Mth 111 Lab 6 The Math of Retirement Name

Work must be neat and organized for full credit. Circle or box your answers. 10 pts.

When Franz and Doug were young and learning about exponential models, they each had a similar dream. Retire comfortably in 2010; Doug on his yacht and Franz in his Swiss Chalet.

Doug started saving right away. But Franz went off climbing and kayaking for 20 years before he settled down, got a job and started saving for his dream.

SAVING FOR RETIREMENT

When interest is compounded yearly on a single initial deposit the final balance is found using $B(t) = B_0(1 + R)^t$ where B_0 is the initial deposit and t is the number of years that interest accrues. But suppose additional payments are made. Then the mathematics becomes a bit more complicated.

t *(yrs)*	**B(t)** = *balance at time t* *(R = APR with yearly compounding, P = payment at the end of the year)*
0	*current balance = B(0) =* B_0
1	B_0 + $R B_0$ + P = $B_0(1 + R) + P$ *past balance* + *interest* + *payment* = *current balance*
2	$B_0(1 + R) + P$ + $R B_0$ + P = $B_0(1 + R)^2 + P(1+R) + P$ *past balance* + *interest* + *payment* = *current balance*
3	$B_0(1 + R)^2 + P(1+R) + P$ + $R B_0$ + P = $B_0(1 + R)^3 + P(1+R)^2 + P(1+R)^1 + P$ *past balance* + *interest* + *payment* = *current balance*

Yuk! This formula is getting rather cumbersome. Fortunately $P(1+R)^2 + P(1+R)^1 + P$ can be simplified.

1) Show that $P[(1 + R)^2 + (1+R)^1 + 1] = P\left[\frac{(1+ R)^3 - 1}{R}\right]$

2) Use the above simplification to write the balance as a function of t. i.e. Give B(t).

B(t) =

Doug's Retirement Plan

1970 Doug began saving for retirement by investing in an IRA that paid 8% interest per year. From 1970 – 1995 (25 years) Doug faithfully added $2400 at the end of each year to his retirement account. In 1996, Doug quit adding to his retirement so as to help put his children through College.

3) Use your function to find B(25), the balance of Doug's account at the end of 1995. Note, $B_0 = 0$.

4) From 1996 to 2010 Doug made no additional payments. However, he still earned interest during those years on his current balance. To determine his retirement balance in 2010 use his ending balance in 1995 as the new initial balance in B(t) with a $0 payment and find B(15).

5) What total did Doug pay into his account? i.e. What is the sum of all Doug's contributions?

Franz's Retirement Plan

From 1970 - 1990, Franz was too busy traveling the world climbing and kayaking to save any money. Finally, in 1990, he got real job and began investing into an IRA that paid 8% interest per year. During the next 20 years (1990 – 2010) he planned to made payments of $6000 at the end of each year to his retirement account.

6) What total will Franz pay into his account? i.e. What is the sum of all Franz's contributions?

7) Using the function B(t) you found in (2) what will Franz's balance be in 2010?

8) Who spent more, Doug or Franz?

9) How long will it take for Franz to save $500,000?

10) (a) Use P =$2400, R = 8% and $B_0 = 0$
Graph B(t) from 1970 − 1995.

(b) What does this indicate to you about saving for retirement?