Mth 111 Solving Linear Equations ***Franz Helfenstein*** **Name**

In a **linear equation**

- Constant terms are possible.
- All variable terms reduce to (**coefficient**) × (**variable**).
- All **variables** are to the **first power**.
- No **mixed products** of variables are allowed.
- No **division by variables** are allowed.

Examples of Terms

3	2.54	π	$\sqrt{2}$	10^{-4}	a	b	*are typical <u>constant</u> terms*
$3x$	$2.7x$	ax	$\sqrt{2}x$	$(a + b)x$	$(5+ a)x$	$a^2 x$	*are typical <u>linear</u> terms*
$5x^2$	$3\frac{1}{x}$	$6\sqrt{x}$	xy	$\frac{5x}{6y}$	$3x(x + 1)$	$\frac{x}{x+1}$	*are typical <u>non-linear</u> terms*

<u>General Process</u>

Linear equations use a routine step-by-step solution process. Any adjustment to the equation is legitimate as long as it occurs equally on <u>both sides</u> of the equation. As you might expect, some adjustments are more beneficial than others. That is, there is a preferred sequence of manipulations and <u>if we follow the preferred steps</u>, the equation simplifies nicely to x = "solution". This x-value (solution) must make our original equation true. The solution should be substituted into the original equation to double check the solution.

An Efficient Scheme for Solving Linear Equations

(1) Change Subtractions to Additions. e.g. $10 - 2x = 10 + (-2x)$

(2) Remove all parentheses. *(apply Distributive Property)*

(3) Mark each Term with a bracket. e.g. [*term*] + [*term*]

(4) Remove all fractions. *(multiply all terms by LCD then cancel and simplify)*

(5) Shift variable term(s) to one side by adding or subtracting terms.

(6) Shift everything else to the other side by adding or subtracting terms.

(7) If necessary write variable term(s) as (coefficient) · (variable). *(Distributive Law)*

i.e. $\sqrt{2}\,x + 4x = (\sqrt{2} + 4)x$ or $\pi x + ax = (\pi + a)x$

(8) Divide both sides by the variable's coefficient.

(9) Check the answer by substituting solution into the <u>original</u> equation.

Example 1: Solve $2x - 4\,\dfrac{(3x-5)}{3} = \dfrac{4x}{5} + \dfrac{41}{3}$

(1)	$2x + (-4)\,\dfrac{(3x+(-5))}{3} = \dfrac{4x}{5} + \dfrac{41}{3}$	*change subtraction to addition*
(2)	$2x + \dfrac{(-12x)+20}{3} = \dfrac{4x}{5} + \dfrac{41}{3}$	*distribute*
(3)	$\left[2x\right] + \left[\dfrac{(-12x)+20}{3}\right] = \left[\dfrac{4x}{5}\right] + \left[\dfrac{41}{3}\right]$	*bracket each term*
(4)	$15\left[2x\right] + \dfrac{15}{1}\left[\dfrac{(-12x)+20}{3}\right] = \dfrac{15}{1}\left[\dfrac{4x}{5}\right] + \dfrac{15}{1}\left[\dfrac{41}{3}\right]$	*multiply every term by LCD*
(4)	$15[2x] + 5[(-12x) + 20] = 3[4x] + 5[41]$	*cancel*
(4)	$30x + (-60x) + 100 = 12x + 205$	*simplify*
	$-30x + 100 = 12x + 205$	*simplify*
(5)	$\underline{+30x \quad = \quad +30x}$	*shift variable terms to one side*
	$100 = 42x + 205$	
(6)	$\underline{-205 \quad = \quad -205}$	*shift non-variables to other side*
	$-105 = 42x$	
(8)	$x = \dfrac{-105}{42} = \dfrac{-5}{2}$	*divide out x's coefficient*
(9)	Check by TI ✓	*so,* $\boxed{x = -2.5}$

Example 2: Solve $\dfrac{ax+1}{bx+1} = 1$

(3)	$\left[\dfrac{ax+1}{bx+1}\right] = \left[2\right]$	*bracket terms*
(4)	$\left[\dfrac{bx+1}{1}\right]\left[\dfrac{ax+1}{bx+1}\right] = \left[\dfrac{bx+1}{1}\right]\left[2\right]$	*multiply every term by LCD*
(4)	$ax + 1 = [bx + 1][2]$	*cancel*
(4)	$ax + 1 = 2bx + 2$	*simplify*
(5)	$\underline{-2bx \quad = \quad -2bx}$	*shift variable terms to one side*
	$ax - 2bx + 1 = 2$	
(6)	$\underline{-1 \quad = \quad -1}$	*shift non-variables to other side*
	$ax - 2bx = 1$	
(7)	$(a - 2b)x = 1$	*write as (coefficient) × (variable)*
(8)	$x = \dfrac{1}{a-2b}$	*divide out x's coefficient*
(9)	Checking by substitution is difficult	*so,* $\boxed{x = \dfrac{1}{a-2b}}$

Solve for x:

1) $3(5x + 4) - 15 = 2x - 3$

2) $4x - 3(3x + 2) = 7 - 5x$

3) $4x + 2(3x + 2) = 2(7 + 5x) - 10$

4) $3(ax + b) = 7ax - c$

5) $\frac{2x}{3} + \frac{3}{4} = \frac{5x}{2} + 3$

6) $\frac{3 - x}{2} + \frac{3}{4} = 2x - 5\frac{2x - 5}{6}$

7) $7 - \frac{3(x - 5)}{2} = 5x - \frac{6x - 5}{3}$

8) $\frac{x + y}{x - y} = 2y - 1$

9) $\frac{3 - x}{4} + \frac{2}{3} = 4 - \frac{3 \cdot (2x - 5)}{2}$

10) $\frac{2x}{3} - 3 \cdot \frac{5 - 3x}{2} = \frac{2x}{5} + 5$