An Efficient Scheme for Solving Linear Equations (DFOOFD)

(D) Distribute. Be especially mindful a – (b) or a – b(c) expressions.

(F) Remove all Fractions: *Bracket each term* [ ] *then multiply all terms by LCD and simplify*

(O) Shift variable term(s) to One side *(Additive Property)*

(O) Shift everything else to the Other side *(Additive Property)*

(F) Form (coefficient) · (variable). *(Factor out variable)*

i.e. $xy^2 + 4x = (y^2 + 4)x$ or $\pi x + ax = (\pi + a)x$

(D) Divide both sides by the variable's coefficient *(Multiplicative property)*

(C) Check the answer

Example 1: Solve $2x - 4\,\frac{(3x - 5)}{3} = \frac{4x}{5} + \frac{41}{3}$

(1)	$2x + (-4)\,\frac{(3x + (-5))}{3} = \frac{4x}{5} + \frac{41}{3}$	*change subtraction to addition*
(2)	$2x + \frac{(-12x) + 20}{3} = \frac{4x}{5} + \frac{41}{3}$	*and distribute*
(3)	$15\left[2x\right] + \frac{15}{1}\left[\frac{(-12x)+20}{3}\right] = \frac{15}{1}\left[\frac{4x}{5}\right] + \frac{15}{1}\left[\frac{41}{3}\right]$	*bracket terms and mult by LCD*
(4)	15[2x] + 5[(-12x) + 20] = 3[4x] + 5[41]	*cancel*
(5)	30x + (-60x) + 100 = 12x + 205	*simplify*
(6)	-30x + 100 = 12x + 205	*simplify*
(7)	-105 = 42x	*add/sub to shift terms*
(8)	$x = \frac{-105}{42} = \frac{-5}{2}$	*divide out x's coefficient*
(9)	Check by TI ✓	*so,* *x = -2.5*

Example 2: Solve $\frac{ax + 1}{bx + 1} = 2$

(1)	$\left[\frac{bx+1}{1}\right]\left[\frac{ax+1}{bx+1}\right] = \left[\frac{bx+1}{1}\right]\left[2\right]$	*bracket terms and multiply by LCD*
(4)	ax + 1 = [bx + 1][2]	*cancel*
(4)	ax + 1 = 2bx + 2	*simplify*
	ax – 2bx = 1	*add/sub to shift terms*
(7)	(a – 2b)x = 1	*write as (coefficient) × (variable)*
(8)	$x = \frac{1}{a - 2b}$	*divide out x's coefficient*
(9)	Checking by substitution is difficult	*so,* $x = \frac{1}{a - 2b}$

Mth 111 Solving Equations *Franz Helfenstein* **NAME**

In your own words, write your solution procedure for Linear Equations. Consider applying DFOOFD

Solve for x:

1) $3(5x + 4) - 15 = 2x - 3$

2) $4x - 3(3x + 2) = 7 - 5x$

3) $4x + 2(3x + 2) = 2(7 + 5x) - 10$

4) $3(ax + b) = 7ax - c$

5) $\frac{2x}{3} + \frac{3}{4} = \frac{5x}{2} + 3$

6) $\frac{3 - x}{2} + \frac{3}{4} = 2x - 5\frac{2x - 5}{6}$

7) $7 - \frac{3(x - 5)}{2} = 5x - \frac{6x - 5}{3}$

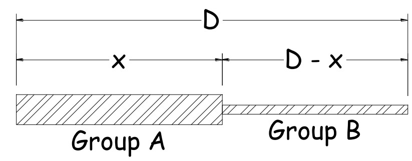
8) $\frac{x + y}{x - y} = 2y - 1$

In your own words, write your solution procedure for <u>Quadratic</u> Equations.

9) $2x + 15 = x(x + 4)$

10) $3[x(2x - 5) - 15] = 4(25 - 4x)$

Surprisingly (perhaps), there are situations where compromise can be reached by solving a linear equation. Suppose the ideal is to give the entire Domain (D) a gold watch but it costs too much. A cheaper alternative is to give Group A a gold watch and Group B a silver watch.

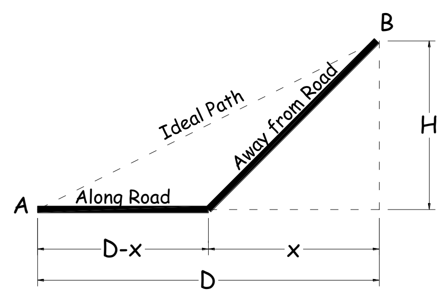


11) Here's one specific example. It costs \$35/ft for stainless pipe and \$15/ft for galvanized. What length (x) will maximize stainless pipe when 50' of pipe is needed on a budget of \$1000?

Write the mathematical model (eqn) for this scenario then solve the equation and solve the problem.

12) Here is a specific example that leads to a quadratic equation. Suppose vertical/horizontal sections cost \$20/ft while diagonals cost \$50/ft. What value of x maximizes the diagonal portion when D = 400', H = 300' ft and only \$20,000 is available?

Write the mathematical model (eqn) for this scenario then solve the equation and solve the problem.



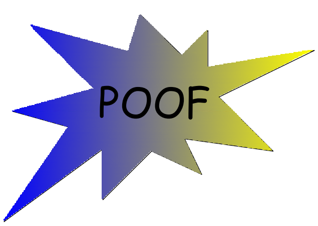

On Halloween, Buddy had obtained a huge stash of candy and was heading home when he met with an unfortunate event.

A monster accosted him and stole half his stash plus one more just for good measure.

Buddy quickly ate one of his candies then hurried for home at a rush. Dang, he was accosted by another monster; bigger and uglier than the first one. This monster stole half his remaining stash and then one more for good measure.

Buddy quickly downed another one of his candies to fortify himself and ran for home. A big cloud of thick, dark, foul, smelling smoke appeared from nowhere and then the smoke coalesced into a hideous witch. The witch cackled as she took half his rapidly shrinking stash plus one more for good measure. Buddy was now beside himself. He stuffed one of his few remaining candies in his mouth and sprinted for his house. He was almost home when he ran smack into another candy-stealing creature.

Like before, this monster stole half his puny stash and one more for good measure. When Buddy dug into his bag to check, all that remained were 3 candies. Buddy headed home in deep despair when amazingly. . .



the beautiful, sweet, glowing Good-Fairy-God-Mother-of-All appeared and said she could return all the candies if Buddy could tell her the number he had started with and how many were stolen. But he must do it by beginning with "x" candies.

Can you help Buddy? Let x = Buddy's Stash

Mth 111 Solving Equations *Franz Helfenstein* NAME ANSWER KEY

In your own words, write your solution procedure for Linear Equations. Consider applying DFOOFD

(D) Distribute. Be especially mindful a – (b) or a – b(c) expressions.

(F) Remove all Fractions: *Bracket each term [ ] then multiply all terms by LCD and simplify*

(O) Shift variable term(s) to One side *(Additive Property)*

(O) Shift everything else to the Other side *(Additive Property)*

(F) Form (coefficient) · (variable). *(Factor out variable)*

i.e. $\sqrt{2}\,x + 4x = (\sqrt{2} + 4)x$ or $\pi x + ax = (\pi + a)x$

(D) Divide both sides by the variable's coefficient *(Multiplicative property)*

(C) Check the answer

Solve for x:

1) $3(5x + 4) - 15 = 2x - 3$

$x = 0$

2) $4x - 3(3x + 2) = 7 - 5x$

x = No Solution

3) $4x + 2(3x + 2) = 2(7 + 5x) - 10$

x = All Real Numbers

4) $3(ax + b) = 7ax - c$

$x = \dfrac{3b + c}{4a}$

5) $\dfrac{2x}{3} + \dfrac{3}{4} = \dfrac{5x}{2} + 3$

$x = \dfrac{-27}{22}$

6) $\dfrac{3 - x}{2} + \dfrac{3}{4} = 2x - 5\,\dfrac{2x - 5}{6}$

$x = \dfrac{-23}{10}$

7) $7 - \dfrac{3(x - 5)}{2} = 5x - \dfrac{6x - 5}{3}$

$x = \dfrac{77}{27}$

8) $\dfrac{x + y}{x - y} = 2y - 1$

$x = \dfrac{y^2}{y - 1}$

In your own words, write your solution procedure for Quadratic Equations.

Shift all terms to one side and simplify into the Quadratic Form $ax^2 + bx + c = 0$. Then use factoring or the Quadratic Formula (QF) to solve for x.

9) $2x + 15 = x(x + 4)$

$x^2 + 2x - 15 = 0$ $\quad$ $(x + 5)(x - 3) = 0$

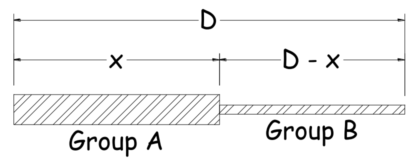
$x = -5, 3$

10) $3[x(2x - 5) - 15] = 4(25 - 4x)$

$6x^2 + x - 145 = 0$

$x = -5, \dfrac{29}{6}$

Surprisingly (perhaps), there are situations where compromise can be reached by solving a linear equation. Suppose the ideal is to give the entire Domain (D) a gold watch but it costs too much. A cheaper alternative is to give Group A a gold watch and Group B a silver watch.



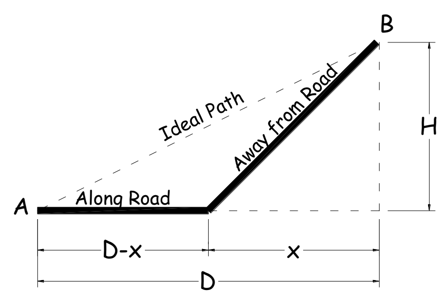
11) Here's one specific example. It costs \$35/ft for stainless pipe and \$15/ft for galvanized. What length (x) will maximize stainless pipe when 50' of pipe is needed on a budget of \$1000?

Write the mathematical model (eqn) for this scenario then solve the equation and solve the problem.

$35x + 15(50 - x) = 1000$ $\quad$ $x = 12.5$ $\quad$ so 12'5' stainless pipe and 37.5' galvanized pipe.

12) Here is a specific example that leads to a quadratic equation. Suppose it costs \$15/ft to install a gas line along the road but \$60/ft away from the road. What value of x maximizes the diagonal portion when D = 120', H = 50' ft and only \$5,000 is available?

Write the mathematical model (eqn) for this scenario then solve the equation and solve the problem.



$15(120-x) + 50\sqrt{50^2 + x^2} = 5000$ $\quad$ $D - x \approx 81.9'$, $x \approx 38.1'$

Buddy's Candy:

Let x = original stash	Monster steals half	Monster steals 1	Buddy eats 1	Monster steals half	Monster steals 1	Buddy eats 1	Monster steals half	Monster steals 1	Buddy eats 1	Monster steals half	Monster steals 1	3 remain
x	$\frac{1}{2}x$	$\frac{1}{2}x - 1$	$(\frac{1}{2}x - 2)$	$\frac{1}{2}(\ \ )$	$\frac{1}{2}(\ \ ) - 1$	$[\frac{1}{2}(\ \ ) - 2]$	$\frac{1}{2}[\ \ ]$	$\frac{1}{2}[\ \ ] - 1$	$\{\frac{1}{2}[\ \ ] - 2\}$	$\frac{1}{2}\{\ \ \}$	$\frac{1}{2}\{\ \ \} - 1$	$= 3$

$\frac{1}{2}\{\frac{1}{2}[\frac{1}{2}(\frac{1}{2}x - 2) - 2] - 2\} - 1 = 3$ $\quad$ $x = 92$