Mth 111 Sample Exam 2 *Franz Helfenstein* **Name**

You must show the solution process not merely the answer to receive full credit. Write in a neat and organized fashion. Circle or box-in your answers. **Simplify and write exact values where possible.** 100 pts.

1) Give the equation of the line through (1, -2) & (-13, 19) 4 pts

2) Solve for x <u>using algebraic manipulation</u>: $12 - 7\frac{5x-2}{3} = \frac{x}{4} + 5$ 5 pts

3) Solve for x using algebraic manipulation: $(2x - 5)(x + 2) = 2(3x + 1) + 27$ 5 pts

4) Solve for y: $7 + \frac{5x-y}{2} = \frac{5y-7x+4}{3}$ 5 pts

5) Solve for y: $5x + 3y = ax - by + c$ 5 pts

6) Solve for x: $3e^{mx+b} - 5 = 16$ 4 pts

7) Solve for x: $\frac{\ln(mx+b)}{2} + 8 = 11$ 4 pts

8) Solve <u>using the TI</u>: $5e^{-x^2} = x$. Give the answer with 3 significant digits. 2 pts

9*) A starfish population was initially 723. 25 days later the population had decreased to 515.

Using $P(t) = P_0e^{rt}$, determine r. 3 pts

10) $P(t) = P_0e^{-rt}$ models radioactive decay. You start with 100 grams of radioactive X with a half-life of 14 min.

(a) What is the value for P_0? 1 pts
(b) What is the value for r? 1 pts
(c) How much X will be left in 1 hour (60 min)? 2 pts
(d) How many minutes until X decays to 10 gms? 2 pts

11) The May fly hatch can be modeled by $M(t) = 430\ t\ e^{-0.23t}$ Where t = 0 on June 1.

(a) What is the maximum population? Hint: Graph it. 2 pts

(b) How many days until the population drops to 50? 2 pts

12) Function 'f' represents the number of people a dog has contacted (P) vs the number of minutes (T) the dog was in a park. The dog was in the park for 20 min. 4 pts

(a) Give the independent variable _____

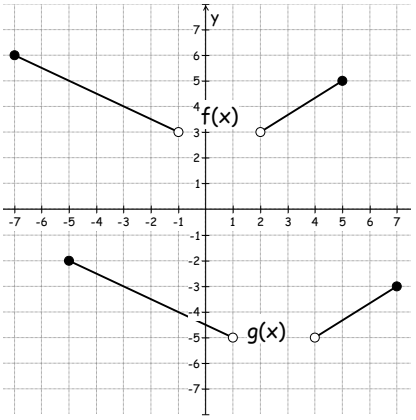
(b*) What does f(7) = 8 mean in terms of this function?_____

(c) Which of these correctly describes this relationship? (Circle one)

(i) T = f(N) (ii) y = f(x) (iii) P = f(T) (iv) P = f(x) (v) P = g(t)

(d) Give the domain of this function.

13) Use the graph to answer the following: 1 pt each



(a) f(1) = (b) f(5) = (c) f(g(-3)) =

(d) g(g(-1)) =

(e) What is the <u>domain</u> of f(x)?

(f) Give all x-values for which f(x) = 5. x =

(g*) Circle the correct version of g(x) as a translated version of f(x).

$g = f(x + 2) - 8$ $g = f(x - 2) - 8$ $g = f(2) - 8$ $g = 2 + 2f(x) - 4$

(h) Find the average rate of change of f(x) from $x_1 = -7$ to $x_2 = 5$

14a) Simplify to a single term. $\ln(ab) + \ln(c) =$

(b) Simplify to a single term. $\ln ax^2 - \ln bx =$

(c) Simplify to an <u>integer</u>. $\log_5 800 - \log_5 32 =$

(d) Simplify to a single term. $3\ln 2x - \ln x =$

(e) Simplify. $\ln e^{6x+1} =$

(f) Simplify. $e^{\ln 4x - 7} =$

(g) Simplify to positive exponents. $a^7 a^5 b^3 b^{-8} =$

(h) Simplify to positive exponents. $(5x^3)^4 =$

(i) Simplify to positive exponents. $x^2\sqrt{x^7} =$

15) $f(x) = \sqrt{x + 4}$ $g(x) = x^2 - 5$ Simplify the following: 3 pts each

(a) f(x + 6) =

(b) $g(x^{-1}) =$

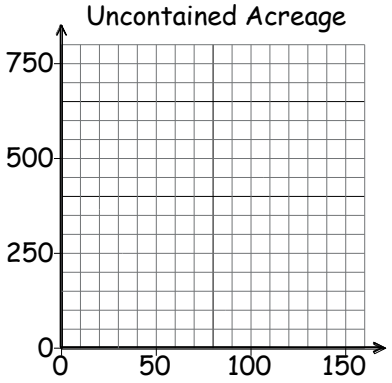
(c) f(g(x)) =

(d) g(x + h) =

(e) $f^2(x) - 2g(x) =$

16) Find the inverse of $y = \dfrac{5}{3x + 2}$ 5 pts

17) $N(t) = 95te^{-0.06t}$ models the uncontained acreage of a forest fire with t = days since the lightning strike that started the fire. 1 pt each



(a) Draw the graph of N(t).

(b) How many days after the lightning strike does the uncontained acreage peak? Round your answer to the nearest day.

(c*) The USFS switches to mop-up operations when uncontained acreage drops to 50 ac. How many days is that? Round your answer to the nearest day.

18) Spawning Salmon are declining exponentially. Let t = years with t = 0 being 2000. Coquille River population is given. 2 pts ea

t	0	1	2	3	4
Salmon	753	623	546	450	371

(a) Run <u>exponential regression</u> on this data to determine the function which best matches the data. Convert it to the form $y = Ae^{kt}$. Write both forms here.

(b) Predict the Salmon spawning population in 2010 (t = 10).

(c) Predict when the Salmon spawning population will drop to just 50.

Mth 111 Exam 2 *Franz Helfenstein* **Name** KEY

You must show the solution process not merely the answer to receive full credit. Write in a neat and organized fashion. Circle or box-in your answers. **Simplify and write exact values where possible.** 100 pts.

1) Give the equation of the line through (1, -2) & (-13, 19) 4 pts

$m = \frac{19 - -2}{-13 - 1} = \frac{21}{-14} = \frac{-3}{2}$

$b = 19 - \frac{-3}{2}(-13) = -\frac{1}{2}$

$y = -\frac{3}{2}x - \frac{1}{2}$

2) Solve for x <u>using algebraic manipulation</u>: $12 - 7\frac{5x-2}{3} = \frac{x}{4} + 5$ 5 pts

$12[12] + \frac{4}{12}\left[\frac{-35x+14}{3}\right] = \frac{3}{12}\left[\frac{x}{4}\right] + 12[5]$

$144 + -140x + 56 = 3x + 60$

$-140x + 200 = 3x + 60$

$140 = 143x$

$x = \frac{140}{143}$ ✓

3) Solve for x using algebraic manipulation: $(2x - 5)(x + 2) = 2(3x + 1) + 27$ 5 pts

$2x^2 - x - 10 = 6x + 2 + 27$

$2x^2 - 7x - 39 = 0$

$(2x - 13)(x + 3) = 0$

$x = -3, \ 13/2$

4) Solve for y: $6[7] + \frac{3}{6}\left[\frac{5x - y}{2}\right] = \frac{2}{6}\left[\frac{5y - 7x + 4}{3}\right]$ 5 pts

$42 + 15x - 3y = 10y - 14x + 8$

$34 + 29x = 13y$

$y = \frac{29x + 34}{13}$

5) Solve for y: $5x + 3y = ax - by + c$ 5 pts

$3y + by = ax - 5x + c$

$y = \frac{ax - 5x + c}{b + 3}$

6) Solve for x: $3e^{mx+b} - 5 = 16$ 4 pts

$\frac{+5}{3}$

$\ln e^{mx+b} = \ln 7$

$mx + b = \ln 7$

$x = \frac{(\ln 7) - b}{m}$

7) Solve for x: $\frac{\ln(mx+b)}{2} + 8 = (11-8)^2$ 4 pts

$e^{\ln(mx+b)} = e^6$

$mx + b = e^6$

$x = \frac{e^6 - b}{m}$

8) Solve <u>using the TI</u>: $5e^{-x^2} = x^2$. Give the answer with 3 significant digits. 2 pts

$x \approx \pm 1.15$

$5e^{-x^2} = x^2$ $x = 1.196$

9*) A starfish population was initially 723. 25 days later the population had decreased to 515.

Using $P(t) = P_0e^{rt}$, determine r. 3 pts

$515 = 723e^{r \cdot 25}$ $\frac{\ln(\frac{515}{723})}{25} = r \approx -0.0136$

10) $P(t) = P_0e^{-rt}$ models radioactive decay. You start with 100 grams of radioactive X with a half-life of 14 min.

(a) What is the value for P_0? 1 pts 100 gms

(b) What is the value for r? 1 pts $\ln 2/14 \approx 0.0495$

(b) How much X will be left in 1 hour? 2 pts

$P = 100e^{-R \cdot 60} \approx 5.13$ gms

(c) How many minutes until X decays to 5 gms? 2 pts

$5 = 100e^{-Rt}$ $t \approx 60.5$ min to 10 g ~47 min

11) The May fly hatch can be modeled by $M(t) = 430\ t\ e^{-0.23t}$ Where t = 0 on June 1.

(a) What is the maximum population? Hint: Graph it. 2 pts ~688

(b) How many days until the population drops to 50? 2 pts ~23 days

12) Function 'f' represents the number of people a dog contacted (P) vs the number of minutes (T) the dog was in the park. The dog was in the park for 20 min. 4 pts

(a) Give the independent variable T

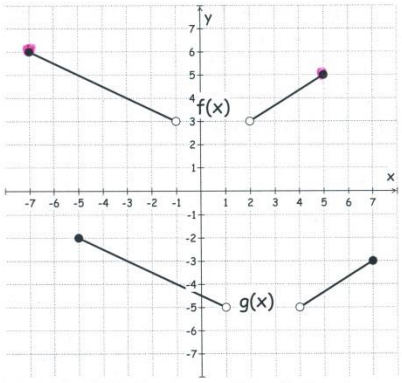
(b*) What does $f(7) = 8$ mean in terms of this function? (T, P) In 7 minutes, the dog had contacted 8 people.

(c) Which of these correctly describes this relationship? (Circle one)

(i) T = f(N) (ii) y = f(x) (iii) P = f(T) (iv) P = f(x) (v) P = g(t)

(d) Give the domain of this function. [0, 20 min]

13) Use the graph to answer the following: 1 pt each



(a) f(1) = DNE (b) f(5) = 5 (c) f(g(-3)) = 4

(d) g(g(-1)) = -2, 5

(e) What is the domain of f(x)? $[-7, -1) \cup (2, 5]$

(f) Give all x-values for which f(x) = 5. x = -5, 5

(g*) Circle the correct version of g(x) as a translated version of f(x).

g(x) = f(x − 2) - 8 g(x) = f(x − 2) - 8 g(x) = f(2) - 8 g(x) = 2 + f(x) 2 7

(h) Find the average rate of change of f(x) from $x_1 = -7$ to $x_2 = 5$

$$m = \frac{5-6}{5--7} = \frac{-1}{12}$$

14) 2 pts each

(a) Simplify to a single term. $\ln(ab) + \ln(c) =$ $\ln(abc)$

(b) Simplify to a single term. $\ln ax^2 - \ln bx =$ $\ln \frac{ax^2}{bx} = \ln\left(\frac{ax}{b}\right)$

(c) Simplify to an integer. $\log_5 800 - \log_5 32 =$ $\log_5 25 = 2$

(d) Simplify to a single term. $3\ln(2x) - \ln x =$ $\ln \frac{(2x)^3}{x} = \ln 2^3x^2$

(e) Simplify. $\ln e^{6x+1} =$ $6x+1$

(f) Simplify. $e^{\ln 4x-7} =$ $4x-7$

(g) Simplify to positive exponents. $a^7 a^5 b^3 b^{-8} =$ a^{12}/b^5

(h) Simplify to positive exponents. $(5x^3)^4 =$ $5^4 x^{12}$

(i) Simplify to positive exponents. $x^2\sqrt{x^7} =$ $x^{11/2}$

15) $f(x) = \sqrt{x+4}$ $g(x) = x^2 - 5$ Simplify the following: 3 pts each

(a) $f(x+6) = \sqrt{(x+6)+4} = \sqrt{x+10}$

(b) $g(x^{-1}) = x^{-2} - 5 = \frac{1}{x^2} - 5$

(c) $f(g(x))) = \sqrt{x^2-5+4} = \sqrt{x^2-1}$

(d) $g(x+h) = (x+h)^2 - 5$

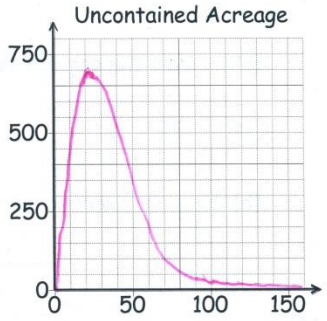
(e) $f^2(x) - 2g(x) = x+4-2(x^2-5) = -2x^2+x+14$

16) Find the inverse of $y = \frac{5}{3x+2}$ 5 pts

$x = \frac{5}{3y+2}$ $y = \frac{\frac{5}{x}-2}{3}$ $y^{-1} = \frac{5}{3x} - \frac{2}{3}$ or $y^{-1} = \frac{5-2x}{3x}$

$3y+2 = \frac{5}{x}$

17) $N(t) = 95te^{-0.06t}$ models the uncontained acreage of a forest fire with t = days since the lightning strike that started the fire. 1 pt each



(a) Draw the graph of N(t).

(b) How many days after the lightning strike does the uncontained acreage peak? Round your answer to the nearest day. ~17 days

(c*) The USFS switches to mop-up operations when uncontained acreage drops to 50 ac. How many days is that? Round your answer to the nearest day. ~85 days

BONUS

Spawning Salmon are declining exponentially. Let t = years with t = 0 being 2000. Coquille River population is given. 2 pts ea

t	0	1	2	3	4
Salmon	753	623	546	450	371

(a) Run exponential regression on this data to determine the function which best matches the data. Convert it to the form $y = Ae^{kt}$ $y = 754e^{kt}$ $k = \ln(0.840) \sim -.174$

(b) What will the spawning Salmon population be in 2010 (t = 10)? ~132

(c) When will the spawning Salmon population drop to 50? ~2015 - 2016