Introduction to Functions
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Name
Understanding functions and function notation is crucial to understanding much of mathematics. This activity should help you with an initial understanding of functions.

1) Explain how one of these could represent a function while the other would not. Be explicit.
y Fred's Life


2) Which of these represent a function and which do not. Explain your answer. Be explicit.

| ID\# (x) | DOB ( $\mathrm{y}_{1}$ ) |  |  | Allergies ( $\mathrm{Y}_{2}$ ) |  |  | $\operatorname{SSN}\left(\mathrm{Y}_{3}\right)$ |  |  | Meds ( $Y_{4}$ ) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 101 | Fcn: | Yes | No | Fcn: | Yes | No | Fcn: | Yes | No | Fcn: | Yes | No |
| 102 |  |  |  |  |  |  |  |  |  |  |  |  |
| 103 |  |  |  |  |  |  |  |  |  |  |  |  |
| 104 |  |  |  |  |  |  |  |  |  |  |  |  |
| 105 |  |  |  |  |  |  |  |  |  |  |  |  |
| 106 |  |  |  |  |  |  |  |  |  |  |  |  |

3) Which of these represent a function and which do not. Explain your answer. Be explicit.

| $y=m x+b$ |  |  | $x^{2}+y^{2}=1$ |  |  | $y=a x^{2}+b x+c$ |  |  | $y=2 \pm \sqrt{3 x}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Fcn | Yes | No | Fcn: | Yes | No | Fcn: | Yes | No | Fcn: | Yes | No |

4) Which of these represent a function and which do not.

| $x$ : race time $y$ : Bib \# | $x: \operatorname{Bib} \#$ <br> $y$ : race time |  |  | $x$ : gross income $y$ : Fed tax in 2010 |  |  | $x$ : age <br> $y$ : hair color |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Fcn: Yes No | Fcn: | Yes | No | Fcn: | Yes | No | Fcn: | Yes | No |
| $x$ : MD's age <br> $y$ : office wait time | $\begin{aligned} & x: \text { Name } \\ & y: D O B \end{aligned}$ |  |  | $\begin{gathered} x: D O B \\ y: \text { age } 1 / 1 / 2012 \\ \hline \end{gathered}$ |  |  | $x$ : Fed tax paid <br> $y$ : Gross Income |  |  |
| Fcn: Yes No | Fcn: | Yes | No | Fcn: | Yes | No | Fcn: | Yes | No |

5) 

(a) $f(-3)=$
(b) $f(2)=$
(c) $f(10)=$
(d) $(f+g)(2)=$
(e) $(f+g)(10)=$
(f) $f(6) g(10)=$
(g) $(f / g)(0)=$
(h) $2 f(4)+5=$

Domain of $f(x)$ :
Range of $f(x)$ :

6) $f(x)=3 x^{2}-2 \quad g(x)=x+1 \quad p(x)=\frac{x+1}{x}$
(a) $f(-2)=$
(b) $f(0)=$
(c) $\mathrm{p}(0)=$
(d) $(f+g)(5)=$
(e) $f(t)=$
(f) $g(a+b)=$
(g) $p(x-1)=$
(h) $f(x)+2=$
(i) $g^{2}(x)=$
7) You are tasked with collecting and organizing data then finding a function that corresponds to that data. First you must organize the data into independent vs dependent ( $x$ vs $y$ ). Fill in the indicated variables. Write 'unrelated' if the variables are unrelated.

Elevation \& 10K Race Time

Age \& 10K Race Time

Hair Color \& 10K Race Time
$x$-var: $\qquad$ $y$-var: $\qquad$
$x$-var: $\qquad$ $y$-var: $\qquad$
$x$-var: $\qquad$ $y$-var: $\qquad$
8) Consider this data set:


| $x$ | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 5 | 8 | $7,9,11,13,15$ | 14 | 17 |

Does the data represent a function?

If you run Linear Regression (try it if you can) you get
$y=3 x+2$ which is a function.

Do you think your regression function/equation is a good descriptor for this data? Why/why not?

Understanding functions and function notation is crucial to understanding much of mathematics. This activity should help you with an initial understanding of functions.

1) Explain how one of these could represent a function while the other would not. Be explicit.
y Fred's Life


Fred could both elope with Beth and have an affair with Sally so this is not a function


Fred takes one unique path through his life. This is a function.
2) Which of these represent a function and which do not. Explain your answer. Be explicit.

| ID\# (x) | DOB ( $\mathrm{y}_{1}$ ) | Allergies ( $\mathrm{Y}_{2}$ ) | $\operatorname{SSN}\left(Y_{3}\right)$ | Meds ( $\mathrm{Y}_{4}$ ) |
| :---: | :---: | :---: | :---: | :---: |
| 101 | Fcn: Yes No | Fcn: Yes No | Fcn: Yes No | Fcn: Yes No |
| 102 | ID is a unique | A person may have | Legally, each person | A person may be |
| 103 | dentifier and each | multiple allergies. | should have one | taking multiple |
| 104 | person can have only one DOB. However, if | Hence not a function. | unique SSN. Hence a function. | medications. Hence not a function. |
| 105 | 'Born Again' were |  |  |  |
| 106 | included then it's not a function |  |  |  |

3) Which of these represent a function and which do not. Explain your answer. Be explicit.

| $y=m x+b$ | $x^{2}+y^{2}=1$ | $y=a x^{2}+b x+c$ | $y=2 \pm \sqrt{3 x}$ |
| :---: | :---: | :---: | :---: |
| Fcn: Yes No | Fcn: Yes No | Fcn: Yes No | Fcn: Yes No |
| The only line that is not |  |  |  |
| a function is a vertical |  |  |  |
| line. Since $y=m x+b$ |  |  |  |
| cannot be vertical, this |  |  |  |
| is a function. |  |  |  | | This a circle which |
| :---: |
| does not pass the VLT. |
| Also, $y= \pm \sqrt{1-x^{2}}$ which |
| has 2 outputs for each |
| $x \neq \pm 1$. e.g. $x=0, y= \pm 1$ |$\quad$| This is a parabola which |
| :---: |
| passes the VLT. Hence a |
| function. | | This is a parabola turned |
| :---: |
| sideways which does not |
| pass the VLT. Hence not |
| a function. Specifically, |
| $x=3, y=-1$ or 5 |

4) Which of these represent a function and which do not.

| $x$ : race time y: Bib \# | $x:$ Bib \# $y$ : race time | $x$ : gross income <br> $y$ : Fed tax in 2010 | $x$ : age <br> $y$ : hair color |
| :---: | :---: | :---: | :---: |
| Fcn: Yes No | Fcn: Yes No | Fcn: Yes No | Fcn: Yes No |
| Two or more racers could cross finish line at the same time. Hence not a function. | Each racer has one unique finish time. Hence a function. | 2 Taxpayers with same gross income may have different deductions / tax liabilities. Hence not a function. | Variables are unrelated. Hence not a function. |


| $x:$ MD's age <br> $y:$ office wait time | $x:$ Name <br> $y: D O B$ | $x:$ DOB <br> $y:$ age $1 / 1 / 2012$ | $x:$ Fed tax paid <br> $y:$ Gross Income |
| :---: | :---: | :---: | :---: |
| Fcn: Yes No | Fcn: Yes No | Fcn: Yes No | Fcn: Yes No |
| These variables should <br> be unrelated. Hence not <br> a function. | Assuming Name is a <br> unique identifier this is a <br> function. If Name <br> First Name only, this <br> would not be a function. | Age $=1 / 1 / 2012-D O B$ is <br> a unique value. Hence a <br> function. | People who pay no tax <br> could have a wide variety <br> of incomes. Hence not a <br> function. |

5) 

(a) $f(-3)=1$
(b) $f(2)=1$
(c) $f(10)=D N E$
(d) $(f+g)(2)=10$
(e) $(f+g)(10)=$ DNE
(f) $f(6) g(10)=(6)(9)=54$
(g) $(f / g)(0)=0 / 8=0$
(h) $2 f(4)+5=9$

Domain of $f(x):-4 \leq x<10$ or $x \in[-4,10)$
Range of $f(x):-3 \leq y \leq 6$ or $y \in[-3,6]$

6) $f(x)=3 x^{2}-2 \quad g(x)=x+1 \quad p(x)=\frac{x+1}{x}$
(a) $f(-2)=10$
(b) $f(0)=-2$
(c) $p(0)=D N E$
(d) $(f+g)(5)=79$
(e) $f(t)=3 t^{2}-2$
(f) $g(a+b)=(a+b)+1=a+b+1$
(g) $p(x-1)=\frac{(x-1)+1}{(x-1)}=\frac{x}{x-1}$
(h) $f(x)+2=(3 \times 2-2)+2=3 x^{2}$
(i) $g^{2}(x)=(x+1)^{2}$
7) You are tasked with collecting and organizing data then finding a function that corresponds to that data. First you must organize the data into independent vs dependent ( $x$ vs $y$ ). Fill in the indicated variables. Write 'unrelated' if the variables are unrelated.

Elevation \& 10K Race Time $\quad x$-var: Elevation $y$-var: Racetime
Age \& 10K Race Time
$x$-var: Age
$y$-var: Racetime
Hair Color \& 10K Race Time
$x$-var: Unrelated $y$-var: $\qquad$
8) Consider this data set:


| $x$ | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 5 | 8 | $7,9,11,13,15$ | 14 | 17 |

Does the data represent a function? No
If you run Linear Regression (try it if you can) you get $y=3 x+2$ which is a function.

Do you think your regression function/equation is a good descriptor for this data? Why/why not?
No. The function $y=3 x+2$ does not do a good job of representing the variation associated with $x=3$.

