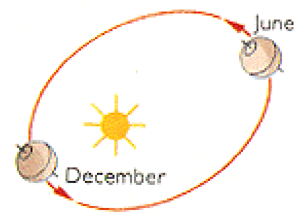
Modeling Temperature Data

Name(s):______________________________

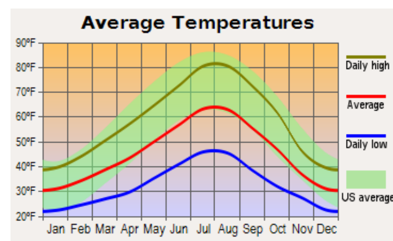
Since the trigonometric functions are periodic, they are a particularly useful tool when modeling cyclic behavior. For example, variables that depend on the seasons may be modeled with trigonometric functions because the seasons repeat every year just like the sine function repeats every 2π. Another neat application of trig curves (in particular, sine and cosine) is modeling temperature data. In this project, we will mathematically model the average annual high temperature of Bend, OR.

Why would a periodic function like sine or cosine be particularly useful for modeling temperatures? For one, temperatures are cyclical. In the Northern Hemisphere, it tends to get cold in the winter and warmer in the summer. But why? In the southern hemisphere, our winter is their summer, even though the earth is closest to the sun in December[a].

The reason is actually pretty cool (or hot): The Northern Hemisphere's summer is when the North Pole is tilted towards the sun[b], while the Southern Hemisphere has winter (and vice versa). Thus, even though the Northern Hemisphere is very far away from the sun during our summer, we're tilted toward the sun. Therefore, it's warmer in our half than in the Southern Hemisphere. The reverse is true in the winter...check out the diagram at right to help you visualize.

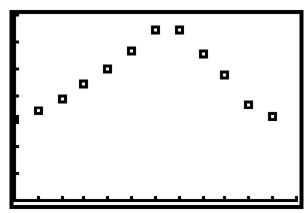


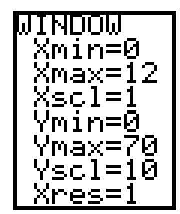
Since temperatures are tied to the seasons, and seasons are tied to the orbit of the earth, and the orbit of the earth is elliptical...well, that makes it a prime candidate for a periodic function! Here is a listing of average annual high temperatures for Bend OR: Data from http://www.city-data.com/city/Bend-Oregon.html



Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Avg High Temp	38	42	46	54	61	68	77	83	77	67	55	43

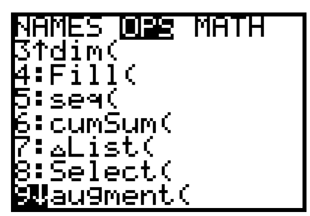
Start by constructing a scatter plot of this data in your TI. To make the data quantitative, let January = 0, February = 1, and so forth, until December = 11. *See List Shortcuts below.*





Any window is fine but if you set your π to these dimensions your graph should match.

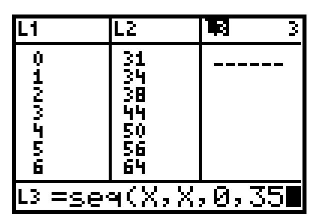
The data could be sinusoidal! Suppose we want to see three cycles. That's a lot of data entry even for Mr. Magic Fingers. Let's take some TI short cuts.



List Shortcuts

Assume L_1 = 0-11 and L_2 = one temp cycle. We want L_3 = 0-35 & L_4 = 3 temp cycles.

Fill a List with a sequence use seq(. In the L_3 header type seq(X, X, 0, 35, 1). This fills the L_3 with a sequence from 0 to 35 in increments of 1. To find 'seq(' use the LIST → OPS menu.



To cycle L_2 three times use augment(which joins two Lists together. In the Home screen type augment(L_2, L_2) ↵ L_4. This joins L_2 with L_2 and stores it in L_4. Then type augment(L_4, L_2) ↵ L_4 to add a third cycle to L_4. To find 'augment(' use the LIST → OPS menu.

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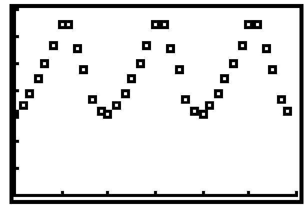
[a] According to scientists, however, this will reverse in 10,500 years. Thus, at that time, the earth will be closer to the sun in June. Further, the North Pole will be tilted toward the sun. You think summer's hot now? Glad we won't be around for **that**. Whew...

[b] The earth tilts at approximately 23.45°from vertical.

You can plot L_3 vs L_4 in the window [0, 36] × [0, 70] to see the three cycles.



Now that we are convinced this is a good candidate for a sinusoidal model let's see if we can model the temperature as a function in the general form T(t) = A sin(bt + c).

Amplitude

Find the amplitude (A-value) of this curve. Think about this for a minute. Using a wave's max & min how do you compute the amplitude? Show how you found A.

½(max − min) = (½)(83 − 38) = 22.5 A = ___22.5____

Period

Find the frequency (b–value) of this curve. Think about this for a minute. This curve repeats every 12 months. Use that fact to b. Show how you found b. *Hint: When t reaches 12, (bt) must begin to repeat.*

$(b)(12) = 2\pi \rightarrow b = \pi/6$ b = ___π/6____

Phase Shift

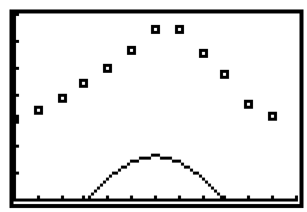
Assume the temp is at it's lowest (minimum) on Jan 1 when t = 0. Use that fact to find the c–value. Show how you found c. *Hint: When t = 0, sin(bt + c) must be at its minimum. When is the sin t at its minimum.*

$\sin(3\pi/2) \rightarrow$ min; $(b)(0) + c = 3\pi/2 \rightarrow c = 3\pi/2$ c = ___3π/2____

Try it out!

Before you go any further, be sure your calculator is set to *radian* ζ. *Why?*

Enter your model into Y_1. Your π should be set to the original size since the data fits in it quite well. Now go ahead and σ it!



π [0, 12] × [0, 70]

Oops! That curve didn't fit well at all! What went wrong?

Perhaps you already had an inkling that something was amiss. Before you read any further, try and deduce what the problem is.

Well, nothing went wrong! The curve actually has the correct shape we just forgot that it's not centered on the x-axis! We need to tweak our curve a little bit, that's all. We need to use some of our old MTH 111 translations (shifts) to *lift it upward*.

Vertical Translation

Remember that the sine and cosine curves are normally centered on the ***x*** – axis. That is, they oscillate at a fixed distance above and below the x-axis. So, *T(t) = A sin(bt + c)* will be centered around the x-axis too. The graph of our current model (Y_1) is currently centered on the ***x*** – axis although it shouldn't be! It should be centered higher up. Call that value k. What form must the model have now?

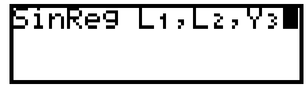
What is the value of k? k = ___________60.5

OK! Adjust your function in Y_1 and re-graph it. If it fits, write your model here: (*keep π in your model*)

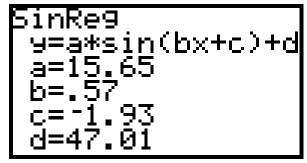
T(t) =

Your TI can also do regression to find what it calls a 'best-fit' curve. The TI regression feature will try to match the form $y(x) = a \sin(bt + c) + d$. Let's do this now, and compare its best-fit curve with our model.

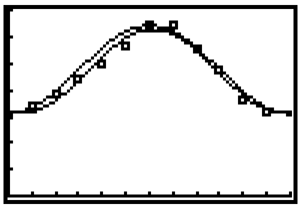
(1) Press □ → CALC and select the SinReg command (*choice* **C:**).



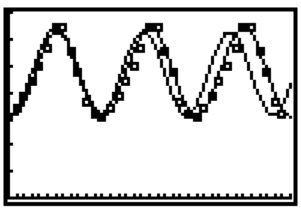
(2) Let's assume your original data is in L_1 and L_2. To save the results in the o list include a Y-variable in the arguments. The Y-variable is found using □. Be careful not to overwrite your model in Y1.



Now, take a look at how this σ looks with our data.



Graph both your model and the regression model? Which appears to be the better model? *Why?*

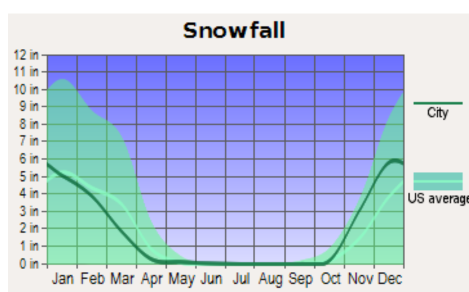


Now change your π and STAT PLOT to see all three cycles. You do not need to change the functions (models). Graph both your model and the regression model? Which appears to be the better model now? *Why?*

Why does the TI's regression function become skewed as it progresses while our mathematical model stays consistently in line with our data?

Find the equivalent mathematical model using a *cosine function* for the same data. *Hint: You've done most of the work already.*

Would a sine function make a reasonable mathematical model for average **monthly** snowfall levels? *Why/Why not?*



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Source http://www.city-data.com/city/Bend-Oregon.html