Mth 111 Outcomes for Exam 2 **NAME**

After studying, place a check mark next to those outcomes you feel you understand and/or are proficient with. Place a question mark next to those outcomes which you feel your skills/understanding is questionable. Turn in with your test.

To be successful in Mth 111 you should be able to …

Prerequisite Material

1. Solve a linear equation algebraically.
2. Solve a quadratic equation algebraically. (QF is adequate)
3. Graph a line from its equation.
4. Find the equation of a line from two points.
5. Find the equation of a line from a graph of the line.
6. Find the equation of a line using regression.

Functions I (include algebraic form, graphic form, tabular form)

1. Explain the concept of a function. i.e. What is a function?
2. Determine if a relationship is a function. i.e. vertical line test
3. Understand function notation in algebraic, graphic and tabular sense.
4. Evaluate functions with change of variable, at a value, with new expression. e.g. $f(x) \to f(t), f(2), f(a + b)$
5. Give the domain and range of a function from its algebraic, graphic or tabular form.
6. Give increasing or decreasing intervals.
7. Find local maximums or minimums.
8. Find the roots (zeros) of a function.
9. Graph piecewise functions.
10. Rewrite a piecewise graph in algebraic format.
11. Rewrite an implicit function in explicit form. i.e. $F(x,y) = 0 \to y = f(x)$.
12. Graph a function in a 'friendly' window (appropriate window).
13. Simplify the different quotient. i.e. Simplify $\frac{f(x + h) - f(x)}{h}$
14. Compute the average rate of change. (i.e. avg slope)
15. Transform a function graphically. i.e. $y = f(x)$ vs. $y = a\, f(b(x \pm h)) \pm k$

Mathematical Models

1. Interpret a mathematical model in algebraic or graphic form.
2. Identify the independent vs. the dependent variable.

Quadratics

1. Graph a quadratic and identify the four critical points: roots, vertex and y-intercept.
2. Switch between the key quadratic forms:

   $$y = ax^2 + bx + c \quad \leftrightarrow \quad y = a(x - h)^2 + k \quad \leftrightarrow \quad y = a(x - r_1)(x - r_2)$$

3. Find the equation of a quadratic from:

   (a) two roots and a third point (b) vertex and a third point., (c) three random points.

Functions II (include algebraic form, graphic form, tabular form)

1. Perform various operations among functions. e.g. $f(x) + g(x)$, $[f(x)][g(x)]$, $f(g(x))$, $f^2(x)$, etc.
2. Find the inverse of a function algebraically.
3. Find the inverse of a function graphically.
4. Determine if the inverse of a function is a function. i.e. Vertical/Horizontal Line test.
5. Show that $f(x) \circ f^{-1}(x) = f^{-1}(x) \circ f(x) = x$ algebraically.
6. Distinguish between $f^{-1}(x)$ vs. $[f(x)]^{-1}$

Solving Equations Using the Graphing Calculator

1. Solve $f(t) = g(t)$ by the intersection method.
2. Solve $f(t) = 0$ by the root method.

Exponents and Exponential Equations (include algebraic form, graphic form, tabular form)

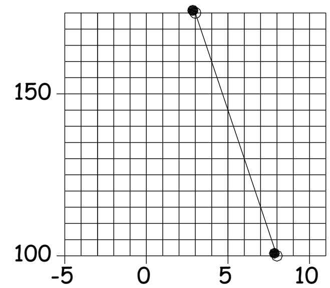
1. Apply the Rules of Exponents to simplify expressions. e.g. $(3x^2)^3 = 3^3 x^6$
2. Interpret an exponential model in algebraic or graphic form.
   a. Interpret $P(t) = P_0e^{-kt}$
   b. Interpret $P(t) = P_0e^{rt}$
   c. Interpret $y = a b^t$
3. Solve exponential equations algebraically. e.g. Solve for x: $3 e^{mx + b} = 10$

Logarithms and Logarithmic Equations (include algebraic form, graphic form, tabular form)

1. Apply the Rules of Logarithms to simplify expressions. e.g. $\ln (4x) - \ln x = \ln 4$
2. Solve logarithmic equations algebraically. e.g. Solve for x: $\ln (mx + b)^2 = 10$

Mth 111 Warmup for Exam 2 *Franz Helfenstein* **Name**

1) Use the indicated points to find the equation of this line in slope-intercept form.



2) Find the equation of the line that passes through the midpoint of the line segment shown here and is perpendicular to this line segment.

3) Solve for x and check your answer: $3 - \frac{x-1}{2} = \frac{x}{4}$

4) Solve for x and check your answer: $\frac{2x-4}{3} - 5 = 12 - \frac{7x-3}{2}$

5) Solve for y: $10 - \frac{2x-5y}{3} = 12 - 5\frac{3y-8x}{2}$

6) Solve for x: $\frac{3x^2}{2} - \frac{5}{4} = \frac{5-7x}{4}$

7) For $f(x) = 1 - 3x^2$, compute and simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$

8) For $f(t) = t^2$, simplify $4\,[\,f(x-1)\,] + 10$

9) $f(x) = xe^{(2x)}$ Simplify the following: (a) $f(3t) =$ (b) $f(a+b) =$

10) Simplify to an equivalent expression.

(a) Combine factors and convert to all positive exponents.

$$\frac{(a^2x^{-3})^3\,(ax^4)^2}{(a^{-4}x^0)^2} =$$

(b) Simplify to $a^n a^m$

$$a^{mx+b} =$$

(c) Simplify to x^n

$$f(x) = x^4;\;\; f^3(x^2) =$$

11) A bacteria population at 12:00 was 100. 5 hours later the population had increased to 140.

a) Using $P(t) = P_0e^{kt}$ with 12:00 corresponding to t = 0, determine P_0.

b) Determine k accurate to 4 significant digits.

c) Using your model what will be the population in 15 hours?

d) In how many hours will the population reach 250,000?

12) Mathematics often requires solving complex equations where algebraic methods are insufficient. e.g. This equation would be rather difficult to solve algebraically: $xe^x = 2x^3 (x + 1)$

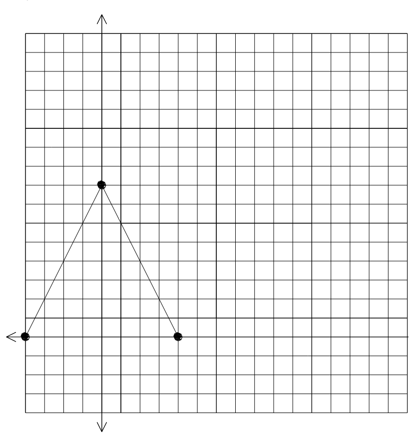
<u>Outline a procedure</u> for solving such equations, then give <u>all solutions</u> to the equation (there are three)!

13) $f(x) = 3x^2 – 12$; $g(t) = \sqrt{t + 1}$

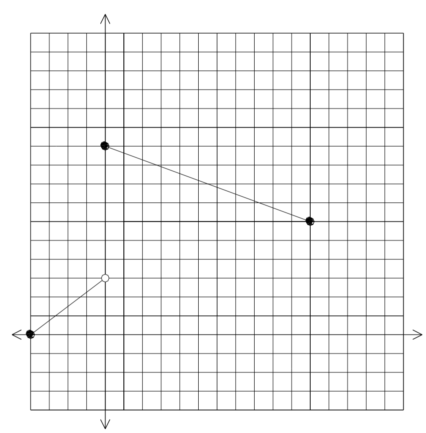
(a) Compute f(g(x)) (b) Compute $f^{\,2}(x)$ (c) Compute $[g(3)/f(2)]^2$

14) Find the inverse of the following function: f(x) = -3x/4 + 24

15) f(x) is shown. (a) Describe 2f(x – 8) – 4 then sketch the result. (b) Describe 8 – f(x) then sketch the result. (c) Describe $f(\frac{x-4}{3})$ then sketch the result.



16) g(x) is shown. Find $g^{-1}(x)$ in 2 ways. (a) Draw the inverse and compute the algebraic form. (b) compute the algebraic form for g(x) and compute the inverse from that.



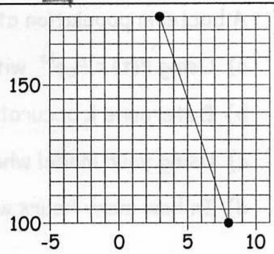
17) Find the average rate of change from x=-2 to x = 6 for y = $(\frac{1}{4})(x + 4)(x)(x – 8)$. Hint: Use the Table feature of you TI. What does this number represent?

18) Find the parabola that passes through (–6, 0) and has a vertex at (8, 6)

KEY

Mth 111 Warmup for Exam 2 *Franz Helfenstein* **Name**

1) Use the indicated points to find the equation of this line in slope-intercept form.



$(3,175)$ $(8,100)$

$y = -15x + 220$

2) Find the equation of the line that passes through the <u>midpoint</u> of the line segment shown here and is <u>perpendicular</u> to this line segment.

$mp = \left(\frac{3+8}{2}, \frac{175+100}{2}\right)$ $m = \frac{1}{15}$

$\left(\frac{11}{2}, \frac{175}{2}\right)$

$y = \frac{1}{15}x + \frac{1307}{15}$

3) Solve for x: $3 + \frac{-x+1}{2} = \frac{x}{4}$

$4 \cdot 3 + 4 \cdot \frac{-x+1}{2} = \frac{x}{4} \cdot 4$ $12 + 2 - 2x = x$

$14 = 3x$

$12 + 2(1-x) = x$

$x = \frac{14}{3}$ ✓

4) Solve for x and check your answer: $\frac{2x-4}{3} - 5 = 12 + \frac{-7x+3}{2}$

$6 \cdot \frac{2x-4}{3} + 6(-5) = 6 \cdot 12 + 6 \cdot \frac{3-7x}{2}$ $25x = 119$

$4x - 8 + (-30) = 72 + 9 - 21x$

$x = \frac{119}{25}$ ✓

5) Solve for y: $10 + \frac{-2x+5y}{3} = 12 + (-5)\frac{3y-8x}{2}$

$6 \cdot 10 + 6 \cdot \frac{5y-2x}{3} = 6 \cdot 12 + 6(-5)\frac{3y-8x}{2}$ $55y = 124x + 12$

$60 + 10y - 4x = 72 + (-45y) + 120x$

$y = \frac{124x+12}{55}$

6) Solve for x: $\frac{3x^2}{2} - \frac{5}{4} = \frac{5-7x}{4}$

$4 \cdot \frac{3x^2}{2} - 4 \cdot \frac{5}{4} = 4 \cdot \frac{5-7x}{4}$ $6x^2 + 7x - 10 = 0$

$6x^2 - 5 = 5 - 7x$ $(6x-5)(x+2) = 0$

$x = -2, \frac{5}{6}$

7) For $f(x) = 1 - 3x^2$, compute and simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$

$\frac{[1-3(x+h)^2] - [1-3x^2]}{h} = \frac{1 + (-3)[x^2 + 2xh + h^2] - 1 + 3x^2}{h} = \frac{(-6x-3h)h}{h} = -6x - 3h$

8) For $f(t) = t^2$, simplify $4\,[\,f(x-1)\,] + 10$

$4[(t-1)^2] + 10 = 4t^2 - 8t + 8 + 10 = 4t^2 - 8t + 18$

9) $f(x) = xe^{(2x)}$ Simplify the following: (a) $f(3t) =$ (b) $f(a+b) =$

(a) $3te^{6t}$

(b) $(a+b)e^{2a+2b}$ or $(a+b)e^{2a} \cdot e^{2b}$

10) Simplify to an equivalent expression.

(a) Combine factors and convert to all <u>positive</u> exponents. (b) Simplify to $a^n b^m$ (c) Simplify to x^n

(a) $\frac{(a^2x^{-3})^3\,(ax^4)^2}{(a^{-4}x^0)^2}$ $= a^{16}/x$

(b) $(a^3b^4)^2\,(a^3b^2)^4 =$ $a^{18}b^{16}$

(c) $f(x) = x^4;\ f^3(x^2) =$ $[(x^2)^4]^3 = x^{24}$

11) A bacteria population at 12:00 was 100. 5 hours later the population had increased to 140.

a) Using $P(t) = P_0 e^{kt}$ with 12:00 corresponding to t = 0, determine P_0. $P_o = 100$

b) Determine k accurate to 4 significant digits. $140 = 100e^{k \cdot 5}$ $k \sim 0.06729$

c) Using your model what will be the population in 15 hours? ~274.4

d) In how many hours will the population reach 250,000? 116.3 hrs

12) Mathematics often requires solving complex equations where algebraic methods are insufficient. e.g. This equation would be rather difficult to solve algebraically: $xe^x = 2x^3 (x + 1)$

Outline a procedure for solving such equations, then give all solutions to the equation!

Left Hand Side → y_1 Find intersections $x \cong 0, 0.7834, 6.413$
Right Hand Side → y_2 x-values = solutions
Graph

13) $f(x) = 3x^2 - 12$, $g(t) = \sqrt{t+1}$

(a) Compute f(g(x)) (b) Compute $f^2(x)$ (c) Compute $[g(3)/f(2)]^2$

$3(\sqrt{t+1})^2 - 12 = \boxed{3t-9}$ $\boxed{(3x^2-12)^2 = 9x^4 - 72x^2 + 144}$ $\frac{\sqrt{4}}{3 \cdot 2^2 - 12} = \boxed{\phi}$

14) Find the inverse of the following function: $f(x) = -3x/4 + 24$

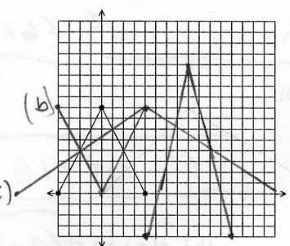
$f^{-1}(x) = \frac{4x-96}{-3} = -\frac{4x+96}{3}$

15) f(x) is shown. (a) Describe $2f(x - 8) - 4$ then sketch the result. (b) Describe $8 - f(x)$ then sketch the result. (c) Describe $f(\frac{x-4}{3})$ then sketch the result.

(a) → 8, ↕x2, ↓4

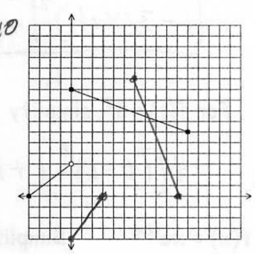
(b) ↻, ↑8

(c) → 4, ↔ x3



16) g(x) is shown. Find $g^{-1}(x)$ in 2 ways. (a) Draw the inverse and compute the algebraic form. (b) compute the algebraic form for g(x) and compute the inverse from that.

$g^{-1}(x) = \begin{cases} \frac{4}{3}x - 4, & 0 \le x < 3 \\ -\frac{11x}{4} + 27.5, & 6 \le x \le 10 \end{cases}$



17) Find the average rate of change from x=-2 to x = 6 for $y = (\frac{1}{4})(x + 4)(x)(x - 8)$. What does this number represent? Hint: Use Tables

(-2, 10)
(6, -30)

$m = \frac{-30-10}{6--2} = \boxed{-5}$ Slope from x = -2 to x = 6 on curve.

18) Find the parabola that passes through (-6, 0) and has a vertex at (8, 6) = (h, k)

$y = a(x-8)^2 + 6$ $a = -3/98$
$0 = a(-6-8)^2 + 6$

$\boxed{y = \frac{-3}{98}(x-8)^2 + 6}$