QUICK SOLUTIONS

1	H = 42' 2.5"	*12*	$h \approx 89.6'$
2	D = 90' 3"	*13*	$A = 2{,}042\ ft^2$
3	$H \approx 3{,}594$ ft	*14*	2.44 cu-yds per foot of retaining wall
4	$\theta = \approx 3.94°$	*15*	$\theta_1 \approx 32.0°$; $\theta_2 \approx 14.0°$
5	$\theta = \approx 65.11°$	*16*	$z \approx 126'$
6	$\theta = \approx 72.13°$	*17*	1° latitude ≈ 69 mi; 1° longitude ≈ cos (|latitude|) 69 mi
7	$c \approx 28.8'$	18	Redmond and Madras are about 32 miles apart. Redmond and Sisters are about 23 miles apart.
8	$c \approx 51.0'$	19	≈ 40 km
9	$y = -0.449x + 6$; $a \approx 13.4$	20	$a \approx 280'$
10	$d_1 \approx 306$ mi	21	$c \approx 1908$ yds; Direction home is S 63° W
11	$A \approx 13.09\ ft^2$; $P \approx 14.80'$		

COMPLETE SOLUTIONS

Problem 1 Find the height of the flag pole.

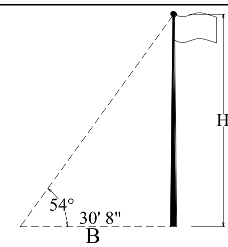
Notice there is a convenient right-triangle associated with the problem. Since the height and base (H & B) form a "t" this must be a tangent relationship.

Set up a tangent ratio $\tan(54°) = \frac{opp}{adj} = \frac{H}{B} = \frac{H}{30'\ 8''} = \frac{H}{30 + 8/12}$

Solve for H $H = (30 + 8/12)\tan(54°) \approx 42.2090'$

Convert to ft-in $0.209\text{ ft }(12\text{ in/ft}) \approx 2.5''$

Answer $H = \boxed{42'\ 2.5''}$



Problem 2 Find the length of the guy wire.

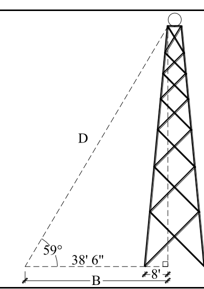
Notice there is a convenient right-triangle associated with the problem. Since the diagonal and the base (D & B) bracket the reference angle this must be a cosine relationship.

Set up a cosine ratio $\cos(59°) = \frac{adj}{hyp} = \frac{B}{D} = \frac{38'\ 6'' + 8'}{D} = \frac{46.5'}{D}$

Solve for D $D = \frac{46.5'}{\cos(59°)} \approx 90.28'$

Convert to ft-in $.28\text{ ft }(12\text{ in/ft}) \approx 3''$

Answer $D = \boxed{90'\ 3''}$



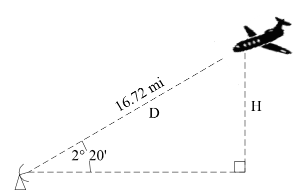
Problem 3 Find the plane's elevation in feet.

Notice there is a convenient right-triangle associated with the problem. Since the diagonal and the side opposite the reference angle (D & H) are involved this must be a sine relationship.

Set up a sine ratio $\sin(2°\ 20') = \frac{opp}{hyp} = \frac{H}{D} = \frac{H}{16.72\text{ mi}}$

Solve for H $H = (16.72\text{ mi})\sin(2°\ 20') \approx 0.6807$ mi

Convert to ft $0.6807\text{ mi }(5280\text{ ft/mi}) \boxed{\approx 3{,}594\text{ ft}}$



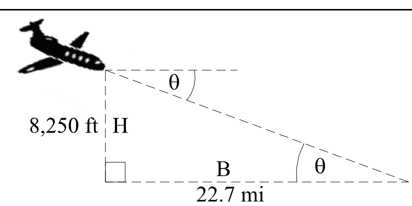
Problem 4 Find the plane's angle of descent.

Notice there is a convenient right-triangle associated with the problem. Since the height and base (H & B) form a "t" this must be a tangent relationship.

Set up a tangent ratio $\tan(\theta) = \frac{opp}{adj} = \frac{H}{B} = \frac{8250\text{ ft}}{22.7\text{ mi}}$

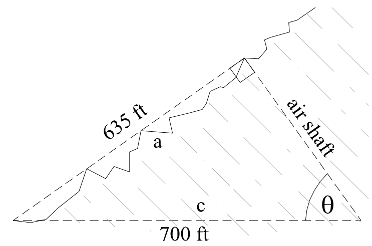
Convert to like units $\tan(\theta) = \frac{825\not{0}\ \not{ft}}{(22.7\ \not{mi})(528\not{0}\ \not{ft}/\not{mi})}$

Solve for θ $\theta = \tan^{-1}(825/22.7/528) \boxed{\approx 3.94°}$



Problem 5

700' into a tunnel, an air shaft must be drilled so as to reach the surface 635' uphill from where the tunnel starts. For optimum efficiency, the air shaft must take the shortest route to the surface. What should be the drilling angle? Note: the shortest route will intersect the surface at 90°. *Why?*



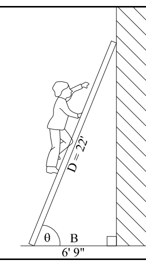
Since the diagonal and the side opposite the reference angle (c & a) are involved this must be a sine relationship.

Set up a sine ratio $\sin(\theta) = \frac{\text{opp}}{\text{hyp}} = \frac{a}{c} = \frac{635 \text{ ft}}{700 \text{ ft}}$

Solve for θ $\theta = \sin^{-1}(635/700)$ $\boxed{\approx 65.11°}$

Problem 6 Find the angle the ladder makes with the ground.

Notice there is a convenient right-triangle associated with the problem. Since the diagonal and the base (D & B) bracket the reference angle this must be a cosine relationship.



Set up a cosine ratio $\cos(\theta) = \frac{\text{adj}}{\text{hyp}} = \frac{B}{D} = \frac{6' 9''}{22'}$

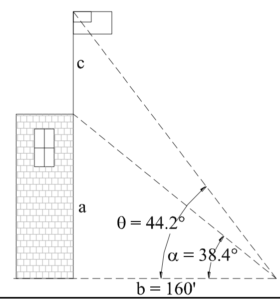
Convert to ft $\frac{6' 9''}{22'} = \frac{6 + 9/12 \text{ ft}}{22 \text{ ft}} = \frac{6.75 \cancel{\text{ft}}}{22 \cancel{\text{ft}}} = \frac{6.75}{22}$

Solve for θ $\theta = \cos^{-1}(6.75/22)$ $\boxed{\approx 72.13°}$

Problem 7

Marc is standing 160 feet from a tall building. From there, he measures the angle (from the horizontal) to the top of the building at 38.4°. There is a flag pole on the edge of the roof. Marc measures the angle to the top of the flag pole too. That angle measures 44.2°. What is the length of the flag pole?

1) Draw a labeled diagram and fill it with right-triangles.



2) Find 'a' using $\tan(\alpha) = \frac{a}{b}$ $a = b \tan(\alpha) = 160' \tan(38.4°) \approx 126.81'$

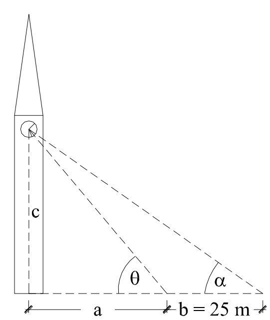
3) Find 'a+c' using $\tan(\theta) = \frac{a+c}{b}$ $a+c = b \tan(\theta) = 160' \tan(44.2°) \approx 155.59'$

4) Find 'c' $c = (a+c) - a = 155.59' - 126.81'$ $\boxed{\approx 28.8'}$

Problem 8

While Lydia is walking toward a tall Clock Tower she measures the angle (from the horizontal) to the clock at 34.6°. After she moves 25 m closer, she measures the angle again. It now measures 46.2°. How high is the clock above the ground?

1) Draw a labeled diagram and fill it with right-triangles.
   $\alpha = 34.6°$, $\theta = 46.2°$



2) Note: $\tan(\alpha) = \frac{c}{a+b}$ & $\tan(\theta) = \frac{c}{a}$.
   Solve each for 'c'.

$$c = (a + b)\tan(\alpha)$$
$$c = a \tan(\theta)$$

3) Eliminate 'c'

$$a \tan(\theta) = (a + b)\tan(\alpha)$$

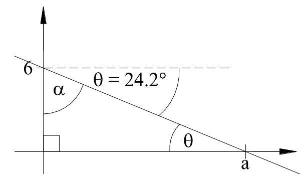
4) Solve for 'a'

$$a \tan(\theta) = a \tan(\alpha) + b \tan(\alpha)$$
$$a \tan(\theta) - a \tan(\alpha) = b \tan(\alpha)$$
$$a[\tan(\theta) - \tan(\alpha)] = b \tan(\alpha)$$
$$a = \frac{b \tan(\alpha)}{\tan(\theta) - \tan(\alpha)} \approx 48.9'$$

5) Solve for 'c'. $c = a \tan(\theta)$ $\boxed{\approx 51.0'}$

Problem 9

(a) Find the equation for the line (in slope-intercept form) which crosses the y-axis at (0, 6) at an angle of -24.2°. That is, the line is angled downward. (b) Find the x-intercept.



1) Draw a labeled diagram. We need the slope and y-intercept.

2) Find the y-intercept, b. $b = 6$

3) Find the slope, m. $m = -\tan(\theta) = -\tan(24.2°) \approx -0.449$

4) Write Equation, $y = mx + b$. $\boxed{y = -0.449x + 6}$

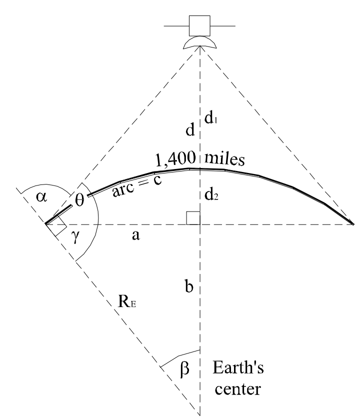
5) Find the x-intercept using $\tan(\theta) = \frac{b}{a}$. $a = b/\tan(\theta) = 6/\tan(24.2°) \boxed{\approx 13.4}$

Notice that we could also have used the equation $y = -0.449x + 6$ to find the x-intercept. The x-intercept occurs when $y = 0$. Thus we solve for x when $y = 0$.

$$0 = -0.449x + 6 \quad \rightarrow \quad 0.449x = 6 \quad \rightarrow \quad x = 6/0.449 \approx 13.4$$

Problem 10

A satellite is tracked from two stations that are 1,400 statute miles apart on the Earth. At the point that the satellite is exactly midway between the stations, the "line of sight" to the satellite is offset 72° 18' 48" from directly overhead. How high is the satellite's orbit above the Earth?



1) Draw a labeled diagram and fill it with right-triangles.

2) Find β using $\textit{arc length} = \frac{\beta}{360°} 2\pi R_E$
$$\frac{\beta}{360} 2\pi R_E = 700 \text{ mi}$$
$$\beta = \frac{360°\ 700 \text{ mi}}{2\pi\ 3960 \text{ mi}} \approx 10.1280°$$

3) Find 'a' using $\sin(\beta) = \frac{a}{R_E}$ $a = (R_E) \sin(\beta) \approx 696.360$ mi

4) Find 'b' using $\cos(\beta) = \frac{b}{R_E}$ $b = (R_E) \cos(\beta) \approx 3898.29$ mi

5) Find θ using complements and supplements. $\alpha = 72° 18' 48"$
$$\gamma = 90° - \beta \approx 79.8720°$$
$$\theta = 180° - \gamma - \alpha \approx 27.8147°$$

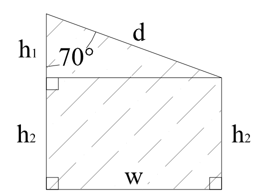
6) Find 'd' using $\tan(\theta) = \frac{d}{a}$ $d = a \tan(\theta) \approx 367.378$ mi

7) Find d_1 using $d = d_1 + d_2$ and $R_E = b + d_2$ $d_1 = d - d_2 = d - (R_E - b) \approx 306$ mi

Earth radius ≈ 3,960 mi.

Problem 11

A trapezoid window has a width of 4', a height of 4' and the interior angle of 70° (a) Find its area and (b) find its perimeter.



$w = 4'$, $h_1 + h_2 = 4'$

1) Draw a labeled diagram with right-triangles.

2) Find h_1 using $\tan(70°) = \frac{w}{h_1}$ $h_1 = w/\tan(70°) = 4'/\tan(70°)$
$$h_1 \approx 1.456'$$

3) Find h_2 $h_2 = 4' - h_1 \approx 2.544'$

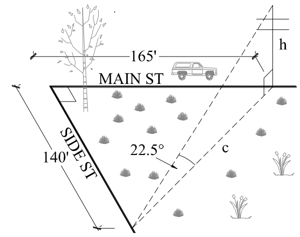
4) Find area $A = \frac{(h_1 + h_2) + h_2}{2} \cdot w \approx \boxed{13.09 \text{ ft}^2}$

5) Find d using $\sin(70°) = \frac{w}{d}$ $d = w/\sin(70°) \approx 4.257'$

6) Find perimeter $4' + 4' + h_2 + d \approx \boxed{14.80'}$

Problem 12

A telephone pole is located 165' down a main street. Perpendicular to that main street is a side street. 140' down that side street a sighting is taken to the top of the pole @ 22.5°. How tall is the pole?



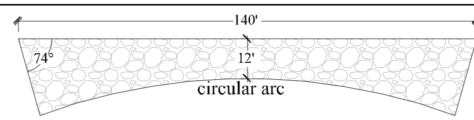
1) Draw a labeled diagram with right-triangles.
2) Use the Pyth. Thm to find c. $c^2 = 140^2 + 165^2 = 46{,}825$

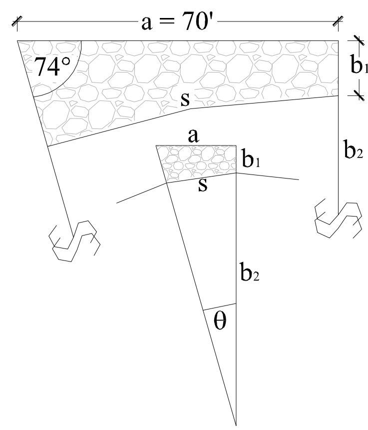
   $c = \sqrt{46825} \approx 216.4'$

3) Find h using $\tan(22.5) = \frac{h}{c}$ $h = c \tan(22.5) \approx \boxed{89.6'}$

Problem 13

A hi-way bridge spans 140'. The underside is formed with a circular arc. The sides are canted inward at 74°. The bridge is 12' thick in the center and 24' (diagonally) along the sides. Find the bridge's cross-sectional area.





1) Draw a labeled diagram with right-triangles.
2) Area = $\text{Area}_{\text{triangle}} - \text{Area}_{\text{sector}}$ $A_{total} = 2(A_t - A_s)$
3) Find θ using complements $\theta = 90° - 74° = 16°$
4) Find sector radius (b_2) using $\tan(\theta) = \frac{a}{b_1 + b_2}$

   Solve for b_2 $b_1 + b_2 = a/\tan(\theta)$

   $b_2 = [a/\tan(\theta)] - b_1 = [70'/\tan 16°)] - 12' \approx 232.12'$

5) Find sector area (A_s) using $A_s = \frac{\theta}{360°}\pi r^2$

   Solve for A_s $A_s = \frac{16°}{360°}\pi (b_2)^2 \approx 7{,}523 \text{ ft}^2$

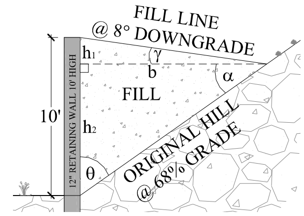
6) Find triangle area (A_t) using $A_t = (½)BH$

   $A_t = (½)a(b_1 + b_2) \approx 8{,}544 \text{ ft}^2$

7) Find Area using $A = 2(A_t - A_s)$ $A = 2(8{,}544 \text{ ft}^2 - 7{,}523 \text{ ft}^2) = \boxed{2{,}042 \text{ ft}^2}$

Problem 14

A 10 ft vertical retaining wall is built at the base of a hill which has a 68% grade. The area behind the wall is to be filled so that the final grade slopes from the retaining wall back towards the hill at an 8° down-slope. How much fill (cu-yds) is required for each foot of length of the retaining wall?



$h_1 + h_2 = 10'$

1) Draw a labeled diagram with right-triangles.
2) Area = (½)(h_1 + h_2) b. So, we must find b.
3) Find α using tan (α) = 68% $\qquad \alpha = \tan^{-1}(0.68) \approx 34.2°$
4) Find γ using tan (γ) = 8% $\qquad \gamma = \tan^{-1}(0.08) \approx 4.6°$
5) Note: $\tan(\gamma) = \frac{h_1}{b}$, $\tan(\alpha) = \frac{h_2}{b}$., $h_1 + h_2 = 10'$. Solve for b.

$$h_1 = b\tan(\gamma),\ h_2 = b\tan(\alpha)$$

$$h_1 + h_2 = 10' = b\tan(\gamma) + b\tan(\alpha) = b[\tan(\gamma) + \tan(\alpha)]$$

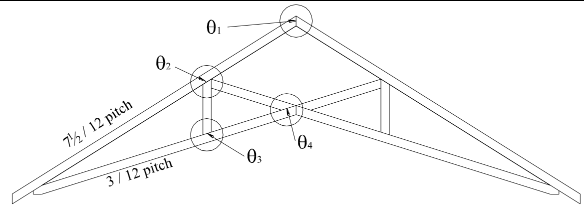
$$b = \frac{10'}{\tan(\gamma) + \tan(\alpha)} \approx 13.16'$$

6) Find triangle area $\qquad A = (½)(10')(b) \approx 65.79\ ft^2$
7) Find vol per ft of length $\qquad V = A\cdot L = (65.79\ ft^2)(1\ ft) = 65.79$ cu-ft

$$(65.79 \text{ cu-ft})\frac{1 \text{ cu-yd}}{27 \text{ cu-ft}} \approx \boxed{2.44 \text{ cu-yds per foot of retaining wall}}$$

Problem 15

A scissor truss is made by joining a number of components. Find the cutting angles θ_1, & θ_2 for a scissor truss with a top rafter @ 7 ½ / 12 pitch and a bottom rafter @ 3/12 pitch.



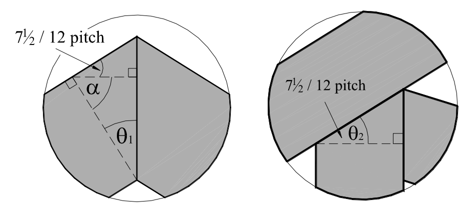
1) Draw a diagram for θ_1, & θ_2.

2) θ_1 & α are complements. α and the 7 ½ / 12 pitch angle are also complements. Thus, θ_1 has a 7 ½ / 12 pitch. Hence tan (θ_1) = 7.5/12. *Why?*

$$\theta_1 = \tan^{-1}(7.5/12) \approx \boxed{32.0°}$$

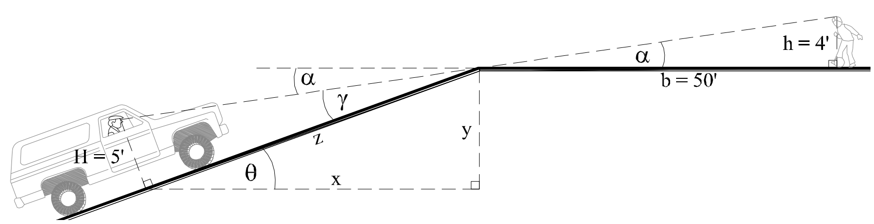
3) θ_2 has a 3/12 pitch. Hence tan (θ_2) = 3/12.

$$\theta_2 = \tan^{-1}(3/12) \approx \boxed{14.0°}$$



Problem 16

A driver is going up a road sloped at 12% which eventually flattens out. A 4' tall child is playing in the road 50' from the edge of the slope. The driver's eyes are 5' above the road bed when the vehicle is on flat ground. At what point will the driver begin to see the child?

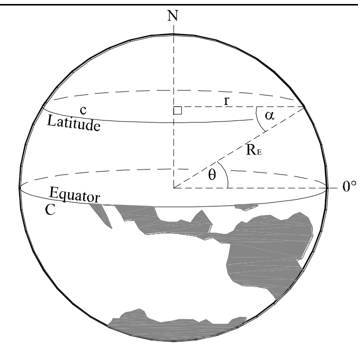


1) Draw a diagram. $\alpha \neq \gamma$
2) $\theta = \tan^{-1}(12\%) \approx 6.8°$
3) $\tan(\alpha) = \frac{h}{b} = \frac{4'}{50'} = 0.08$
4) Solve for α. $\qquad \alpha = \tan^{-1}(0.08) \approx 4.6°$
5) $\alpha + \gamma = \theta$. *Why?* $\qquad \gamma = \theta - \alpha \approx 2.3°$
6) $\tan(\gamma) = \frac{H}{z}$ $\qquad z = H/\tan(\gamma) = 5'/\tan(\gamma) \approx \boxed{126'}$

Problem 17

(a) Find the conversion from one degree latitude to surface miles. This is constant throughout Earth.

(b) Find the conversion (at θ° latitude) from one degree longitude to surface miles. This varies with latitude.



$R_E \approx$ 3,960 mi, RE ≈ 6,370 km

1) Draw a diagram. Notice that 'C', the circumference along the equator, is much larger than 'c', the circumference along latitude θ. Also, the circumference along every longitude is always C.

2) $C = 2\pi R_E = 360°$. So, $1° = 2\pi R_E/360 \approx$ 69 mi. That is, $\boxed{1° \text{ latitude} \approx 69 \text{ mi}}$

3) Since R_E intersects parallel lines, $\theta = \alpha$.

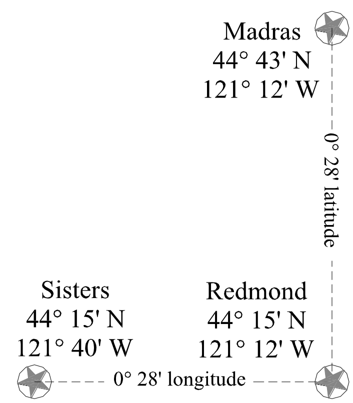
4) $\cos(\alpha) = \cos(\theta) = \dfrac{r}{R_E} \rightarrow r = R_E \cos(\theta)$

5) $c = 2\pi r = 360°$. So, $1° = 2\pi r/360 = \dfrac{2\pi R_E \cos(\theta)}{360} \approx 69 \cos(\theta)$ mi. That is,

$$\boxed{1° \text{ longitude} \approx \cos(|\text{latitude}|)\ 69 \text{ mi}}$$

Problem 18

A map shows Madras @ 44° 43' N, 121° 12' W, Redmond @ 44° 15' N, 121° 12' W and Sisters @ 44° 15' N, 121° 40' W. Thus, there are 0° 28' of latitude between Redmond and Madras and 0° 28' longitude between Redmond and Sisters.



28' latitude uses the conversion 1° latitude ≈ 69 mi.

$$28' = (28/60)\text{deg} \cdot 69 \text{ mi/deg} \approx 32 \text{ mi.}$$

Thus, Redmond and Madras are about 32 miles apart.

But 28' longitude uses the conversion 1° latitude ≈ cos (|latitude|) 69 mi.

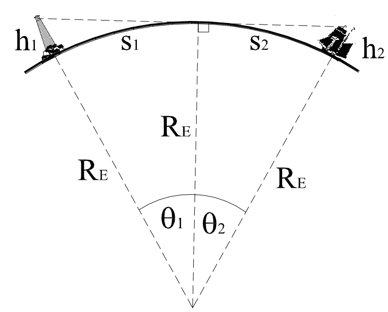
$$28' = (28/60)\text{deg} \cdot 69 \text{ mi/deg} \cos(44° 20') \approx 23 \text{ mi.}$$

So Redmond and Sisters are about 23 miles apart.

Problem 19

A Ship at sea has a lookout in the crows nest 20 m above the water looking for a lighthouse that is situated on a rock outcropping. The light of the lighthouse is 45 m above the water. At what distance will the lookout see the light directly?

1) Draw a diagram. Notice the distance asked for is $s_1 + s_2$ which are arcs of a circle of radius R_E. Thus, we need to find θ_1 & θ_2.



$R_E \approx$ 3,960 mi, RE ≈ 6,370 km

2) Find θ_1. $\cos(\theta_1) = \dfrac{R_E}{R_E + h_1} = \dfrac{6370 \text{ km}}{6370 \text{ km} + 45\text{m}}$

$$\theta_1 = \cos^{-1}\left[\frac{6370 \text{ km}}{6370.045 \text{ km}}\right] \approx 0.2154°$$

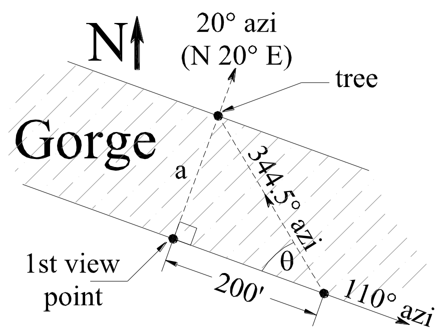
3) Find θ_2. $\cos(\theta_2) = \dfrac{R_E}{R_E + h_2} = \dfrac{6370 \text{ km}}{6370 \text{ km} + 20\text{m}}$

$$\theta_2 = \cos^{-1}\left[\frac{6370 \text{ km}}{6370.020 \text{ km}}\right] \approx 0.1436°$$

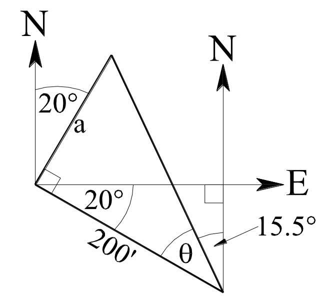
4) $S = s_1 + s_2 = \dfrac{\theta}{360°} 2\pi r = \dfrac{\theta_1 + \theta_2}{360°} 2\pi R_E \approx \dfrac{0.2149° + 0.1432°}{360°} 2\pi\ 6370 \text{ km} \approx \boxed{40 \text{ km}}$

Problem 20

Valerie needs to calculate the distance across the Crooked River Gorge so her company can build a bridge for a movie set. As she looks across the gorge she spots a tree on the other side directly across from where she is standing. Her compass gives a reading of 20° azi (aka N 20° E) toward the tree. Valerie turns clockwise 90° (so she is looking up the gorge) and walks 200 feet along the rim of the gorge. At that point, she takes a second compass reading toward the same tree. That reading is 344.5° azi (N 15.5° W). How wide is the gorge to the nearest foot?



1) Although the initial diagram would suffice, a second diagram that shows a compass reference at each view point makes it much clearer.



2) To find a, we need θ. θ + 15.5° + 20° = 90° *Why?*

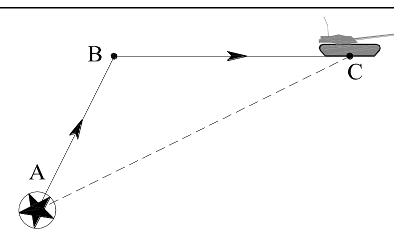
$$\theta = 90° - 15.5° - 20° = 54.5°$$

3) Find a. $\tan(\theta) = \frac{a}{200'}$

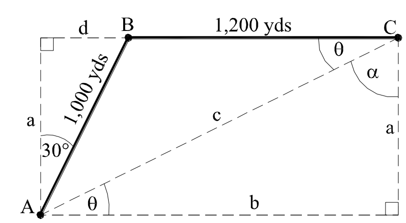
$$a = 200 \tan(\theta) = 200' \tan(54.4°) \approx 280'$$

Problem 21

A tank travels 1,000 yds from pt A to pt B on a N 30° E bearing and then changes direction and travels due east 1,200 yds from pt B to pt C. What direction and how far must the tank travel to return to its starting point most directly?



1) A more detailed diagram is required.



2) Find a $\cos(30°) = \frac{a}{1000 \text{ yds}}$

$$a = 1000 \cos(30°) \approx 866.0 \text{ yds}$$

3) Find d. $\sin(30°) = \frac{d}{1000 \text{ yds}}$

$$d = 1000 \sin(30°) = 500 \text{ yds}$$

4) Find b $b = d + 1{,}200 \text{ yds} = 500 \text{ yds} + 1200 \text{ yds} = 1{,}700 \text{ yds}$

5) Find c using $a^2 + b^2 = c^2$ $c = \sqrt{a^2 + b^2} = \sqrt{3640000} \approx \boxed{1908 \text{ yds}}$

6) Find θ using $\tan(\theta) = \frac{a}{b}$ $\theta = \tan^{-1}\left[\frac{a}{b}\right] \approx 27°$

7) Find α $\alpha = 90° - \theta = 90° - 27° = 63°$. Thus, direction home is $\boxed{\text{S } 63° \text{ W}}$