Mth 111 Practice for Exam 2 *Franz Helfenstein* **Name**

You must show the solution process not merely the answer to receive full credit. Write in a neat and organized fashion. Circle or box-in your answers. S**implify and write exact values where possible.** 100 pts.

1) Give the equation of the line through (2, -2) & (-13, 23)

2) Solve for x using algebraic manipulation: $15 - 7\frac{x-1}{2} = \frac{x}{4} + 3$

3) Solve for x using algebraic manipulation: $(x - 5)(x + 2) = 4(3x + 11)$

4) Solve for y: $10 - \frac{5x - y}{3} = \frac{3y - 2x + 4}{5}$

5) Solve for y: $ax + by = 3x - 2y + 8$

6) Solve for x: $4e^{ax + b} - 5 = 11$

7) Solve for x: $\frac{\ln(ax + b)}{2} + 7 = 6$

8) Solve using the TI: $e^{-x/2} = 2 + \ln x$. Give the answer with 4 significant digits.

9) An insect population was 600 on May 1. 5 days later the population had increased to 1,430.

Use $P(t) = P_0e^{rt}$ with May 1 corresponding to t = 0. Determine r.

10) $P(t) = P_0e^{-kt}$ models radioactive decay. Suppose you start with 200 grams of radioactive Silicon (Si) with a half-life of 140 yrs.

(a) What is the value for P_0?

(b) How much Silicon will be left in 500 yrs?

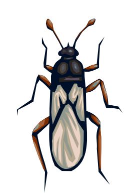
(c) How many years until the Silicon decays to 10 gms?

11) The mosquito population for Swamp Camp can be modeled by $M(t) = 500\, t\, e^{-0.12t}$ Where t = 0 on June 1.

(a) What is the maximum mosquito population? Hint: Graph it.

(b) How many days until the mosquito population drops below 1 of the nasty little buggers?

12) Function 'f' represents the number of bugs counted (N) at time of day (T) with time in minutes starting at midnight for the next 24 hrs.



(a) Give the independent variable ________________________________

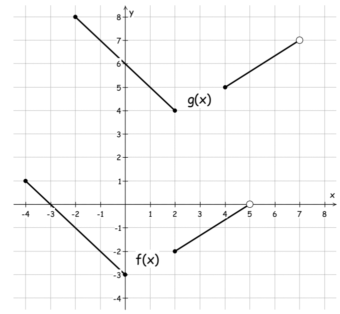
(b) What does f(7) = 8 mean in terms of this function?____________________

(c) Which of these correctly describes this relationship? (Circle one)

(i) N = f(T) (ii) y = f(x) (iii) T = f(N) (iv) N = f(x) (v) $N = 3T^2 + 4$

(d) Give the domain of this function.

13) Use the graph to answer the following:



(a) $f(-2)$ = (b) $f(5)$ = (c) $g(f(2))$ =

(d) How many roots does $f(x)$ have and <u>what are they</u>?

(e) What is the <u>domain</u> of $f(x)$?

(f) Give all x-values for which $g(x) = 5$. x =

(g) Circle the correct version of $g(x)$ as a translated version of $f(x)$.

$g(x) = f(x + 2) + 7$ $g(x) = f(x - 2) + 7$ $g(x) = f(2) + 7$ $g(x) = 2 + f(x) + 7$

(h) Find the average rate of change of $f(x)$ from $x_1 = -4$ to $x_2 = 2$

14)

(a) Simplify to a single term.
$\ln(ax) + \ln(b) =$

(b) Simplify to a single term.
$\ln ax - \ln bx =$

(c) Simplify to an <u>integer</u>.
$\log_3 405 - \log_3 5 =$

(d) Simplify to a single term.
$2\ln x - \ln x =$

(e) Simplify.
$\ln e^{4x} =$

(f) Simplify.
$e^{\ln 3x} =$

(g) Simplify to positive exponents.
$a^3 a^5 b^{-4} =$

(h) Simplify to positive exponents.
$(2x^2)^3 =$

(i) Simplify to positive exponents.
$\sqrt{x^9} =$

15) $f(x) = \sqrt{x - 1}$ $g(x) = x^2 + 1$ Simplify the following:

(a) $f(x + 1) =$

(b) $g(x^{-1}) =$

(c) $f(g(x))) =$

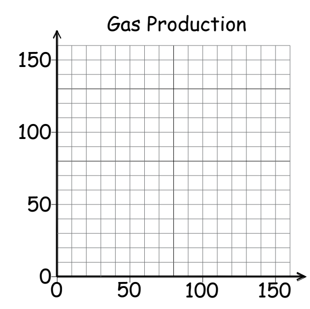
(d) $f^2(x) =$

(e) $10 - 5g(x) =$

(f) $g(g(x))$

16) Find the inverse of $y = \dfrac{2}{x + 1}$

17) $N(t) = 9.62te^{-0.035t}$ models natural gas production in a gas field (t = days).
t = 0 corresponds to the <u>field coming on line</u>.



(a) Draw the graph of $N(t)$.

(b) How many days after the field comes on line does the production peak? Round your answer to the nearest day.

(c*) The investors stop production when it drops to 10. How many days is that? Round your answer to the nearest day.

An Oil Field supply is declining exponentially. Let t = years with t = 0 being 2000. Supply is given in <u>millions</u> of barrels (bbl).	

t	0	1	2	3	4	5
bbl	50	42	36	30	25	24

(a) Run exponential regression on this data to determine the function which closely matches the data. Convert it to the form $y = Ae^{kt}$

(b) What will the oil supply be in 2010 (t = 10)?

(c) When will the oil supply drop to 1,000,000 bbl (bbl = 1 in above chart)?

Mth 111 Exam 2 *Franz Helfenstein* **Name** KEY

You must show the solution process not merely the answer to receive full credit. Write in a neat and organized fashion. Circle or box-in your answers. **Simplify and write exact values where possible.** 100 pts.

1) Give the equation of the line through (2, -2) & (-13, 23) 4 pts

$m = -5/3$

$b = 4/3$

$y = \frac{-5}{3}x + \frac{4}{3}$

2) Solve for x <u>using algebraic manipulation</u>: $15 - 7\frac{x-1}{2} = \frac{x}{4} + 3$ 5 pts

$60 - 14x + 14 = x + 12$

$15x = 62$

$x = 62/15$

3) Solve for x using algebraic manipulation: $(x - 5)(x + 2) = 4(3x + 11)$ 5 pts

$x^2 - 3x - 10 = 12x + 44$

$x^2 - 15x - 54 = 0$

$(x-18)(x+3) = 0$

$x = -3, 18$

4) Solve for y: $10 + \frac{5x - y}{3} = \frac{3y - 2x + 4}{5}$ LCD=15 5 pts

$150 + 25x - 5y = 9y - 6x + 12$

$138 + 31x = 14y$

$y = \frac{31x + 138}{14}$

5) Solve for y: $ax + by = 3x - 2y + 8$ 5 pts

$by + 2y = (b+2)y = 3x - ax + 8$

$y = \frac{3x - ax + 8}{b+2}$

6) Solve for x: $4e^{ax+b} - 5 = 11$ 4 pts

$e^{ax+b} = 4$

$\ln e^{ax+b} = \ln 4$

$x = \frac{(\ln 4) - b}{a}$

7) Solve for x: $\frac{\ln(ax+b)}{2} + 7 = 6$ 4 pts

$\ln(ax+b) = -2$

$e \quad e$

$x = \frac{e^{-2} - b}{a}$

8) Solve <u>using the TI</u>: $e^{-x/2} = 2 + \ln x$. Give the answer with 4 significant digits. 2 pts

$x \sim 0.3176$

9*) An insect population was 600 on May 1. 5 days later the population had increased to 1,430.

Use $P(t) = P_0e^{rt}$ with May 1 corresponding to t = 0. Determine r. 3 pts

$1430 = 600\, e^{5r}$

$r = \ln\left(\frac{1430}{600}\right)/5 \sim 0.1737$

10) $P(t) = P_0e^{-kt}$ models radioactive decay. Suppose you start with 200 grams of radioactive Silicon (Si) with a half-life of 140 yrs.

(a) What is the value for P_0? 2 pts 200gms

(b) How much Silicon will be left in 500 yrs? 2 pts

$k = \ln 2/140$

16.8 gms

(c) How many years until the Silicon decays to 10 gms? 2 pts 605.1 yrs

$10 = 200\, e^{-kt}$

11) The mosquito population for Swamp Camp can be modeled by $M(t) = 500\, t\, e^{-0.12t}$ Where t = 0 on June 1.

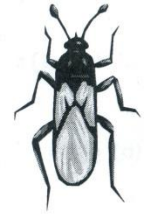
(a) What is the maximum mosquito population? Hint: Graph it. 2 pts

~ 1533

(b) How many days until the mosquito population drops below 1 of the nasty little buggers? 2 pts

$\sim$ 89 days

12) Function 'f' represents the number of bugs counted (N) at time of day (T) with time in minutes starting at midnight for the next 24 hrs. 4 pts



(a) Give the independent variable T = time of day minutes

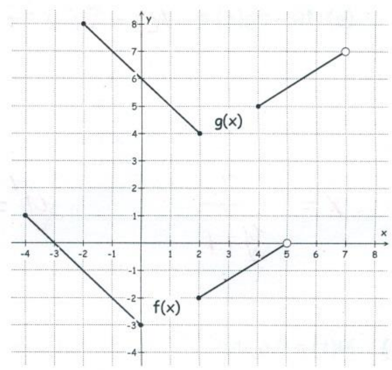
(b*) What does f(7) = 8 mean in terms of this function? At 7am there were 8 bugs

(c) Which of these correctly describes this relationship? (Circle one)

(i) N = f(T) (ii) y = f(x) (iii) T = f(N) (iv) N = f(x) (v) $N = 3T^2 + 4$

(d) Give the domain of this function. $t \in [0, 1440]$

13) Use the graph to answer the following: 1 pt each



(a) f(-2) = -1 (b) f(5) = ϕ (c) g(f(2)) = g(-2) = 8

(d) How many roots does f(x) have and what are they?

1 @ x = -3

(e) What is the domain of f(x)?

$x \in [-4, 0] \cup [2, 5)$

(f) Give all x-values for which g(x) = 5. x = 1, 4

(g*) Circle the correct version of g(x) as a translated version of f(x).

g(x) = f(x + 2) + 7 g(x) = f(x – 2) + 7 g(x) = f(2) + 7 g(x) = 2 + f(x) + 7

(h) Find the average rate of change of f(x) from $x_1 = -4$ to $x_2 = 2$

$y_1 = f(-4) = 1$

$y_2 = f(2) = -2$

$m = -1/2$

14) 2 pts each

(a) Simplify to a single term.
ln (ax) + ln (b) =

ln(abx)

(b) Simplify to a single term.
ln ax – ln bx =

ln(a/b)

(c) Simplify to an integer.
$\log_3 405 - \log_3 5 =$ $\log_3 81 =$ 4

(d) Simplify to a single term.
2ln x – ln x =

ln x

(e) Simplify.
$\ln e^{4x} =$

4x

(f) Simplify.
$e^{\ln 3x} =$

3x

(g) Simplify to positive exponents.
$a^3 a^5 b^{-4} =$

a^8/b^4

(h) Simplify to positive exponents.
$(2x^2)^3 =$

2^3x^6

(i) Simplify to positive exponents.
$\sqrt{x^9} =$

$x^{9/2}$

15) $f(x) = \sqrt{x-1}$ $\quad$ $g(x) = x^2 + 1$ $\quad$ Simplify the following: $\quad$ 3 pts each

(a) $f(x + 1) =$ $\sqrt{(x+1)-1} = \sqrt{x}$

(b) $g(x^{-1}) =$ $x^{-2} + 1 = \frac{1}{x^2} + 1$

(c) $f(g(x)) =$ $\sqrt{(x^2+1)-1} = x$

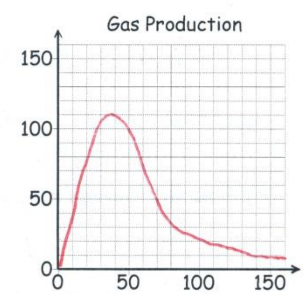
(d) $f^2(x) =$ $\sqrt{x-1}^{\,2} = x-1$

(e) $10 - 5g(x) =$ $10 - 5(x^2+1) = 5 - 5x^2$

16) Find the inverse of $y = \frac{2}{x+1}$ $\quad$ 5 pts

$x = \frac{2}{y+1}$ $\quad$ $y^{-1} = \frac{2}{x} - 1$ or $\frac{2-x}{x}$

17) $N(t) = 9.62te^{-0.035t}$ models natural gas production in a gas field (t = days). t = 0 corresponds to the field coming on line. $\quad$ 1 pt each



(a) Draw the graph of N(t).

(b) How many days after the field comes on line does the production peak? Round your answer to the nearest day.

28.6 ~ 29 days

(c*) The investors stop production when it drops to 10. How many days is that? Round your answer to the nearest day.

~ 140 days

BONUS

An Oil Field supply is declining exponentially. Let t = years with t = 0 being 2000. Supply is given in <u>millions</u> of barrels (bbl). 2 pts each	

t	0	1	2	3	4	5
bbl	50	42	36	30	25	24

(a) Run exponential regression on this data to determine the function which closely matches the data. Convert it to the form $y = Ae^{kt}$

$y = 48.99(0.857)^t \rightarrow y = 48.99e^{-0.1545t}$

(b) What will the oil supply be in 2010 (t = 10)? 10.45 Mbbl

(c) When will the oil supply drop to 1,000,000 bbl (bbl = 1 in above chart)?

~25.2 yrs