sin(x^2)$$

Note: This syntax is valid for most data structures.

Sets

A set is an expression sequence enclosed in curly braces (**{ }**).

> $\left\{4, 12\,i, \sin\left(\frac{2}{3}\right)\right\}:$

A Maple set has the basic properties of a mathematical set.

- Each element is unique. Repeated elements are stored only once.
- The order of elements is not stored.

For example:

> $\{c, a, a, a, b, c, a\}$

$$\{a, b, c\}$$

Using Sets

To perform mathematical set operations, use the set data structure.

> $\{2, 6, 5, 1\} \cup \{2, 8, 6, 7\}$

$$\{1, 2, 5, 6, 7, 8\}$$

Note: The union operator is available in 1-D Math input as **union**. For more information, refer to the **?union** help page.

For more information on sets, refer to the **?set** help page.

Lists

A list is an expression sequence enclosed in brackets (**[ ]**).

> $L := [2, 3, 3, 1, 0]$

$$L := [2, 3, 3, 1, 0]$$

Note: Lists preserve both the **order** and **repetition** of elements.

Accessing Entries

To refer to an element in a list:

- Use square brackets.

For example:

> $L[-2..-1]$

$$[1, 0]$$

For more information, see *Accessing Elements (page 338)*.

Using Lists

Some commands accept a list (or set) of expressions.

For example, you can solve a list (or set) of equations using a context menu or the **solve** command.

> $solve([x - y^2 = -2, x + y = 0])$

$$\{x = 2, y = -2\}, \{x = -1, y = 1\}$$

For more information, see *Solving Equations and Inequations (page 112)*.

For more information on sets and lists, refer to the **?set** help page.

Arrays

Conceptually, the Array data structure is a generalized list. Each element has an index that you can use to access it.

The two important differences are:

- The indices can be any integers.
- The dimension can be greater than one.

Creating and Using Arrays

To define an Array, use the **Array** constructor.

Standard **Array** constructor arguments are:

- Expression sequences of ranges - Specify the indices for each dimension
- Nested lists - Specify the contents

For example:

> $a := Array(1..3, 1..3, [[1,2,3],[4,5,6],[7,8,9]])$

$$a := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

> $b := Array(1..2, 2..5, [[1.2, 4.9, 6.3, 7.1], [9.2, 5.5, 2.4, 1.7]])$

$$b := Array(1..2, 2..5, \{(1,2) = 1.2, (1,3) = 4.9, (1,4) = 6.3, (1,5) = 7.1, (2,2) = 9.2, (2,3) = 5.5, (2,4) = 2.4, (2,5) = 1.7\}, datatype = anything, storage = rectangular, order = Fortran\_order)$$

To access entries in an Array, use either square bracket or round bracket notation.

Square bracket notation respects the actual index of an Array, even when the index does not start at 1.

> $a[1,1]$

$$1$$

> $a[2,3]$

$$6$$

> $b[2,3]$

$$5.5$$

> $b[1,1]$

```
Error, Array index out of range
```

Round bracket indexing normalizes the dimensions to begin at 1. Since this method is relative, you can access the end of the array by entering -1.

> $a(-1,2)$

$$8$$

> $b(1,1)$

$$1.2$$

The **Array** constructor supports other syntaxes. It also supports many options. For more information on the **Array** constructor and the Array data structure, refer to the **?Array** help page. For more information on indexing methods, refer to the **?rtable_indexing** help page.

Large Arrays

Only one- and two-dimensional Arrays (with at most 10 indices in each dimension) display in the document. Larger Arrays display as a placeholder.

> $Array(0..100)$

$$\begin{bmatrix} \textit{0 .. 100 Array} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{bmatrix}$$

To view large Arrays:

- Double-click the placeholder.

The **Matrix Browser** displays the Array. For more information, see *Viewing Large Matrices and Vectors (page 162)*.

Tables

Tables are conceptually an extension of the Array data structure, but the table data structure is implemented using hash tables. Tables can be indexed by any values, not only integers.

Defining Tables and Accessing Entries

> $Greek := table([a = \alpha, b = \beta, c = \gamma]):$

> $Greek[b]$

$$\beta$$

You can also assign anything, for example, a list, to each element.

> $Translation := table([one = [un, uno], two = [deux, dos], three = [trois, tres]]):$

> $Translation[two]$

$$[deux, dos]$$

For more information on tables, refer to the **?table** help page.

Matrices and Vectors

Matrices and Vectors are specialized data structures used in linear algebra and vector calculus computations.

> $M := \begin{bmatrix} 12 & 33 \\ 83 & 12 \end{bmatrix}$: $v := \langle 2, 14 \rangle$:

For information on defining Matrices and Vectors, see *Creating Matrices and Vectors (page 159)*.

> $M.v$

$$\begin{bmatrix} 486 \\ 334 \end{bmatrix}$$

> $v^{\%T}.M$

$$\begin{bmatrix} 1186 & 234 \end{bmatrix}$$

> M^{-1}

$$\begin{bmatrix} -\frac{4}{865} & \frac{11}{865} \\ \frac{83}{2595} & -\frac{4}{865} \end{bmatrix}$$

For more information on these data structures, including how to access entries and perform linear algebra computations, see *Linear Algebra (page 159)*.

Functional Operators

A functional operator is a mapping $f: x \rightarrow y(x)$. The value of $f(x)$ is the result of evaluating $y(x)$.

Using functional operators, you can define mathematical functions.

Defining a Function

To define a function of one or two variables:

1. In the **Expression** palette, click one of the function definition items. See Figure 8.1. Maple inserts the function definition.

2. Replace the placeholders, using **Tab** to move to the next placeholder. **Note:** If pressing the **Tab** key indents the text, click the Tab icon in the toolbar. This allows you to move between placeholders.

3. Press **Enter**.

$$f := a \rightarrow y$$
$$f := (a, b) \rightarrow z$$

Figure 8.1: Function Definition Palette Items

For example, define a function that adds 1 to its input.

> $add1 := x \rightarrow x + 1:$

Note: To insert the right arrow, you can enter the characters **->**. In 2-D Math, Maple replaces -> with the right arrow symbol $\rightarrow$. In 1-D Math, the characters are not replaced.

You can evaluate the function **add1** with symbolic or numeric arguments.

> $add1(12); add1(x + y)$

$$13$$

$$x + y + 1$$

Distinction between Functional Operators and Other Expressions

The expression $x + 1$ is different from the functional operator $x \rightarrow x + 1$.

Assign the functional operator $x \rightarrow x + 1$ to *f*.

> $f := x \rightarrow x + 1:$

Assign the expression $x + 1$ to *g*.

> $g := x + 1:$

To evaluate the *functional operator f* at a value of *x*:

- Specify the value as an argument to f.

> $f(22)$

$$23$$

To evaluate the *expression* g at a value of x:

- You **must** use the **eval** command.

> $g(22)$

$$x(22) + 1$$

> $eval(g, x=22)$

$$23$$

For more information on the **eval** command, and on using palettes and context menus to evaluate an expression at a point, see *Substituting a Value for a Subexpression (page 359)*.

Multivariate and Vector Functions

To define a multivariate or vector function:

- Enclose coordinates or coordinate functions in parentheses (**( )**).

For example, a multivariate function:

> $f := (x, y) \rightarrow \frac{x^3}{y^2+1}$:

> $f(0, 0); f(-2.1, 1.9)$

$$0$$

$$-2.008893709$$

A vector function:

> $g := t \rightarrow (\sin(t), \cos(t), t)$:

> $g(0); g\left(\frac{\pi}{2}\right)$

$$0, 1, 0$$

$$1, 0, \frac{1}{2}\pi$$

Using Operators

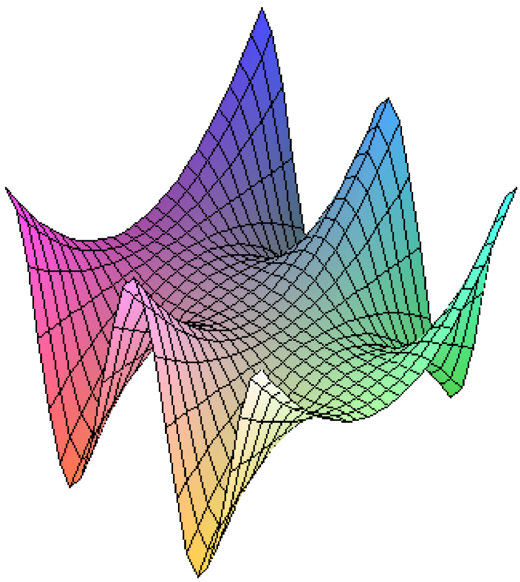
To perform an operation on a functional operator, specify arguments to the operator. For example, for the operator *f*, specify *f*(*x*), which Maple evaluates as an expression. See the following examples.

Plotting:

Plot a three-dimensional operator as an expression using the **plot3d** command.

> $h := (x, y) \rightarrow x^2\cos(y):$

> $plot3d\left(h(x, y), x = -2..2, y = -2\pi..2\pi\right)$



For information on plotting, see *Plots and Animations (page 237)*.

Integration:

Integrate a function using the **int** command.

> $k := x \rightarrow \sin(\cos(x)\pi)$:

> $int\left(k(t), t = 0 .. \frac{\pi}{2}\right)$

$$\frac{1}{2}\pi \operatorname{StruveH}(0, \pi)$$

For information on integration and other calculus operations, see *Calculus (page 175)*.

Strings

A string is a sequence of characters enclosed in double quotes (**" "**).

> $S :=$ "This is a sequence of characters." :

Accessing Characters

You can access characters in a string using brackets.

> $S[11 .. -2]$

"sequence of characters"

Using Strings

The **StringTools** package is an advanced set of tools for manipulating and using strings.

> $with(StringTools)$:

> *Random*(9,'*alnum*')

"8dvpw7bJm"

> *Stem*("impressive")

"impress"

> *Split*("Create a list of strings from the words in a string")

["Create", "a", "list", "of", "strings", "from", "the", "words", "in", "a", "string"]

8.3 Working with Maple Expressions

This section describes how to manipulate expressions using commands. Topics covered include testing the expression type, accessing operands of an expression, and evaluating an expression.

Low-Level Operations

Expression Types

A Maple *type* is a broad class of expressions that share common properties. Maple contains over 200 types, including:

- \`+\`
- **boolean**
- **constant**
- **integer**
- **Matrix**
- **trig**
- truefalse

For more information and a complete list of Maple types, refer to the **?type** help page.

The type commands return **true** if the expression satisfies the type check. Otherwise, they return **false**.

Testing the Type of an Expression

To test whether an expression is of a specified type:

- Use the **type** command.

> *type*(*sin*(*x*), '*trig*')

true

> *type*(*sin*(*x*) + cos(*x*), '*trig*')

false

For information on enclosing keywords in right single quotes ('), see *Delaying Evaluation (page 366)*.

Maple types are not mutually exclusive. An expression can be of more than one type.

> *type*(3, '*constant*')

true

> *type*(3, '*integer*')

true

For information on converting an expression to a different type, see *Converting (page 356)*.

Testing the Type of Subexpressions

To test whether an expression has a subexpression of a specified type:

- Use the **hastype** command.

> *hastype*(*sin*(*x*) + cos(*x*), '*trig*')

true

Testing for a Subexpression

To test whether an expression contains an instance of a specified subexpression:

- Use the **has** command.

> $has(\sin(x+y), x)$

$$true$$

> $has(\sin(x+y), x+y)$

$$true$$

> $has(\sin(x+y), \sin(x))$

$$false$$

The **has** command searches the structure of the expression for an exactly matching subexpression.

For example, the following calling sequence returns **false**.

> $has(x+y+z, x+z)$

$$false$$

To return all subexpressions of a particular type, use the **indets** command. For more information, see *Indeterminates (page 352)*.

Accessing Expression Components

Left and Right-Hand Side

To extract the left-hand side of an equation, inequality, or range:

- Use the **lhs** command.

To extract the right-hand side of an equation, inequality, or range:

- Use the **rhs** command.

For example:

> $y = x + 1$

$$y = x + 1 \tag{8.1}$$

> $lhs(\mathbf{(8.1)})$

$$y \tag{8.2}$$

> $rhs(\mathbf{(8.1)})$

$$x + 1 \tag{8.3}$$

For the following equation, the left endpoint of the range is the left-hand side of the right-hand side of the equation.

> $x = 3\,..5$

$$x = 3\,..5 \tag{8.4}$$

> $lhs(rhs(\mathbf{(8.4)}))$

$$3 \tag{8.5}$$

Numerator and Denominator

To extract the numerator of an expression:

- Use the **numer** command.

To extract the denominator of an expression:

- Use the **denom** command.

> $e := \dfrac{1 + sin(x)^3 - \dfrac{y}{x}}{y^2 - 1 + x}$:

If the expression is not in normal form, Maple normalizes the expression before selecting the numerator or denominator. (For more information on normal form, refer to the **?normal** help page.)

> *numer*(*e*)

$$x + \sin(x)^3 x - y$$

> *denom*(*e*)

$$x\left(y^2 - 1 + x\right)$$

> *denom*(*denom*(*e*))

$$1$$

The expression can be any algebraic expression. For information on the behavior for non-rational expressions, refer to the **?numer** help page.

Components of an Expression

The components of an expression are called its *operands*.

To count the number of operands in an expression:

- Use the **nops** command.

For example, construct a list of solutions to an equation.

> $solutions := \left[solve\left(6x^3 - x^2 + 7, x\right)\right]$

$$solutions := \left[-1, \frac{7}{12} - \frac{1}{12} \mathrm{I}\sqrt{119}, \frac{7}{12} + \frac{1}{12} \mathrm{I}\sqrt{119}\right]$$

Using the **nops** command, count the number of solutions.

> *nops*(*solutions*)

$$3$$

For more information on the **nops** command and operands, refer to the **?nops** help page.

Indeterminates

To find the indeterminates of an expression:

- Use the **indets** command.

The **indets** command returns the indeterminates as a set. Because the expression is expected to be rational, functions such as sin(x), f(x), and sqrt(x) are considered to be indeterminate.

> $indets\left((3+\pi)\, x^2 \sin\left(\sqrt{1+y}\right)\right)$

$$\left\{x, y, \sqrt{1+y}, \sin\left(\sqrt{1+y}\right)\right\}$$

To return all subexpressions of a particular type, specify the type as the second argument. For information on types, see *Testing the Type of an Expression (page 348)*.

> $indets\left((3+\pi)\, x^2 \sin\left(\sqrt{1+y}\right), 'radical'\right)$

$$\left\{\sqrt{1+y}\right\}$$

To test whether an expressions has subexpressions of a specific type (without returning them), use the **has** command. For more information, see *Testing for a Subexpression (page 349)*.

Manipulating Expressions

This section introduces the most commonly used manipulation commands. For additional manipulation commands, see *Iterative Commands (page 380)*.

Simplifying

To simplify an expression:

- Use the **simplify** command.

The **simplify** command applies simplification rules to an expression. Maple has simplification rules for various types of expressions and forms, including trigonometric functions, radicals, logarithmic functions, exponential functions, powers, and various special functions. You can also specify custom simplification rules using a set of *side relations*.

> $simplify\left(5 + 32 - 8^{\left(\frac{1}{3}\right)}\right)$

$$35$$

> $simplify\left(\sin(x)^2 + \ln(2y) + \cos(x)^2\right)$

$$1 + \ln(2) + \ln(y)$$

To limit the simplification, specify the type of simplification to be performed.

> $simplify\left(\sin(x)^2 + \ln(2\,y) + \cos(x)^2, \text{'}trig\text{'}\right)$

$$1 + \ln(2\,y)$$

> $simplify\left(\sin(x)^2 + \ln(2\,y) + \cos(x)^2, \text{'ln'}\right)$

$$\sin(x)^2 + \ln(2) + \ln(y) + \cos(x)^2$$

You can also use the **simplify** command with *side relations*. See *Substituting a Value for a Subexpression (page 359)*.

Factoring

To factor a polynomial:

- Use the **factor** command.

> $factor\left(x^6 - x^5 - 9\,x^4 + x^3 + 20\,x^2 + 12\,x\right)$

$$x\,(x-2)\,(x-3)\,(x+2)\,(x+1)^2$$

> $factor\left(x^3\,y + x^2\,y^2 - 3\,x^3 - x^2\,y + 2\,x\,y^2 - 6\,x^2 - 5\,x\,y + y^2 - 3\,x - 3\,y\right)$

$$(y-3)\,(x+1)^2\,(x+y)$$

Maple can factor polynomials over the domain specified by the coefficients. You can also factor polynomials over algebraic extensions. For details, refer to the **?factor** help page.

For more information on polynomials, see *Polynomial Algebra (page 150)*.

To factor an integer:

- Use the **ifactor** command.

> $ifactor(196911)$

$$(3)^4\,(11)\,(13)\,(17)$$

For more information on integers, see *Integer Operations (page 106)*.

Expanding

To expand an expression:

- Use the **expand** command.

The **expand** command distributes products over sums and expands expressions within functions.

> $expand\left((y-3)\,(x+1)^2\,(x+y)\right)$

$$y x^3 + y^2 x^2 - y x^2 + 2 y^2 x - 5 y x + y^2 - 3 x^3 - 6 x^2 - 3 x - 3 y$$

> $expand(\sin(x+y))$

$$\sin(x)\cos(y) + \cos(x)\sin(y)$$

Combining

To combine subexpressions in an expression:

- Use the **combine** command.

The **combine** command applies transformations that combine terms in sums, products, and powers into a single term.

> $combine(\sin(x)\cos(y) + \cos(x)\sin(y))$

$$\sin(x+y)$$

Recall that a was previously assigned to represent a two-dimensional array (see *Creating and Using Arrays (page 340)*).

> $combine\left((x^a)^2\,x\right)$

$$\begin{bmatrix} x^3 & x^5 & x^7 \\ x^9 & x^{11} & x^{13} \\ x^{15} & x^{17} & x^{19} \end{bmatrix}$$

The **combine** command applies only transformations that are valid for all possible values of names in the expression.

> $combine(4\ln(x) - \ln(y))$

$$4\ln(x) - \ln y$$

To perform the operation under assumptions on the names, use the **assuming** command. For more information about assumptions, see *Assumptions on Variables (page 144)*.

> $combine(4\ln(x) - \ln(y))$ assuming $x > 0, y > 0$

$$\ln\left(\frac{x^4}{y}\right)$$

Converting

To convert an expression:

- Use the **convert** command.

The **convert** command converts expressions to a new form, type (see *Expression Types (page 348)*), or in terms of a function. For a complete list of conversions, refer to the **?convert** help page.

Convert a measurement in radians to degrees:

> $convert(\pi, 'degrees')$

$$180\, degrees$$

To convert measurements that use units, use the Unit Converter or the **convert/units** command.

> $convert(450.2[[kg]], 'units', lb)$

$$992.5211043\, [[lb]]$$

For information on the Unit Converter and using units, see *Units (page 128)*.

Convert a list to a set:

> $convert([a, b, c, d], 'set')$

$$\left\{c, d, \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, Array(1..2, 2..5, \{(1, 2) = 1.2, (1, 3) = 4.9, (1, 4) = 6.3, (1, 5) = 7.1, (2, 2) = 9.2, (2, 3) = 5.5, (2, 4) = 2.4, (2, 5) = 1.7\}, datatype = anything, storage = rectangular, order = Fortran\_order)\right\}$$

Maple has extensive support for converting mathematical expressions to a new function or function class.

> $convert(\cos(x), \exp)$

$$\frac{1}{2} e^{I x} + \frac{1}{2} e^{-I x}$$

Find an expression equivalent to the inverse hyperbolic cotangent function in terms of Legendre functions.

> $convert(\operatorname{arccoth}(z), Legendre)$

$$\mathrm{LegendreQ}\left(0, \frac{1}{z}\right) + \frac{1}{2} \frac{\pi \sqrt{-(z-1)^2}}{z-1}$$

For more information on converting to a class of functions, refer to the **?convert/to_special_function** help page.

Normalizing

To normalize an expression:

- Use the **normal** command.

The **normal** command converts expressions into *factored normal form*.

> $normal\left(\frac{x^2-y^2}{(x-y)^3}\right)$

$$\frac{x+y}{(x-y)^2}$$

You can also use the **normal** command for zero recognition.

> $normal\left(x^3 + 1 - (x+1)^3 + 3x(1+x)\right)$

$$0$$

To expand the numerator and denominator, use the **expanded** option.

> $normal\left(\frac{x^2-y^2}{(x-y)^3}, 'expanded'\right)$

$$\frac{x+y}{x^2 - 2xy + y^2}$$

> $normal\left(\sin\left(1 + \frac{1}{x}\right)\right)$

$$\sin\left(\frac{x+1}{x}\right)$$

Sorting

To sort the elements of an expression:

- Use the **sort** command.

The **sort** command orders a list of values or terms of a polynomial.

> $sort([4, 3, 2.1, -4, 43, 0])$

$$[-4, 0, 2.1, 3, 4, 43]$$

> $sort\left(x + 4x^5 - 7x^2 + 1 + 9x^4 - 5x^3\right)$

$$4x^5 + 9x^4 - 5x^3 - 7x^2 + x + 1$$

> $sort\left(xy - 6y^2x + 2y^3 + 5x - 1\right)$

$$-6xy^2 + 2y^3 + xy + 5x - 1$$

For information on sorting polynomials, see *Sorting Terms (page 152)*.

For more information on sorting, refer to the **?sort** help page.

Evaluating Expressions

Substituting a Value for a Subexpression

To evaluate an expression at a point, you must substitute a value for a variable.

To substitute a value for a variable using context menus:

1. Right-click (**Control**-click, for Macintosh) the expression. Maple displays a context menu.

2. From the context menu, select **Evaluate at a Point**. The **Evaluate at a Point** dialog is displayed. See Figure 8.2.

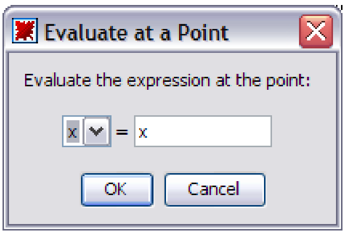


Figure 8.2: Evaluate at a Point

3. In the drop-down list, select the variable to substitute.

4. In the text field, enter the value to substitute for the variable. Click **OK**.

In Worksheet mode, Maple inserts the **eval** command calling sequence that performs the substitution. This is the most common use of the **eval** command.

For example, substitute $x=3$ in the following polynomial.

> $x^3 + 4x^2 - 7x + 2$

$$x^3 + 4x^2 - 7x + 2$$

> $eval\left(x^3 + 4x^2 - 7x + 2, [x=3]\right)$

$$44$$

To substitute a value for a variable using palettes:

1. In the **Expression** palette, click the evaluation at a point item $f(x)\big|_{x=a}$.

2. Specify the expression, variable, and value to be substituted.

For example:

$$> \left.\sqrt{x^2 - x - 3}\,\right|_{x=5}$$

$$\sqrt{17}$$

Substitutions performed by the **eval** function are *syntactical*, not the more powerful *algebraic* form of substitution.

If the left-hand side of the substitution is a name, Maple performs the substitution.

$$> eval\left(cos(a\,b\,c), a = \frac{\pi}{6}\right)$$

$$\cos\left(\frac{1}{6}\,\pi\,b\,c\right)$$

If the left-hand side of the substitution is not a name, Maple performs the substitution only if the left-hand side of the substitution is an operand of the expression.

$$> eval\left(\cos(a\,b), a\,b = \frac{\pi}{6}\right)$$

$$\frac{1}{2}\sqrt{3}$$

$$> eval\left(\cos(a\,b\,c), a\,b = \frac{\pi}{6}\right)$$

$$\cos(a\,b\,c)$$

Maple did not perform the evaluation because $a\,b$ is not an operand of $\cos(a\,b\,c)$. For information on operands, refer to the **?op** help page.

For algebraic substitution, use the **algsubs** command, or the **simplify** command with side relations.

> $algsubs\left(a\,b = \frac{\pi}{6}, \cos(a\,b\,c)\right)$

$$\cos\left(\frac{1}{6}\,c\,\pi\right)$$

> $simplify\left(\cos(a\,b\,c), \left\{a\,b = \frac{\pi}{6}\right\}\right)$

$$\cos\left(\frac{1}{6}\,c\,\pi\right)$$

Numerical Approximation

To compute an approximate numerical value of an expression:

- Use the **evalf** command.

The **evalf** command returns a floating-point (or complex floating-point) number or expression.

> $evalf\left(\cos\left(\frac{\pi}{6}\right)\right)$

$$0.8660254040$$

> $evalf\left(\frac{17}{\sqrt{3}}x^2 + x - e^{\pi}\right)$

$$9.814954579\,x^2 + x - 23.14069264$$

> $evalf(\pi)$

$$3.141592654$$

By default, Maple calculates the result to ten digits of accuracy, but you can specify any number of digits as an index, that is, in brackets (**[ ]**).

> $evalf[40](\pi)$

$$3.141592653589793238462643383279502884197$$

For more information, refer to the **?evalf** help page.

See also *Numerically Computing a Limit (page 176)* and *Numeric Integration (page 185)*.

Evaluating Complex Expressions

To evaluate a complex expression:

- Use the **evalc** command.

If possible, the **evalc** command returns the output in the canonical form **expr1** + **i expr2**.

In 2-D Math input, you can enter the imaginary unit using the following two methods.

- In the **Common Symbols** palette, click the **i** or **j** item. See *Palettes (page 22)*.
- Enter *i* or *j*, and then press the symbol completion key. See *Symbol Names (page 29)*.

> $evalc(\sqrt{1+\mathrm{i}})$

$$\frac{1}{2}\sqrt{2+2\sqrt{2}}+\frac{1}{2}\mathrm{I}\sqrt{-2+2\sqrt{2}}$$

> $evalc(\sin(3+5\,\mathrm{j}))$

$$\sin(3)\cosh(5)+\mathrm{I}\cos(3)\sinh(5)$$

In 1-D Math input, enter the imaginary unit as an uppercase i (**I**).

```
> evalc(2^(1 + I));
```

$$2\cos(\ln(2))+2\,\mathrm{I}\sin(\ln(2))$$

Evaluating Boolean Expressions

To evaluate an expression involving relational operators ($=$, $\neq$, $>$, $<$, $\leq$, and $\geq$):

- Use the **evalb** command.

Note: In 1-D Math input, enter $\neq$, $\leq$, and $\geq$ using the **<>**, **<=**, and **>=** operators.

The **evalb** command uses a three-valued logic system. The return values are **true**, **false**, and **FAIL**. If evaluation is not possible, an unevaluated expression is returned.

> $evalb(x = x)$

$$true$$

> $evalb(x = y)$

$$false$$

> $evalb(3 + 2\,I < 2 + 3\,I)$

$$FAIL$$

Important: The **evalb** command does not perform arithmetic for inequalities involving $<$, $\leq$, $>$, or $\geq$, and does not simplify expressions. Ensure that you perform these operations before using the **evalb** command.

> $evalb(\Re(x) < \Re(x + 1))$

$$\Re(x) < 1 + \Re(x)$$

> $evalb(\Re(x) - \Re(x + 1) < 0)$

$$true$$

Applying an Operation or Function to All Elements in a List, Set, Table, Array, Matrix, or Vector

You can use the tilde character (~) to apply an operation or function to all of the elements in a list, set, table, Array, Matrix, or Vector.

In the following example, each element in the Matrix *M* is multiplied by 2 by adding a tilde character after the multiplication operator($\cdot$) .

> $M := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

$$M := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \tag{8.6}$$

> $M \cdot\sim 2$

$$\begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 16 & 18 \end{bmatrix} \tag{8.7}$$

In the following example, the function **sin** is applied to each element in the Matrix *M*.

> $\sin\sim(M)$

$$\begin{bmatrix} \sin(1) & \sin(2) & \sin(3) \\ \sin(4) & \sin(5) & \sin(6) \\ \sin(7) & \sin(8) & \sin(9) \end{bmatrix} \tag{8.8}$$

The tilde character can also be used to apply a function to multiple data sets, for example,

> $\mathit{diff}\sim\left(z \cdot x^2 + x \cdot y^2, [x, x, y, y, z, z], [y, z, x, z, x, y]\right);$

$$[2\,y, 2\,x, 2\,y, 0, 2\,x, 0] \tag{8.9}$$

You can use values in one data structure type to compute values in another data structure type, as long as both data structures are dimensional and contain the same number of elements. In the following example, the values in an Array are compared to the values in a Matrix that contains the same number of elements.

> $[12, 88, 20] >\sim \langle 3, 100, 25 \rangle$

$$\begin{bmatrix} 3 < 12 \\ 100 < 88 \\ 25 < 20 \end{bmatrix} \tag{8.10}$$

For more information, refer to the **?elementwise** help page.

Levels of Evaluation

In a symbolic mathematics program such as Maple, you encounter the issue of *levels of evaluation.* If you assign **y** to **x**, **z** to **y**, and then 5 to **z**, what is the value of **x**?

At the top-level, Maple *fully evaluates* names. That is, Maple checks if the name or symbol has an assigned value. If it has a value, Maple substitutes the value for the name. If this value has an assigned value, Maple performs a substitution, recursively, until no more substitutions are possible.

For example:

> $x := y$:

> $y := z$:

> $z := 5$:

Maple fully evaluates the name **x**, and returns the value 5.

> x

$$5$$

To control the level of evaluation of an expression:

- Use the **eval** command with an integer second argument.

If passed a single argument, the **eval** command fully evaluates that expression. If you specify an integer second argument, Maple evaluates the expression to that level.

> $eval(x)$

$$5$$

> $eval(x, 1)$

$$y$$

> $eval(x, 2)$

$$z$$

> $eval(x, 3)$

$$5$$

For more details on levels of evaluation, refer to the **?lastnameevaluation**, **?assigned**, and **?evaln** help pages.

Delaying Evaluation

To prevent Maple from immediately evaluating an expression:

- Enclose the expression in right single quotes (**' '**).

Because right single quotes delay evaluation, they are referred to as *unevaluation quotes*.

> $i := 4$:

> i

$$4$$

> $'i'$

$$i$$

Using an Assigned Name as a Variable or Keyword

If you use an assigned name as a variable, Maple evaluates the name to its value, and passes the value to the command. In this example, that causes Maple to return an error message.

> $\sum_{i=1}^{n} i^2$

```
Error, (in sum) summation variable previously assigned, second argument
 evaluates to 4 = 1 .. n
```

Note: In general, it is recommended that you unassign a name to use it as a variable. See *Unassigning a Name Using Unevaluation Quotes (page 368)*.

To use an assigned name as a variable:

- Enclose the name in unevaluation quotes. Maple passes the name to the command.

> $\sum_{'i'=1}^{n} 'i'^2$

$$\frac{1}{3}(n+1)^3 - \frac{1}{2}(n+1)^2 + \frac{1}{6}n + \frac{1}{6}$$

Important: It is recommended that you enclose keywords in unevaluation quotes.

For example, if you enclose the keyword **left** in unevaluation quotes, Maple uses the name, not its assigned value.

> $left := 3:$

> $limit\left(\frac{1}{x}, x=0, 'left'\right)$

$$-\infty$$

Full Evaluation of an Expression in Quotes

Full evaluation of a quoted expression removes one set of right single quotes.

> $i := 4:$

> ' ' i' + 1'

$$'i' + 1 \tag{8.11}$$

> **(8.11)**

$$i + 1 \tag{8.12}$$

> **(8.12)**

$$5 \tag{8.13}$$

For information on equation labels and equation label references, see *Equation Labels (page 97)*.

Enclosing an expression in unevaluation quotes delays evaluation, but does not prevent automatic simplification.

> ' $q - i + 3\,q$'

$$4\,q - i \tag{8.14}$$

Unassigning a Name Using Unevaluation Quotes

To unassign a name:

- Assign the name enclosed in unevaluation quotes to itself.

> i :='i':

> i

$$i$$

You can also unassign a name using the **unassign** command. For more information, see *Unassigning Names (page 95)*.

9 Basic Programming

You have used Maple interactively in the previous chapters, sequentially performing operations such as executing a single command. Because Maple has a complete programming language, you can also use sophisticated programming constructs.

In Maple, you can write programs called procedures, and save them in modules. These modules can be used and distributed in the same way as Maple packages.

Important: It is strongly recommended that you use the Worksheet mode and 1-D Math input when programming or using programming commands. Hence, all input in this chapter is entered as 1-D Math.

9.1 In This Chapter

Section	**Topics**
Flow Control (page 370) - Basic programming constructs	• Conditional Execution (**if** Statement) • Repetition (**for** Statement)
Iterative Commands (page 380) - Specialized, efficient iterative commands	• Creating a sequence • Adding and Multiplying Expressions • Selecting Expression Operands • Mapping a Command over a Set or List • Mapping a Binary Command over Two Lists or Vectors
Procedures (page 383) - Maple programs	• Defining and Running Simple Procedures • Procedures with Inputs • Procedure Return Values • Displaying Procedure Definitions • Displaying Maple Library Procedure Definitions • Modules
Programming in Documents (page 387) - Display methods for Maple code	• Code Edit Region • Startup Code • Document Blocks

9.2 Flow Control

Two basic programming constructs in Maple are the **if** statement, which controls the conditional execution of statement sequences, and the **for** statement, which controls the repeated execution of a statement sequence.

Conditional Execution (if Statement)

You can specify that Maple perform an action only if a condition holds. You can also perform an action, from a set of many, depending on which conditions hold.

Using the **if** statement, you can execute one statement from a series of statements based on a boolean (**true**, **false**, or **FAIL**) condition. Maple tests each condition in order. When a condition is satisfied, Maple executes the corresponding statement, and then exits the **if** statement.

Syntax

The **if** statement has the following syntax.

```
> if conditional_expression1 then
      statement_sequence1
  elif conditional_expression2 then
      statement_sequence2
  elif conditional_expression3 then
      statement_sequence3
  ...
  else
      statement_sequenceN
  end if;
```

The conditional expressions (*conditional_expression1*, *conditional_expression2*, ...) can be any boolean expression. You can construct boolean expressions using:

- Relational operators - <, <=, =, >=, >, <>
- Logical operators - **and**, **or**, **xor**, **implies**, **not**
- Logical names - **true**, **false**, **FAIL**

The statement sequences (*statement_sequence1*, *statement_sequence2*, ..., *statement_sequenceN*) can be any sequence of Maple statements, including **if** statements.

The **elif** clauses are optional. You can specify any number of **elif** clauses.

The **else** clause is optional.

Simple if Statements

The simplest **if** statement has only one conditional expression.

```
> if conditional_expression then
      statement_sequence
  end if;
```

If the conditional expression evaluates to **true**, the sequence of statements is executed. Otherwise, Maple immediately exits the **if** statement.

For example:

```
> x := 1173:
```

```
> if not isprime(x) then
      ifactor(x);
  end if;
```

$$(3)\,(17)\,(23)$$

else Clause

In a simple **if** statement with an **else** clause, if the evaluation of the conditional expressions returns **false** or **FAIL**, Maple executes the statement sequence in the **else** clause.

For example:

```
> if false then
      "if statement";
  else
      "else statement";
  end if;
```

"else statement"

elif Clauses

In an **if** statement with **elif** clauses, Maple evaluates the conditional expressions in order until one returns **true**. Maple executes the corresponding statement sequence, and then exits the **if** statement. If no evaluation returns **true**, Maple exits the **if** statement.

```
> x := 11:

> if not type(x, integer) then
      printf("%a is not an integer.", x);
  elif x >= 10 then
      printf("%a is an integer with more than one digit.", x);
  elif x >= 0 then
      printf("%a is an integer with one digit.", x);
  end if;

11 is an integer with more than one digit.
```

Order of elif Clauses: An **elif** clause's statement sequence is executed only if the evaluation of all previous conditional expressions returns **false** or **FAIL**, and the evaluation of its conditional expression returns **true**. This means that changing the order of **elif** clauses may change the behavior of the **if** statement.

In the following **if** statement, the **elif** clauses are in the **wrong order**.

```
> if not(type(x, integer)) then
      printf("%a is not an integer.", x);
  elif x >= 0 then
      printf("%a is an integer with one digit.", x);
  elif x >= 10 then
      printf("%a is an integer with more than one digit.", x);
  end if;

11 is an integer with one digit.
```

elif and else Clauses

In an **if** statement with **elif** and **else** clauses, Maple evaluates the conditional expressions in order until one returns **true**. Maple executes the corresponding statement sequence, and then exits the **if** statement. If no evaluation returns **true**, Maple executes the statement sequence in the **else** clause.

```
> x := -12:
```

```
> if not type(x, integer) then
      printf("%a is not an integer.", x);
  elif x >= 10 then
      printf("%a is an integer with more than one digit.", x);
  elif x >= 0 then
      printf("%a is an integer with one digit.", x);
  else
      printf("%a is a negative integer.", x);
  end if;
```

```
-12 is a negative integer.
```

For more information on the **if** statement, refer to the **?if** help page.

Repetition (for Statement)

Using **repetition** statements, you can repeatedly execute a statement sequence. You can repeat the statements in three ways.

- Until a counter variable value exceeds a limit (**for/from** loop)
- For each operand of an expression (**for/in** loop)
- Until a boolean condition does not hold (**while** loop)

for/from Loop

The **for**/**from** loop statement repeats a statement sequence until a counter variable value exceeds a limit.

Syntax

The **for**/**from** loop has the following syntax.

```
> for counter from initial by increment to final do
      statement_sequence
  end do;
```

The behavior of the **for/from** loop is:

1. Assign the *initial* value to the name **counter**.

2. Compare the value of **counter** to the value of *final*. If the **counter** value **exceeds** the *final* value, exit the loop. (This is the *loop bound test*.)

3. Execute the *statement_sequence*.

4. Increment the **counter** value by the value of *increment*.

5. Repeat steps 2 to 4, until Maple exits the loop.

The **from**, **by**, and **to** clauses are optional and can be in any order between the **for** clause and the **do** keyword. Table 9.1 lists the default clause values.

Table 9.1: Default Clause Values

Clause	**Default Value**
from *initial*	1
by *increment*	1
to *final*	**infinity** (∞)

Examples

The following loop returns the square root of the integers 1 to 5 (inclusive).

```
> for n to 5 do
     evalf(sqrt(n));
  end do;
```

$$1.$$

$$1.414213562$$

$$1.732050808$$

$$2.$$

$$2.236067977$$

When the value of the counter variable **n** is **strictly greater than 5**, Maple exits the loop.

```
> n;
```

$$6$$

The previous loop is equivalent to the following **for/from** statement.

```
> for n from 1 by 1 to 5 do
      evalf(sqrt(n));
  end do;
```

$$1.$$

$$1.414213562$$

$$1.732050808$$

$$2.$$

$$2.236067977$$

The **by** value can be negative. The loop repeats until the value of the counter variable is **strictly less than** the **final** value.

```
> for n from 10 by -1 to 3 do
      if isprime(n) then
         print(n);
      end if;
  end do;
```

$$7$$

$$5$$

$$3$$

```
> n;
```

$$2$$

for/in Loop

The **for**/**in** loop statement repeats a statement sequence for each component (*operand*) of an expression, for example, the elements of a list.

Syntax

The **for**/**in** loop has the following syntax.

```
> for variable in expression do
      statement_sequence
  end do;
```

The **for** clause must appear first.

The behavior of the **for/in** loop is:

1. Assign the first operand of *expression* to the name *variable*.

2. Execute the *statement_sequence*.

3. Assign the next operand of *expression* to *variable*.

4. Repeat steps 2 and 3 for each operand in *expression*. If there are no more operands, exit the loop. (This is the *loop bound test*.)

Example

The following loop returns a floating-point approximation to the **sin** function at the angles (measured in degree) in the list **L**.

```
> L := [23.4, 87.2, 43.0, 99.7]:
```

```
> for i in L do
      evalf(sin(i*Pi/180));
  end do;
```

$$0.3971478907$$

$$0.9988061374$$

$$0.6819983602$$

$$0.9857034690$$

while Loop

The **while** loop repeats a statement sequence until a boolean expression does not hold.

Syntax

The **while** loop has the following syntax.

```
> while conditional_expression do
      statement_sequence
  end do;
```

A **while** loops repeats until its boolean expression *conditional_expression* evaluates to **false** or **FAIL**. For more information on boolean expressions, see *Conditional Execution (if Statement) (page 370)*.

Example

The following loop computes the digits of 872,349 in base 7 (in order of *increasing* significance).

```
> x := 872349:
```

```
> while x > 0 do
    irem(x, 7);
    x := iquo(x, 7);
  end do;
```

$$2$$

$$x := 124621$$

$$0$$

$$x := 17803$$

$$2$$

$$x := 2543$$

$$2$$

$$x := 363$$

$$6$$

$$x := 51$$

$$2$$

$$x := 7$$

$$0$$

$$x := 1$$

$$1$$

$$x := 0$$

To perform such conversions efficiently, use the **convert/base** command.

```
> convert(872349, base, 7);
```

$$[2, 0, 2, 2, 6, 2, 0, 1]$$

For information on non-base 10 numbers, see *Non-Base 10 Numbers (page 108)*.

General Loop Statements

You can include a **while** statement in a **for/from** or **for/in** loop.

The general **for**/**from** loop has the following syntax.

```
> for counter from initial by increment to final
  while conditional_expression do
     statement_sequence
  end do;
```

The general **for**/**in** loop has the following syntax.

```
> for variable in expression
  while conditional_expression do
     statement_sequence
  end do;
```

After testing the loop bound condition at the beginning of each iteration of the **for** loop, Maple evaluates *conditional_expression*.

- If *conditional_expression* evaluates to **false** or **FAIL**, Maple exits the loop.
- If *conditional_expression* evaluates to **true**, Maple executes *statement_sequence*.

Infinite Loops

You can construct a loop for which there is no exit condition, for example, a **while** loop in which the *conditional_expression* always evaluates to **true**. This is called an *infinite loop*. Maple indefinitely executes an infinite loop unless it executes a **break**, **quit**, or **return** statement or you interrupt the computation. For more information, refer to the **?break**, **?quit**, **?return**, and **?interrupt** help pages.

Additional Information

For more information on the **for** statement and looping, refer to the **?do** help page.

9.3 Iterative Commands

Maple has commands that perform common selection and repetition operations. These commands are more efficient than similar algorithms implemented using library commands. Table 9.2 lists the iterative commands.

Table 9.2: Iterative Commands

Command	Description
seq	Create sequence
add	Compute numeric sum
mul	Compute numeric product
select	Return operands that satisfy a condition
remove	Return operands that do not satisfy a condition
selectremove	Return operands that satisfy a condition and separately return operands that do not satisfy a condition
map	Apply command to the operands of an expression
zip	Apply binary command to the operands of two lists or vectors

Creating a Sequence

The **seq** command creates a sequence of values by evaluating a specified expression over a range of index values or the operands of an expression. See Table 9.3.

Table 9.3: The seq Command

Calling Sequence Syntax	Examples
seq(*expression*, *name* = *initial* .. *final***);**	`> seq(exp(x), x=-2..0);` $e^{-2}, e^{-1}, 1$
seq(*expression*, *name* **in** *expression***);**	`> seq(u, u in [Pi/4, Pi^2/2, 1/Pi]);` $\frac{1}{4}\pi, \frac{1}{2}\pi^2, \frac{1}{\pi}$

Adding and Multiplying Expressions

The **add** and **mul** commands add and multiply sequences of expressions over a range of index values or the operands of an expression. See Table 9.4.

Table 9.4: The add and mul Commands

Calling Sequence Syntax	Examples

add(*expression*, *name* = *initial* .. *final***);**
 	
mul(*expression*, *name* = *initial* .. *final***);**
	`> add(exp(x), x = 2..4);`
	$e^2 + e^3 + e^4$
	`> mul(2*x, x = 1 .. 10);`
	3715891200

add(*expression*, *name* **in** *expression***);**
 	
mul(*expression*, *name* **in** *expression***);**
	`> add(u, u in [Pi/4, Pi/2, Pi]);`
	$\frac{7}{4}\pi$
	`> mul(u, u in [Pi/4, Pi/2, Pi]);`
	$\frac{1}{8}\pi^3$

The endpoints of the index range (**initial** and **final**) in the **add** and **mul** calling sequence must evaluate to numeric constants. For information on symbolic sums and products, refer to the **?sum** and **?product** help pages.

Selecting Expression Operands

The **select**, **remove**, and **selectremove** commands apply a boolean-valued procedure or command to the operands of an expression. For information on operands, refer to the **?op** help page.

- The **select** command returns the operands for which the procedure or command returns **true**.
- The **remove** command returns the operands for which the procedure or command returns **false** or **FAIL**.
- The selectremove command returns two expressions of the same type as the input expression. - The first consists of the operands for which the procedure or command returns **true**. - The second consists of the operands for which the procedure or command returns **false** or **FAIL**.

The structure of the output is the same as the structure of the input. See Table 9.5.

For information on Maple procedures, see *Procedures (page 383)*.

Table 9.5: The select, remove, and selectremove Commands

Calling Sequence Syntax	Examples
select(*proc_cmd*, *expression***);**	`> select(issqr, {198331, 889249, 11751184, 9857934});` $\{889249, 11751184\}$
remove(*proc_cmd*, *expression***);**	`> remove(var -> degree(var) > 3, 2*x^3*y - y^3*x + z );` z
selectremove(*proc_cmd*, *expression***);**	`> selectremove(x -> evalb(x > round(x)), [sin(0.), sin(1.), sin(3.)]);` $[0.1411200081], [0., 0.8414709848]$

For information on optional arguments to the selection commands, refer to the **?select** help page.

Mapping a Command over a Set or List

The **map** command applies a name, procedure, or command to each element in a set or list. See Table 9.6.

Table 9.6: The map Command

Calling Sequence Syntax	Examples

map(*name_proc_cmd*, *expression***);**	`> map(f, {a, b, c});`
	 $\{f(a), f(b), f(c)\}$
	 `> map(u -> int(cos(x), x = 0 .. u), [Pi/4, Pi/7, Pi/3.0]);`
	 $\left[\frac{1}{2}\sqrt{2}, \cos\left(\frac{5}{14}\pi\right), 0.8660254038\right]$

For information on mapping over the operands of other expressions, optional arguments to the **map** command, and other mapping commands, refer to the **?map** help page.

Mapping a Binary Command over Two Lists or Vectors

The **zip** command applies a name or binary procedure or command component-wise to two lists or vectors.

By default, the length of the returned object is that of the shorter list or vector. If you specify a value as the (optional) fourth argument, it is used as the value of the missing elements of the shorter list or vector. In this case, the length of the return value is that of the longer list or vector. See Table 9.7.

Table 9.7: The zip Command

Calling Sequence Syntax	Examples

zip(*proc_cmd*, *a*, *b*);
zip(*proc_cmd*, *a*, *b*, *fill*);
	`> zip(f, [i, j], [k, l]);`
	$[f(i,k), f(j,l)]$
	`> zip(AiryAi, [1, 2], [0], 1);`
	$\left[-\frac{1}{2}\,\frac{3^{1/6}\,\Gamma\left(\frac{2}{3}\right)}{\pi}, \mathrm{AiryAi}(2,1)\right]$

For more information on the **zip** command, refer to the **?zip** help page.

Additional Information

For more information on looping commands, refer to the corresponding command help page.

9.4 Procedures

A Maple procedure is a program consisting of Maple statements. Using procedures, you can quickly execute the contained sequence of statements.

Defining and Running Simple Procedures

To define a procedure, enclose a sequence of statements between **proc(...)** and **end proc** statements. In general, you assign a procedure definition to a name.

The following procedure returns the square root of 2.

```
> p := proc() sqrt(2); end proc;
```

$$p := \textbf{proc}(\ )\ \text{sqrt}(2)\ \textbf{end proc}$$

Note: Maple returns the procedure definition.

To improve readability of procedures, it is recommended that you define a procedure using multiple lines, and indent the lines using space characters. To begin a new line (without evaluating the incomplete procedure definition), press **Shift+Enter**. When you have finished entering the procedure, press **Enter** to create the procedure.

For example:

```
> p := proc()
      sqrt(2);
  end proc:
```

To run the procedure **p**, enter its name followed by parentheses (**( )**).

```
> p();
```

$$\sqrt{2}$$

Procedures with Inputs

You can define a procedure that accepts user input. In the parentheses of the **proc** statement, specify the parameter names. For multiple parameters, separate the names with commas.

```
> geometric_mean := proc(x, y)
      sqrt(x*y);
  end proc:
```

When the user runs the procedure, the parameter names are replaced by the argument values.

```
> geometric_mean(13, 17);
```

$$\sqrt{221}$$

```
> geometric_mean(13.5, 17.1);
```

$$15.19374871$$

For more information on writing procedures, including options and local and global variables, refer to the **?procedure** help page.

Procedure Return Values

When you run a procedure, Maple returns **only** the last statement result value computed. Maple does not return the output for each statement in the procedure. It is irrelevant whether you use semicolons or colons as statement separators.

```
> p := proc(a, b)
      a + b;
      a - b:
  end proc:
```

```
> p(1, 2);
```

$$-1$$

Displaying Procedure Definitions

Unlike simple Maple objects, you cannot display the value of a procedure by entering its name.

```
> geometric_mean;
```

$$geometric\_mean$$

You must evaluate the name of the procedure using the **print** (or **eval**) command.

```
> print(geometric_mean);
```

$$\mathbf{proc}(x, y)\ \ \mathrm{sqrt}(x * y)\ \ \mathbf{end\ proc}$$

Displaying Maple Library Procedure Definitions

Maple procedure definitions are a valuable learning tool. To learn how to program in Maple, it is recommended that you examine the procedures available in the Maple library.

By default, the **print** command returns only the **proc** and **end proc** statements and (if present) the description fields of a Maple procedure.

```
> print(assign);
```

proc(*a*) ... **end proc**

To display a Maple library procedure definition, first set the value of the **interface verboseproc** option to **2**. Then re-execute the **print** calling sequence.

```
> interface('verboseproc' = 2):
```

```
> print(assign);
```

```
proc(a)
    option Copyright (c) 1990 by Waterloo Maple Inc. All rights reserved.;
    local i;
    if 1 <=nargs and type(a, {'::', 'name', 'function'}) then
        a := args[2 .. -1]
    elif nargs = 1 then
        if type(a, {'::', 'name', 'function'} = 'anything') then
            assign/internal(op(a))
        elif type(a, '`=`') then
            if type([lhs(a)], 'list'({'::', 'name', 'function'})) then
                if nops([lhs(a)]) = nops([rhs(a)]) then
                    zip('assign/internal', [lhs(a)], [rhs(a)])
                else
                    error "ambiguous multiple assignment"
                end if
            else
                error "invalid arguments"
            end if
        elif type(a, {'set', 'list'}) then
            map(procname, a)
        else
            error "invalid arguments"
        end if
    else
        seq(procname(i), i = args)
    end if;
    NULL
end proc
```

Modules

Maple procedures associate a sequence of commands with a single command. The module, a more complex programming structure, allows you to associate related procedures and data.

A key feature of modules is that they *export* variables. This means that the variables are available outside the module in which they are created. Most Maple packages are implemented as modules. The package commands are exports of the module.

For more information on modules, refer to the **?module** help page.

9.5 Programming in Documents

To write Maple code, you could simply open a Maple worksheet and start typing. However, if you want to create a readable document with the code interspersed or hidden, there are several options available.

Code Edit Region

The code edit region allows you to program in one contained region, in a natural way. Features include the ability to press **Enter** for line breaking and indentation preservation. Figure 9.1 shows the expanded code edit region.

To insert a new code edit region into your worksheet:

- From the **Insert** menu, select **Code Edit Region**.

Figure 9.1: Code Edit Region

To execute the code within this region, right-click in the region and select **Execute Code**.

You can hide the code in a code edit region by minimizing the region. To minimize, right-click in the region and select **Collapse Code Edit Region**. When the region is minimized, an icon appears with the first line of the code written next to it. It is recommended that you make the first line a comment describing the program or programs contained in the region. See Figure 9.2.

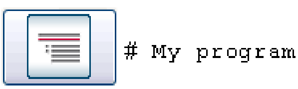


Figure 9.2: Collapsed Code Edit Region

To re-execute the code in the region while it is collapsed, click this icon.

For more information, refer to the **?CodeEditRegion** help page.

Startup Code

Startup code allows you to define commands and procedures that are executed each time the document is opened and after restart is called. This code is completely hidden to others reading the document. For example, use this region to define procedures that will be used throughout the document code but that would take up space and distract readers from the message of the document.

To enter startup code for a document:

1. From the **Edit** menu, select **Startup Code**. Alternatively, click the startup code icon in the toolbar, .

2. Enter commands to be run each time the worksheet is opened or restart is called.

3. Click **Syntax** to check the syntax of the entered code before closing.

4. Click **Save** to save the contents and close the dialog.

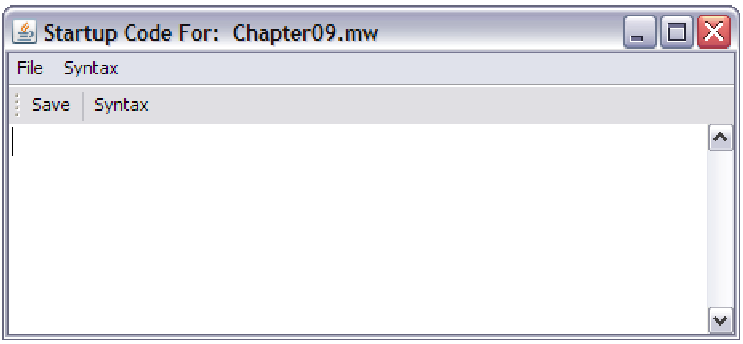


Figure 9.3: Startup Code Editor

For more information, refer to the **?startupcode** help page.

Document Blocks

Document blocks allow you to display the output from commands without showing the commands used. You can intersperse text, 2-D math, and Maple commands in a readable way. For more information, see *Document Blocks (page 51)* in Chapter 1.

10 Embedded Components and Maplets

These graphical components help you to create documents to use and share with colleagues or students, that interact with Maple code within the document without needing the reader to understand that Maple code. Other methods of interaction with Maple are described throughout this guide.

10.1 In This Chapter

Section	Topics
Using Embedded Components (page 389) - Basic interacting with Maple documents containing embedded components	• Interacting with Components • Printing and Exporting
Creating Embedded Components (page 393) - Methods for creating embedded components that work together and with your document	• Inserting Components • Editing Components • Removing Components • Integrating into a Document
Using Maplets (page 400) - Methods for launching a Maplet	• Maplet File • Maple Document
Authoring Maplets (page 402) - Methods for authoring and saving a Maplet	• Maplet Builder • **Maplets** Package • Saving

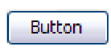
10.2 Using Embedded Components

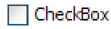
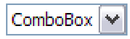
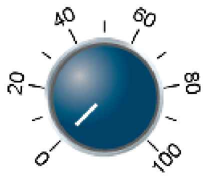
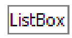
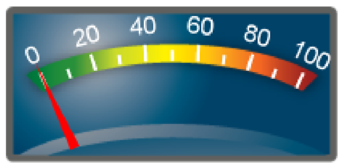
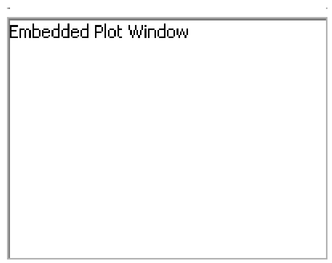
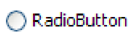
Interacting

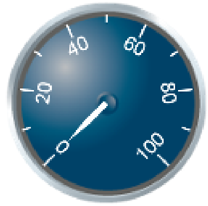
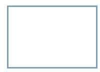
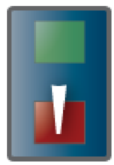
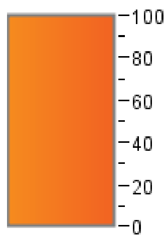
Embedded components allow readers to interact with Maple code through graphical components, rather than commands. They can be used alone, as with a button that you click to execute code, or together, such as a drop-down menu where you select an item, and a change takes place in a plot component.

Component Descriptions

Table 10.1: Embedded Component Descriptions

Component Name and Description	Inserted Image
Button - Click to perform an action; that is, execute code.	

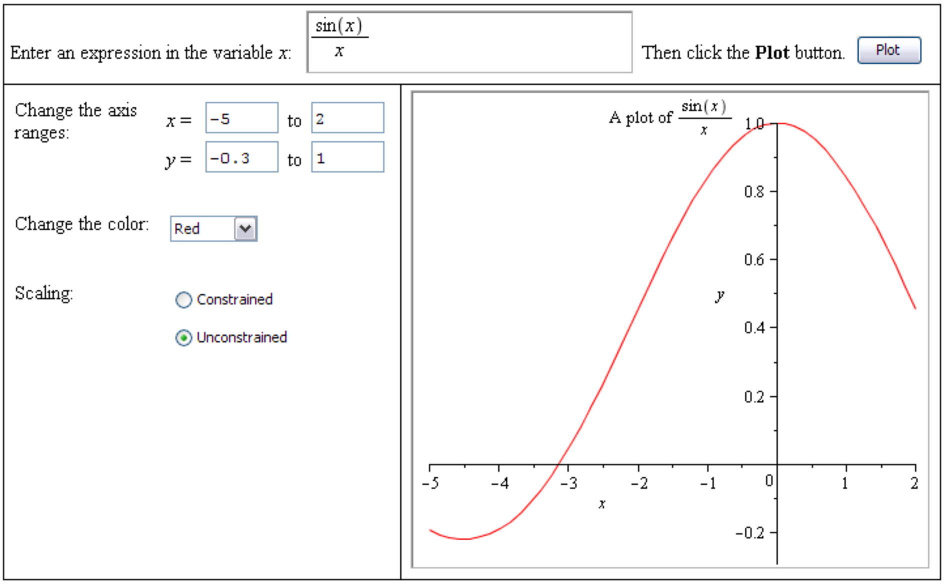
Component Name and Description	Inserted Image
Check Box - Select or de-select. Change the caption, and enter code to execute when the value changes.	
Combo Box - Select one of the listed options from the drop-down menu. Change the items listed, and enter code to execute when the value changes.	
Dial - Select or display an integer or floating-point value. Change the display, and enter code to execute when the value changes.	
Label - Display a label. The value can be updated based on code in the document or another embedded component.	
List Box - Display a list of items. Change the items listed, and enter code to execute when an item is selected.	
Math Expression - Enter or display a mathematical expression. The value can be updated based on code in the document or another embedded component.	
Meter - Select or display an integer or floating-point value. Change the display, and enter code to execute when the value changes.	
Plot - Display a 2-D or 3-D plot or animation. This plot or animation can be interacted with in the same way as other plots (see *Plots and Animations (page 237)*). The value can be updated based on code in the document or another embedded component. You can also enter code to be executed when the **execute code** pointer is used to click or drag in the plot region.	
Radio Button - Use with other radio buttons to select one in a group. Enter code to execute when the value changes.	

Component Name and Description	**Inserted Image**
Rotary Gauge - Select or display an integer or floating-point value. Change the display, and enter code to execute when the value changes.	
Slider - Select or display an integer or floating-point value. Change the display, and enter code to execute when the value changes.	
Text Area - Enter or display plain text. The value can be updated based on code in the document or another embedded component, and you can enter code to execute when the value changes.	
Toggle Button - Select or display one of two options. Change the images displayed, and enter to code to execute when the value changes.	
Volume Gauge - Select or display an integer or floating-point value. Change the display, and enter code to execute when the value changes.	

Example 1 - Using Embedded Components

This example demonstrates several components working together to perform a task. The user inputs an expression, which is plotted when the button is clicked. Plot options are controlled by text areas, a combo box, a math expression, and radio buttons.

For an interactive version of this example, see the .mw version of this manual. In Maple, from the **Help** menu, select **Manuals, Resources, and More... → Manuals → User Manual**.



Printing and Exporting a Document with Embedded Components

Printing: When printing a document, embedded components are rendered as they appear on screen.

Exporting: Exporting a document with embedded components to other formats produces the following results.

- HTML format - components are exported as **.gif** files.
- RTF format - components are rendered as **bitmap** images in the **.rtf** document.
- LaTeX - components are exported as **.eps** files.
- PDF - components are rendered as static images.

10.3 Creating Embedded Components

Embedded Components are graphical components that you can add to your document. They provide interactive access to Maple code without requiring the user to know Maple commands, and include buttons, sliders, math and text input areas, and plot display.

Inserting Components

The graphical interface components can be inserted by using the **Components** palette (Figure 10.1) or by cutting/copying and pasting existing components to another area of the document. Although copied components have most of the same characteristics, they are distinct.

If the **Components** palette is not visible, see *Palettes (page 22)* for instructions on viewing palettes.

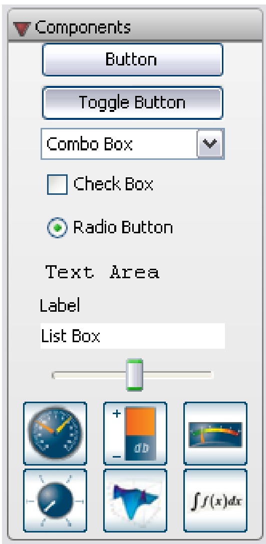


Figure 10.1: Components Palette

Editing Component Properties: General Process

To edit properties of components embedded in the document:

1. Right-click (**Control**-click, for Macintosh) the component to display the context menu.

2. If available, select **Component Properties...**; otherwise, select **Components → Component Properties...**. The related dialog opens.

3. Enter values and contents in the fields as necessary.

4. For actions, such as **Action When Value Changes** in the **Slider** component dialog, click **Edit**. A blank dialog opens allowing you to enter Maple code that is executed when the event occurs. For details, refer to the **?DocumentTools** help page.

Removing Graphical Interface Components

You can remove an embedded component by:

- Using the **Delete** key
- Using the **Backspace** key
- Placing the cursor at the component and selecting from the document menu, **Edit→Delete Element**

Integrating Components into a Document

Use embedded components to display information from calculations, obtain input from a reader, or perform calculations at the click of a button, all without your readers having an understanding of Maple commands. They can be entered in any part of a Maple document, including a document block or table. For details on each component, see its help page.

This simple example inserts a slider with a label that indicates the current value of the slider.

1. Place the cursor in the location where the embedded component is to be inserted.

2. In the **Components** palette, click the **Slider** item. A slider is inserted into the document.

3. In the **Components** palette, click the **Label** item. A label is inserted next to the slider.

4. Right-click (**Control**-click, for Macintosh) the label component. Select **Component Properties**. The **Label Properties** dialog opens. See Figure 10.2.

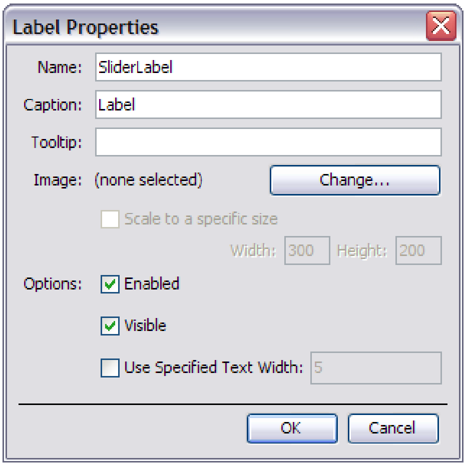


Figure 10.2: Label Properties Dialog

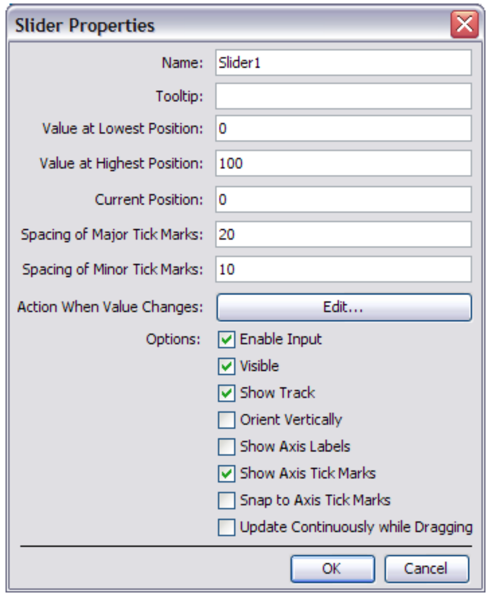


Figure 10.3: Slider Properties Dialog

5. Name the component **SliderLabel** and click **Ok**.

6. Right-click (**Control**-click, Macintosh) the slider component. Select **Component Properties**. The **Slider Properties** dialog opens. See Figure 10.3.

7. Name the component **Slider1.**

8. Enter the value at the lowest position as **0** and the highest as **100.**

9. Enter major tick marks at **20** and minor tick marks at **10**.

10. To define an action, click the **Edit** button for the **Action When Value Changes**. The dialog that opens allows you to program the action of displaying the slider value in the label component. The dialog includes instructions on how to program embedded components. The **use...in/end use;** statement allows you to specify routines using the short form of accessing a package command without invoking the package. For details on this command, refer to the **?use** help page.

11. Before the **end use;** statement at the bottom of the dialog, enter the following command.

Do(%SliderLabel(caption)=%Slider1(value));

12. Click **OK**.

13. Make sure that the **Update Continuously while Dragging** check box is selected.

The value from the slider as you move the arrow indicator populates the **Label** caption field.

For details on this command, refer to the **?DocumentTools[Do]** help page.

Example 2 - Creating Embedded Components

In chapter 7 (see *Embedded Components (page 329)*), you created a document that included embedded components, imported from a task template. Here, we re-create that configuration of components. This example takes two parameters, a and b, as inputs, then plots the function $y = bx + a$ and calculates $\frac{a}{b}$.

1. Create the components.

The table layout is best done after the components are finished, in case the configuration of the components changes as you are working.

Create two **DialComponent**s to set the parameters, a and b, one **GaugeComponent** to display the result, $\frac{a}{b}$, one **PlotComponent** to display the plot, and one **MathContainer-Component** to display the function. Note that you do not need to use the dial and gauge components here, there are others, such as the slider, that could also be used.

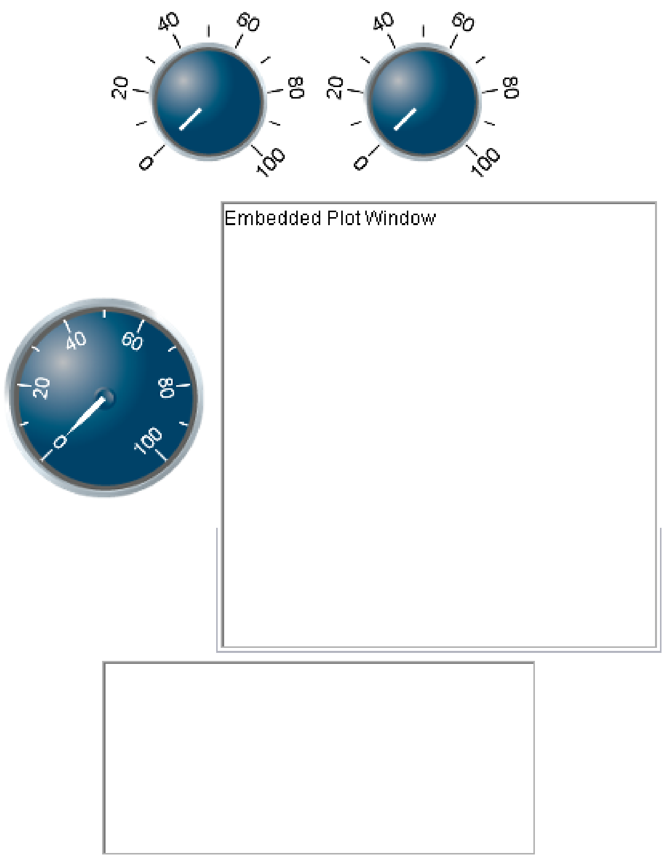


Figure 10.4: The Inserted Components

2. Edit the display of the components.

Open the **Component Properties** dialog for the first **DialComponent**, and notice that it already has a name. This name is used to reference the component from other components, and is unique. Change the display of each of the components as follows:

- **Dial0**: no changes.
- **Dial1**: change the **Value at Highest Position** to 10, the **Spacing of Major Tick Marks** to 1, and the **Spacing of Minor Tick Marks** to 1.
- **RotaryGauge0**: change the **Value at Highest Position** to 40, the **Spacing of Major Tick Marks** to 5, and the **Spacing of Minor Tick Marks** to 1.
- **Plot0**: no changes.

- **MathContainer0**: change the **Width in Pixels** to 200, and the **Height in Pixels** to 45.

Note the names of all of the components, and close each dialog before moving on.

3. Create actions for the components.

Components can perform actions when their values are changed, so the code to execute needs to be in the dials. That way, whenever one of them is changed, the other components are updated to reflect that change.

The following Maple commands retrieve the values of the parameters and display them in the other three components:

```
> parameter1:=Do(%Dial0):
```

```
> parameter2:=Do(%Dial1):
```

```
> Do(%RotaryGauge0=parameter1/parameter2);
```

```
> Do(%Plot0=plot((parameter2*x+parameter1), x=-50..50, y=-
50..50));
```

```
> Do(%MathContainer0=(y=parameter2*x+parameter1));
```

4. Test the actions.

To test these commands, first load the **DocumentTools** package with the following command.

> *with*(*DocumentTools*) :

Execute the commands in the document, and verify that the components you inserted are updated: the gauge should change to the computed value, a plot should appear in the plot component, and the function should display in the math container.

5. Troubleshooting.

The first **Do** command gives an error, because the second parameter is 0. One way to avoid this problem is to change the range of the second dial. In the **Component Properties** dialog for the second **DialComponent**, change the **Value at Lowest Position** from 0 to 1. Alternatively, you could change the code to compensate, with an **if** statement.

6. Copy the actions to the components.

Once the commands work as expected, you can copy them into the components.

- Open the **Component Properties** dialog for the first **DialComponent** and click the **Edit** button for **Action When Value Changes**. Copy and paste the commands into the space between the **use** statements.

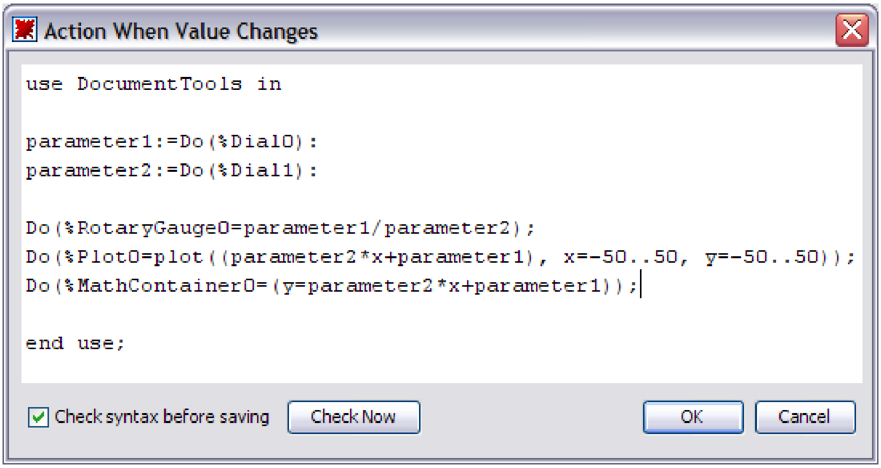
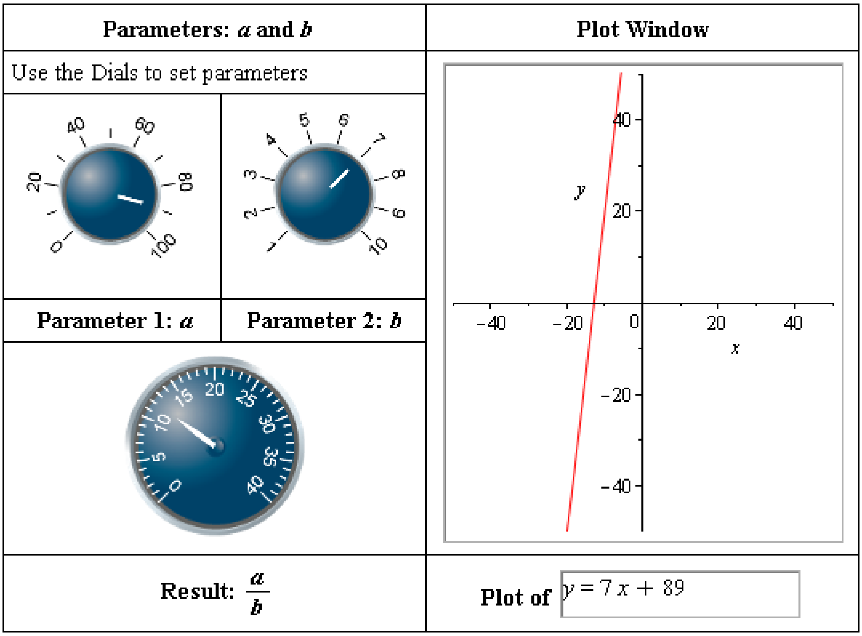


Figure 10.5: DialComponent Action dialog

- Do the same for the second **DialComponent**.

7. Create the layout for the components.

Create a table, and then cut and paste the components into it, along with explanatory text. **Important:** you must cut, not copy, the components, or their names will be changed to avoid duplication. For information on creating and modifying tables, refer to *Tables (page 306)*.



10.4 Using Maplets

A Maplet is a pop-up graphical user interface that provides interactive access to the Maple engine through buttons, text regions, slider bars, and other visual interfaces. You can create your own Maplets, and you can take advantage of the built-in Maplets that cover numerous academic and specialized topics. Built-in Maplets include some assistants and tutors, such as the ODE Analyzer. For more information on this assistant, see *Ordinary Differential Equations (ODEs) (page 121)*.

Maplet applications are launched by executing Maplet code. Maplet code can be saved in a Maplet (**.maplet**) file or Maple document (**.mw**).

Maplet File

To launch a Maplet application saved as a Maplet file:

- In Windows, double-click the file from a Windows file browser.
- In UNIX and on Macintosh, use the command-line interface. At the command-line, enter **maple -q** <*maplet_filename*>.

To view and edit the Maplet code contained within the *.maplet* file:

1. Start Maple.
2. From the **File** menu, select **Open**. Maple displays the **Open** dialog.
3. In the **Files of Type** drop-down list, select **.maplet**.
4. Navigate to the location of the **.maplet** file and select the file.
5. Click **Open**.

Maple Document

To launch a Maplet application for which the Maple code is contained in a Maple document, you need to execute the Maplet code. To display the Maplet application, you must use the **Maplets[Display]** command. **Note:** The Maplet code may be quite large if the Maplet application is complex. In this case, execute the document to ensure user-defined procedures that are referenced in the Maplet application are also defined.

Typical procedure:

1. If present, evaluate user-defined procedures.

```
Myproc:=proc..
```

2. Load the **Maplets[Elements]** package.

```
with( Maplets[Elements] );
```

3. Evaluate the Maplet definition.

```
Maplet_name:=Maplet( Maplet_definition );
```

4. Display the Maplet application.

```
Maplets[Display]( Maplet_name );
```

Important: When a Maplet application is running, you cannot interact with the Maple document.

10.5 Authoring Maplets

To author Maplets, you can use the **Maplet Builder** (GUI-based) or the **Maplets** package (syntax-based). The **Maplet Builder** allows you to drag and drop buttons, sliders, text regions, and other elements to define the Maplet application and set the element properties to perform an action upon selection or update of the element. The **Maplet Builder** is designed to create simple Maplets. The **Maplets** package offers more capabilities, control, and options when designing complicated Maplet applications.

Designing a Maplet application is similar to constructing a house. When building a house, you first construct the skeletal structure (that is, foundation, floors, and walls) and then proceed to add the windows and doors. Constructing a Maplet is no different. First define the rows and columns of the Maplet application and then proceed to add the body elements (such as buttons, text fields, and plot regions).

Simple Maplet

A Maplet application can be defined using the commands in the **Maplets[Elements]** package and then launched using the **Maplets[Display]** command. The following commands define and run a very simple Maplet application that contains the text string "Hello World".

```
> with(Maplets[Elements]):
```

```
> MySimpleMaplet:= Maplet([["Hello World"]]):
```

```
> Maplets[Display](MySimpleMaplet):
```

Figure 10.6: A Simple Maplet

Maplet Builder

To start the **Maplet Builder**:

- From the **Tools** menu, select **Assistants** → **Maplet Builder**.

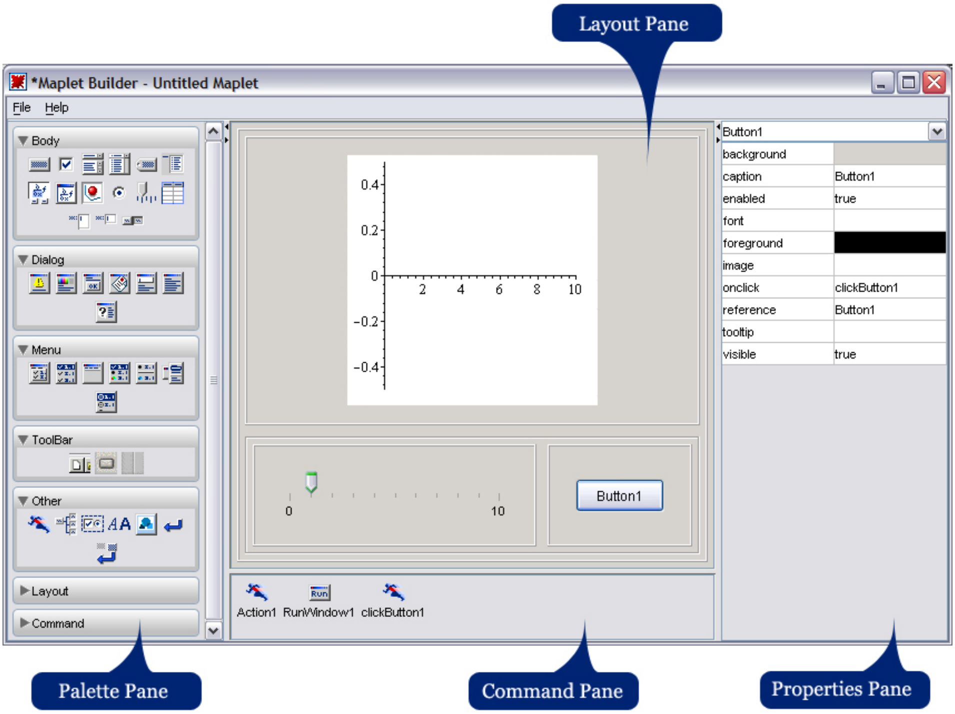


Figure 10.7: Maplet Builder Interface

The **Maplet Builder** is divided into four different panes.

- The **Palette** pane displays palettes, which contain Maplet elements, organized by category. For a description of the elements, see the **?MapletBuilder/Palette** help page. The **Body** palette contains the most popular elements.
- The **Layout** pane displays the visual elements of the Maplet.
- The **Command** pane displays the commands and corresponding actions defined in the Maplet.
- The **Properties** pane displays the properties of an instance of a defined element in the Maplet.

Example 3 - Design a Maplet Using the Maplet Builder

In this example, shown in Figure 10.8, the Maplet user enters a function and plots the result.

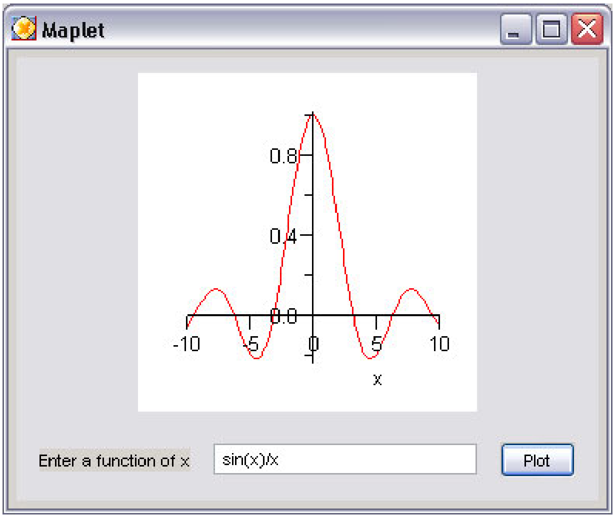
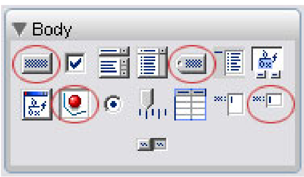


Figure 10.8: Image of the Maplet



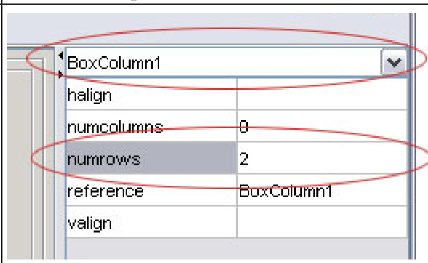
Button element

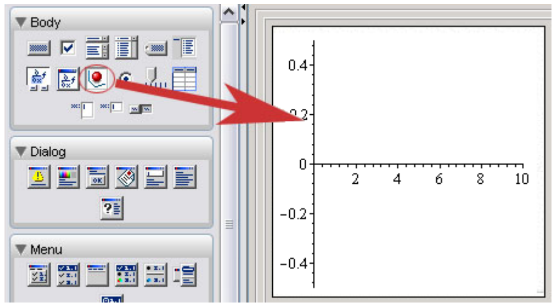
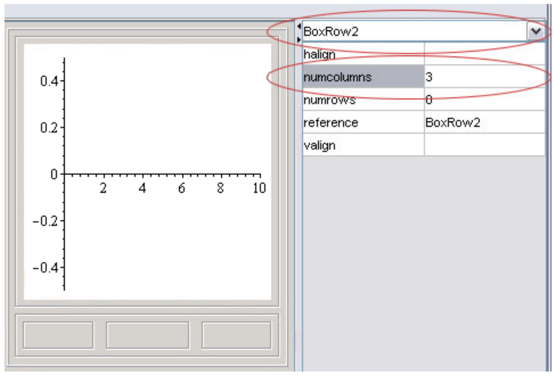
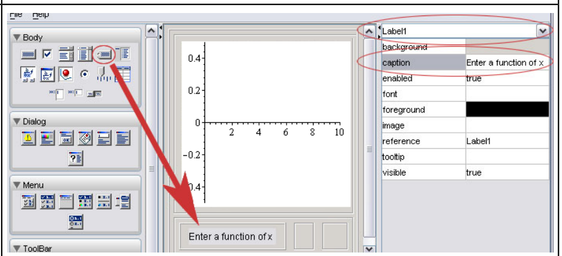
Label element

Plot element

TextField element

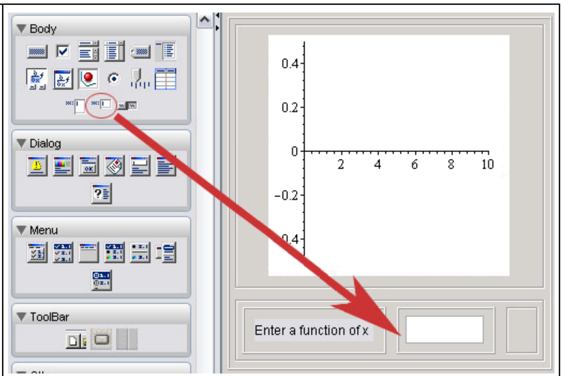
Figure 10.9: Body Elements Used to Define This Maplet

Action	Result in MapletBuilder
Define the number of rows in the Maplet: 1. In the **Properties** pane: a. In the drop-down list, select **Box-Column1**. b. Change the **numrows** field to **2**.	

Add a plot region to row 1: 2. From the **Body** palette, drag the **Plotter** element to the first row in the **Layout** pane.	
Add columns to row 2: 3. In the **Properties** pane: a. In the drop-down list, select **BoxRow2**. b. Change the **numcolumns** field to **3**.	
Add a label to row 2: 4. From the **Body** palette, drag the **Label** element to the left column in the **Layout** pane. 5. In the **Properties** pane: a. In the drop-down list, select **Label1**. b. Change the caption field to **Enter a function of x**.	

<table>
<tr>
<td>Add a text region to row 2:

6. From the Body palette, drag the Text-Field element to the middle column. The TextField element allows the Maplet user to enter input that can be retrieved in an action.

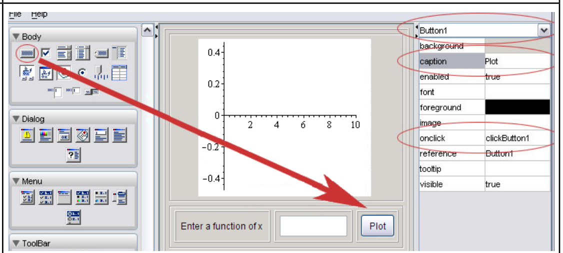
7. If necessary, resize the Maplet Builder to display the entire Layout pane.</td>
<td>
</td>
</tr>
<tr>
<td>Add a button to row 2:

8. From the Body palette, drag the Button element to the right column in the Layout pane.

9. In the Properties pane:

a. In the drop-down list, select Button1.

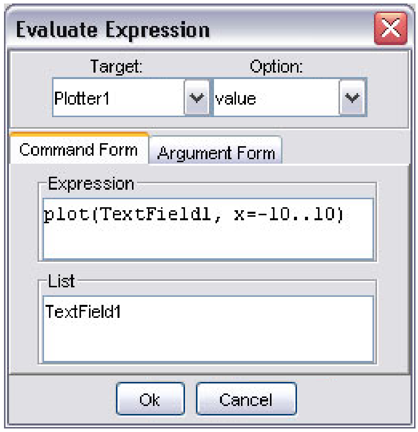
b. Change the caption field to Plot.

c. In the onclick property drop-down list, select <Evaluate>.</td>
<td>
</td>
</tr>
</table>

<table>
<tr>
<td>10. In the Evaluate Expression dialog that displays, the Target drop-down list contains the defined elements to which you can send information, in this case, Plotter1 and TextField1. The List group box, located below the Expression group box, displays the defined elements to which you can retrieve information, in this case, TextField1.

a. In the Target drop-down list, select Plotter1.

b. In the Command Form tab, enter plot(TextField1, x=-10..10) in the Expression group box. (Note: Do not include a semicolon (;) at the end of the plot command.) You can also double-click TextField1 in the List group box to insert this element in the command syntax.

c. Click Ok.</td>
<td>
</td>
</tr>
<tr>
<td colspan="2">Run the Maplet:

11. From the File menu, select Run. You are prompted to save the Maplet. Maplets created with the Maplet Builder are saved as .maplet files.

12. Click Yes and navigate to a location to save this Maplet.</td>
</tr>
</table>

For further information on the **Maplet Builder**, see the **?MapletBuilder** help page. For more examples of designing Maplets using the **Maplet Builder**, see **?MapletBuilder/examples**.

Maplets Package

When designing a complicated Maplet, the **Maplets** package offers greater control. The **Maplets[Elements]** subpackage contains the elements available when designing a Maplet application. After you define the Maplet, use the **Maplets[Display]** command to launch the Maplet.

For more information on the **Maplets** package, refer to the **?MapletsPackage** help page. For more examples of designing Maplets using the **Maplets** package, see the **?Maplets/Roadmap** help page.

Example 4 - Design a Maplet Using the Maplets Package

To introduce the structure of designing Maplets using the **Maplets** package, this example illustrates the equivalent syntax for the *Design a Maplet Using the Maplet Builder (page 404)*.

Load the **Maplets[Elements]** package.

```
> with(Maplets[Elements]):
```

Define the Maplet application. To suppress the display of the data structure associated with the Maplet application, end the definition with a colon.

```
> PlottingMaplet:=Maplet(
    BoxLayout(
     BoxColumn(
       # First Box Row
           BoxRow(
         # Define a Plot region
             Plotter('reference' = Plotter1)
       # End of first Box Row
                  ),
       # Second Box Row
           BoxRow(
         # Define a Label
             Label("Enter a function of x "),
          # Define a Text Field
             TextField('reference' = TextField1),
          # Define a Button
             Button(caption="Plot", Evaluate(value = 'plot(TextField1,
                    x = -10..10)', 'target' = Plotter1))
       # End of second Box Row
             )
      # End of BoxColumn
       )
    # End of BoxLayout
    )
  # End of Maplet
  ):
```

Launch the Maplet.

```
> Maplets[Display](PlottingMaplet);
```

For further examples using both the **MapletBuilder** and **Maplets** package commands, see the Maplets example worksheets. For a listing, refer to the **?examples/index** help page.

Saving

When saving a Maplet, you can save the document as an **.mw** file or you can export the document as a **.maplet** file.

Maple Document

To save the Maplet code as an **.mw** file:

1. From the **File** menu, select **Save**.
2. Navigate to the save location.
3. Enter a filename.
4. Click **Save**.

If the document contains only Maplet code, it is recommended that you export the document as a **.maplet** file.

Maplet File

To export the Maplet code as a **.maplet** file:

1. From the **File** menu, select **Export As**.
2. In the **Files of Type** drop-down list, select **Maplet**.
3. Navigate to the export location.
4. Enter the filename.
5. Click **Save**.

11 Input, Output, and Interacting with Other Products

11.1 In This Chapter

Section	Topics
Writing to Files (page 411) - Saving to Maple file formats	• Saving Data to a File • Saving Expressions to a File
Reading from Files (page 414) -Opening Maple files	• Reading Data from a File • Reading Expressions from a File
Exporting to Other Formats (page 416) - Exporting documents in file formats supported by other software	• Exporting Documents • MapleNet • Maple T.A.
Connectivity (page 420) - Using Maple with other programming languages and software	• Translating Maple Code to Other Programming Languages • Accessing External Products from Maple • Accessing Maple from External Products • Sharing and Storing Maple Worksheet Content with the MapleCloud™

11.2 Writing to Files

Maple supports file formats in addition to the standard **.mw** file format.

After using Maple to perform a computation, you can save the results to a file for later processing with Maple or another program.

Saving Data to a File

If the result of a Maple calculation is a long list or a large array of numbers, you can convert it to Matrix form and write the numbers to a file using the **ExportMatrix** command. This command writes columns of numerical data to a file, allowing you to import the numbers into another program. To convert a list or a list of lists to a Matrix, use the **Matrix** constructor. For more information, refer to the **?Matrix** help page.

$$> L := \begin{bmatrix} -81 & -98 & -76 & -4 & 29 \\ -38 & -77 & -72 & 27 & 44 \\ -18 & 57 & -2 & 8 & 92 \\ 87 & 27 & -32 & 69 & -31 \\ 33 & -93 & -74 & 99 & 67 \end{bmatrix}:$$

> *ExportMatrix*("matrixdata.txt", *L*) :

If the data is a Vector or any object that can be converted to type Vector, use the **ExportVector** command. To convert lists to Vectors, use the **Vector** constructor. For more information, refer to the **?Vector** help page.

> $R := [3, 3.1415, -65, 0]$

$$R := [3, 3.1415, -65, 0] \quad (11.1)$$

> $V := \mathit{Vector}(R)$

$$V := \begin{bmatrix} 3 \\ 3.1415 \\ -65 \\ 0 \end{bmatrix} \quad (11.2)$$

> *ExportVector*("vectordata.txt", *V*) :

You can extend these routines to write more complicated data, such as complex numbers or symbolic expressions. For more information, refer to the **?ExportMatrix** and **?ExportVector** help pages.

For more information on matrices and vectors, see *Linear Algebra (page 159)*.

Saving Expressions to a File

If you construct a complicated expression or procedure, you can save them for future use in Maple. If you save the expression or procedure in the Maple internal format, Maple can retrieve it more efficiently than from a document. Use the **save** command to write the expression to a **.m** file. For more information on Maple internal file formats, refer to the **?file** help page.

$$> qbinomial := (n, k) \rightarrow \frac{\prod_{i=n-k+1}^{n} (1 - q^i)}{\prod_{i=1}^{k} (1 - q^i)}:$$

In this example, small expressions are used. In practice, Maple supports expressions with thousands of terms.

> $expr := qbinomial(10, 4)$

$$expr := \frac{(1 - q^7)(1 - q^8)(1 - q^9)(1 - q^{10})}{(1 - q)(1 - q^2)(1 - q^3)(1 - q^4)} \tag{11.3}$$

> $nexpr := normal(expr)$

$$nexpr := (q^6 + q^5 + q^4 + q^3 + q^2 + q + 1)(q^4 + 1)(q^6 + q^3 + 1)(q^8 + q^6 + q^4 + q^2 + 1) \tag{11.4}$$

You can save these expressions to the file **qbinom.m**.

> **save** *qbinomial*, *expr*, *nexpr*, "qbinom.m"

Clear the memory using the **restart** command and retrieve the expressions using the **read** command.

> *restart*

> **read** "qbinom.m"

> *expr*

$$\frac{(1 - q^7)(1 - q^8)(1 - q^9)(1 - q^{10})}{(1 - q)(1 - q^2)(1 - q^3)(1 - q^4)} \tag{11.5}$$

For more information on writing to files, refer to the **?save** help page.

11.3 Reading from Files

The most common reason for reading files is to load data, for example, data generated in an experiment. You can store data in a text file, and then read it into Maple.

Reading Data from a File

Import Data Assistant

If you generate data outside Maple, you can read it into Maple for further manipulation. This data can be an image, a sound file, or columns of numbers in a text file. You can easily import this external data into Maple using the **Import Data Assistant**, where the supported file formats include files of type Excel®, MATLAB, Image, Audio, Matrix Market, and Delimited.

To launch the Import Data Assistant:

- From the **Tools** menu, select **Assistants**, and then **Import Data**.
- A dialog window appears where you can navigate to your data file. Select the file that you want to import data from, and then select the file type before clicking **Next**.
- From the main window, you can preview the selected file and choose from the applicable options based on the format of the file read in before importing the data into Maple. See **Figure 11.1** for an example.

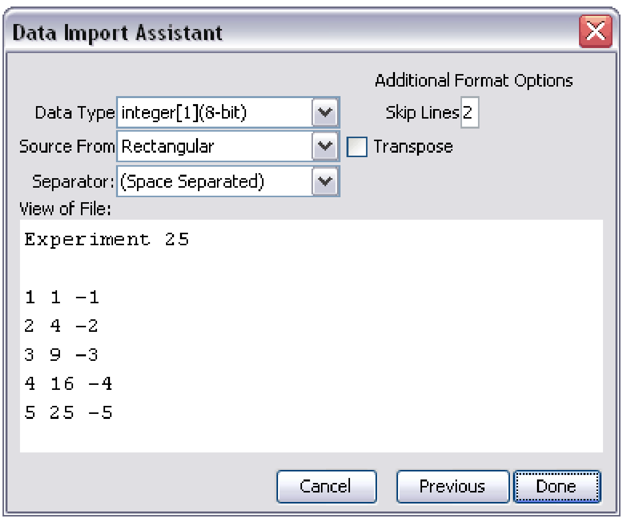


Figure 11.1: Import Data Assistant

ImportMatrix Command

The **Import Data Assistant** provides a graphical interface to the **ImportMatrix** command. For more information, including options not available in the assistant, refer to the **?ImportMatrix** help page.

Reading Expressions from a File

You can write Maple programs in a text file using a text editor, and then import the file into Maple. You can paste the commands from the text file into your document or you can use the **read** command.

When you read a file with the **read** command, Maple treats each line in the file as a command. Maple executes the commands and displays the results in your document but it does *not,* by default, insert the commands from the file in your document.

For example, the file **ks.txt** contains the following Maple commands.

```
S:= n -> sum( binomial( n, beta ) * ( ( 2*beta )! / 2^beta -
beta!*beta ), beta=1..n );

S(19);
```

Note that the file should not contain prompts (>) at the start of lines.

When you read the file, Maple displays the results but not the commands.

$$S := n \rightarrow \sum_{\beta=1}^{n} \operatorname{binomial}(n, \beta) \left(\frac{(2\,\beta)!}{2^{\beta}} - \beta!\,\beta \right)$$

$$10249373616666445980711143287693179829 74 \qquad (11.6)$$

> *filename* := *cat*(*kernelopts*(*datadir*), *kernelopts*(*dirsep*), "ks.txt") :

> **read** *filename*

```
Error, unable to read `C:\Program Files\Maple 14\data\ks.txt`
```

If you set the **interface echo** option to 2, Maple inserts the commands from the file into your document.

> $interface(echo = 2)$:
read *filename*

```
> S:= n -> sum( binomial( n, beta ) * ( ( 2*beta )! / 2^beta -
beta!*beta ), beta=1..n );
```

$$S := n \rightarrow \sum_{\beta=1}^{n} \operatorname{binomial}(n, \beta) \left(\frac{(2\beta)!}{2^{\beta}} - \beta!\,\beta \right)$$

```
> S(19);
```

$$1024937361666644598071114328769317982974 \tag{11.7}$$

For more information, refer to the **?read** and **?interface** help pages.

11.4 Exporting to Other Formats

Exporting Documents

You can save your documents by selecting **Save** or **Save As** from the **File** menu. By selecting **Export As** from the **File** menu, you can also export a document in the following formats: HTML, LaTeX, Maple input, Maplet application, Maple text, plain text, PDF, and Rich Text Format. This allows you to access your work outside Maple.

HTML

The **.html** file that Maple generates can be loaded into any HTML browser. Exported mathematical content can be displayed in one of the following formats: GIF, MathML 2.0 Presentation, MathML 2.0 Content, or Maple Viewer, and is saved in a separate folder. MathML is the Internet standard, sanctioned by the World Wide Web Consortium (W3C), for the communication of structured mathematical formulae between applications. For more information about MathML, refer to the **?MathML** help page.

Maple documents that are exported to HTML translate into multiple documents when using frames. If the frames feature is not selected, Maple creates only one page that contains the document contents.

LaTeX

The **.tex** file generated by Maple is ready for processing by LaTeX. All distributions of Maple include the necessary style files. By default, the LaTeX style files are set for printing the **.tex** file using the **dvips** printer driver. You can change this behavior by specifying an option to the **\usepackage** LaTeX command in the preamble of your **.tex** file. For more information, refer to the **?exporttoLaTeX** help page.

Maple Input

You can export a Maple document as Maple input so that it can be loaded using the Maple Command-line version.

Important: When exporting a document as Maple input for use in Command-line Maple, your document must contain explicit semicolons in 1-D Math input. If not, the exported **.mpl** file does not contain semicolons, and Command-line Maple generates errors.

Maplet Application

The **Export as Maplet** facility saves a Maple document as a **.maplet** file, so that you can run it using the command-line interface or the **MapletViewer**. The MapletViewer is an executable program that can launch saved Maplet applications. It displays and runs Maplet applications independently of the Maple Worksheet interface.

Important: When exporting a document as a Maplet Application for use in Command-line Maple or the MapletViewer, your document must contain explicit semicolons. If not, the exported **.maplet** file does not contain semicolons, and Command-line Maple and the MapletViewer generates errors.

Maple Text

Maple text is marked text that retains the distinction between text, Maple input, and Maple output. Thus, you can export a document as Maple text, send the text file by email, and the recipient can import the Maple text into a Maple session and regenerate the computations in the original document.

PDF

Export a Maple document to a Portable Document Format (PDF) file so that you can open the file in a reader such as Adobe® Acrobat®. The PDF document is formatted as it would appear when the Maple worksheet is printed using the active printer settings.

Note: Images, plots, and embedded components may be resized in the PDF file.

Plain Text

Export a Maple document as plain text so that you can open the text file in a word processor.

Rich Text Format (RTF)

Export a Maple document to a rich text format file so that you can open and edit the file in a word processor.

Note: The generated **.rtf** format is compatible with Microsoft Word and Microsoft WordPad only.

Summary of Translation

Table 11.1: Summary of Content Translation When Exporting to Different Formats

Content	HTML	LaTeX	Maple Input	Maplet Application	Maple Text	Plain Text	Rich Text Format	PDF Format
Text	Maintained	Maintained	Preceded by #	Preceded by #	Preceded by #	Maintained	Maintained	Maintained
1-D Math	Maintained	Maintained	Maintained	Maintained	Preceded by >	Preceded by >	Static image	Static image
2-D Math	GIF or MathML	1-D Math or LaTeX 2e	1-D Math (if possible)	1-D Math (if possible)	1-D Math or character-based typesetting	1-D Math or character-based typesetting	Static image	Either text or shapes depending on option selected
Plot	GIF	Postscript file	Not exported	Not exported	Not exported	Not exported	Static image	Static image
Animation	Animated GIF	Not exported	Not exported	Not exported	Not exported	Not exported	Not exported	Static image
Hidden content	Not exported	Not exported	Not exported	Not exported	Not exported	Not exported	Not exported	Not exported
Manually inserted page break	Not supported	Not supported	Not supported	Not supported	Not supported	Not supported	RTF page break object	Maintained
Hyperlink	Links to help pages become plain text. Links to documents are renamed and converted to HTML links	Plain text	Plain text	Plain text	Plain text	Plain text	Plain text	Plain text
Embedded image or sketch output	GIF	Not exported	Not exported	Not exported	Not exported	Not exported	Static image	Static image
Spreadsheet	HTML table	LaTeX tables	Not exported	Not exported	Not exported	Not exported	RTF table	Static image

Content	HTML	LaTeX	Maple Input	Maplet Application	Maple Text	Plain Text	Rich Text Format	PDF Format
Document style	Approximated by HTML style attributes	LaTeX environments and sections, LaTeX 2e macro calls	Not exported	Not exported	Not exported	Not exported	RTF style	Maintained

MapleNet

Overview of MapleNet

Using MapleNet, you can deploy Maple content on the web. Powered by the Maple computation engine, MapleNet allows you to embed dynamic formulas, models, and diagrams as **live** content in web pages. The MapleNet software is not included with the Maple software. For more information on MapleNet, visit **http://www.maplesoft.com/maplenet**.

MapleNet Documents and Maplets

After you upload your Maple document to the MapleNet server, it can be accessed by anyone in the world using a Web browser. Even if viewers do not have a copy of Maple installed, they can view documents and Maplets, manipulate 3-D plots, and execute code at the click of a button.

Custom Java Applets and JavaServer Pages™ Technology

MapleNet provides a programming interface to the Maple math engine so commands can be executed from a Java applet or using JavaServer Pages technology. Embed MapleNet into your web application, and let Maple handle the math and visualization.

Maple T.A.

Overview of Maple T.A.

Maple T.A. is a web-based automated testing system, based on the Maple engine. Instructors can use pre-written questions or create custom question banks and then choose from these questions to form quizzes and assignments. Maple T.A. automatically grades responses as students complete assignments and tests.

For more information, visit **http://www.maplesoft.com/mapleta**.

Exporting Assignments to Maple T.A.

You can use Maple to create graded questions for use in Maple T.A. For information on creating and testing questions, see *Creating Graded Assignments (page 334)*. Using the Maple T.A. export feature, you can create and test Maple T.A. content.

To export the document:

1. From the **File** menu, select **Export As**.
2. In the **Export As** dialog, specify a filename and the **Maple T.A. (.zip)** file type. The .zip file containing your questions and assignment can be uploaded to Maple T.A. as a course module.

Any document content outside Maple T.A. sections (indicated by green section markers) is ignored by the export process.

For more details, refer to the **?exporttoMapleTA** help page.

11.5 Connectivity

Translating Maple Code To Other Programming Languages

Code Generation

The **CodeGeneration** package is a collection of commands and subpackages that enable the translation of Maple code to other programming languages. Languages currently supported include C, Fortran 77, Java, MATLAB, and Visual Basic.

For details on Code Generation, refer to the **?CodeGeneration** help page.

Accessing External Products from Maple

External Calling

External calling allows you to use compiled C, Fortran 77, or Java code in Maple. Functions written in these languages can be linked and used as if they were Maple procedures. With external calling you can use pre-written optimized algorithms without the need to translate them into Maple commands. Access to the NAG library routines and other numerical algorithms is built into Maple using the external calling mechanism.

External calling can also be applied to functions other than numerical algorithms. Routines exist that accomplish a variety of non-mathematical tasks. You can use these routines in Maple to extend its functionality. For example, you can link to controlled hardware via a serial port or interface with another program. The **Database** package uses external calling to allow you to query, create, and update databases in Maple. For more information, refer to the **?Database** help page.

For more information on using external calling, refer to the **?ExternalCalling** help page.

Mathematica Translator

The **MmaTranslator** package provides translation tools for converting Mathematica® expressions, command operations, and notebooks to Maple. The package can translate Mathematica input to Maple input and Mathematica notebooks to Maple documents. The **Mma** subpackage contains commands that provide translation for Mathematica commands when no equivalent Maple command exists. In most cases, the command achieves the translation through minor manipulations of the input and output of similar Maple commands.

Note: The **MmaTranslator** package does not convert Mathematica programs.

There is a Maplet interface to the **MmaTranslator** package. For more information, refer to the **?MmaToMaple** help page.

Matlab Package

The **Matlab** package enables you to translate MATLAB code to Maple, as well as call selected MATLAB functions from a Maple session, provided you have MATLAB installed on your system.

For more information, refer to the **?Matlab** help page.

Accessing Maple from External Products

Microsoft Excel Add-In

Maple is available as an add-in to Microsoft Excel. This add-in is supported for Excel 2007 and Excel 2003 for Windows, and provides the following features.

- Access to Maple commands from Excel
- Ability to copy and paste between Maple and Excel
- Access to a subset of the Maple help pages
- Maple Function Wizard to step you through the creation of a Maple function call

To enable the Maple Excel Add-in in Excel 2007:

1. Click the Microsoft Office Button and select **Excel Options**.

2. Click **Add-ins**.

3. In the **Manage** box select **Excel Add-ins**, and then **Go**.

4. Navigate to the Excel subdirectory of your Maple installation, select **WMIMPLEX.xla**, and click **OK**.

5. Select the **Maple Excel Add-in** check box.

6. Click **OK**.

To enable the Maple Excel Add-in for versions of Excel prior to Excel 2007:

1. From the **Tools** menu in Excel, choose **Add-Ins**.
2. If the Maple Excel Add-in is not listed:

- Click **Browse** and navigate to the directory in which Maple is installed.
- In the **Excel** directory, select the **WMIMPLEX.xla** file.
- Click **OK**.

3. Select the **Maple Excel Add-in** check box.
4. Click **OK**.

More information is available in the **Using Maple in Excel** help file within Excel.

To view this help file:

1. Enable the add-in.
2. From the **View** menu, select **Toolbars**, and then **Maple**.
3. On the Maple toolbar, click the Maple help icon .

OpenMaple

OpenMaple is a suite of functions that allows you to access Maple algorithms and data structures in your compiled C, Java, or Visual Basic programs. (This is the reverse of external calling, which allows access to compiled C, Fortran 77, and Java code from Maple.)

To run your application, Maple must be installed. You can distribute your application to any licensed Maple user. For additional terms and conditions on the use of OpenMaple, refer to the **extern/OpenMapleLicensing.txt** file in your Maple installation.

For more details on using OpenMaple functions, refer to the **?OpenMaple** help page.

MapleSim

MapleSim™ is a complete environment for modeling and simulating multi-domain engineering systems. During a simulation, MapleSim uses the symbolic Maple computation engine to generate the mathematical models that represent the system behavior.

Because both products are tightly integrated, you can use Maple commands and technical document features to edit, manipulate, and analyze a MapleSim model. For example, you can use Maple commands and tools to manipulate your model equations, develop custom components based on a mathematical model, and visualize simulation results.

MapleSim software is not included with the Maple software. For more information on MapleSim, visit **http://www.maplesoft.com/maplesim**.

Sharing and Storing Maple Worksheet Content

The MapleCloud

You can use the MapleCloud to share worksheet content with other users, view content shared by other users, and store entire standard Maple worksheets or selected content from standard Maple worksheets. Through the MapleCloud palette, you can upload standard Maple worksheet content and allow other users to download a copy of that content. You can also upload and store content in a user-specific area that only you can access.

A list of shared worksheets that you have permissions to view are displayed in the MapleCloud palette. To share content with specific users, you can either create a user group or select an existing group and allow only those group members to access your content. For more information about groups, see ?worksheet,cloud,groups.

Users need an internet connection to use the MapleCloud. To share worksheet content, create, manage and join user groups; and view group-specific content, you must log in to the MapleCloud using a Maplesoft.com, Gmail™, or Google Mail™ account name and password.

A Maplesoft.com membership account gives you access to thousands of free Maple resources and MaplePrimes, which is an active web community for sharing techniques and experiences with Maple and related products. To sign up for a free Maplesoft.com membership account, visit http://www.maplesoft.com/members/sign_up_form.aspx. The MapleCloud is integrated with several of these online features, so it is strongly recommended that you use a Maplesoft.com membership account.

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