Building Linear Models *Franz Helfenstein* **Name**

Building a linear model can be done from (a) data or from (b) rate relationships.

Building a Linear Model Based on Data

Creating the linear model is a 4 step process.

(1) Decide which variable is the independent variable (x) and which is the dependent variable (y). One way to decide is to complete the sentence: _______ depends on _______. If this sentence makes sense while the reverse does not seem to make sense then the variables have been identified.

(2) Plot the data. If you have many points and they appear to form a straight line then a linear model is a good choice. If the points form a curve then a linear model is not a good choice.

(3) Find the equation of the line.

(a) If you have just two data points, use $m = \frac{y_2 - y_1}{x_2 - x_1}$, $b = y_0 - mx_0$. Then $y = mx + b$.

(b) When you have more than two data points, use Regression on your Calculator to find y = mx + b. An alternative to regression is to draw a line that approximates your data and use method (a) with any two points on the line.

(4) Check the viability by using the model to predict values at unknown coordinates. If these values are unrealistic, the model must be refined with new data.

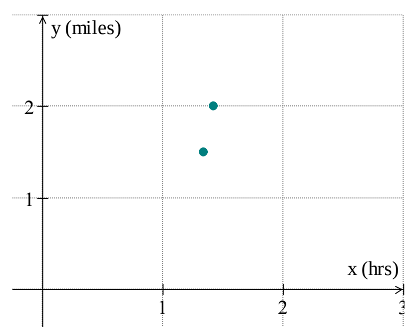
Example

Suppose a runner passes the 1.5 mi mark at 1:20:17 and she passes the 2.0 mi mark at 1:25:03. We know the distance covered depends on the time a racer has been running. Thus time is independent (x) and distance is dependent (y).

Before we can plot the points, it would be convenient to convert Hr:Min:Sec to decimal time. We can use the formula:

$$\text{Decimal Time} = \text{Hr} + \text{Min}/60 + \text{Sec}/60^2$$

This gives (1.33806, 1.5) & (1.41750, 2.0)



Now we compute the linear model.

$m = \frac{2.0 - 1.5}{1.41750 - 1.33806} \approx 6.29$ mph, $b = 2 - (6.29)(1.4175) \approx -6.92$ mi $\boxed{y = 6.29x - 6.92}$

Apply the model to test its viability. To determine when the race started solve for y = 0. 0 = 6.29x – 6.92. x ≈ 1.100 ≈ 1:06:00.

Assuming the runner keeps a steady pace, she should finish the Marathon (26.2 mi) when y = 26.2.

Solve 26.2 = 6.29x – 6.92. x ≈ 5.2655 ≈ 5:15:56.

Her race time is calculated as FinishTime – StartTime = 5:15:56 – 1:06:00 = 4:09:56

Building a Linear Model Based on Rate Relationships

Creating the linear model is a 4 step process.

(1) The slope (m) should be a known rate. e.g. m = mph, fps, gpm, cfs, rpm, $/hr, $/lb, candies/bag, etc.

(2) Determine the independent and dependent variables. $m = \frac{\text{y-units}}{\text{x-units}}$. Thus if m = 500 gpm = 500 $\frac{\text{gal}}{\text{min}}$ then x = min, y = gal.

(3) b must be known and in y-units. b is the value when x = 0. Thus, if m = 500 gpm then b is the number of gallons at time zero.

(4) Then y = mx + b.

(5) Check the viability by using the model to predict values at unknown coordinates. If these values are unrealistic, the model must be refined with new data.

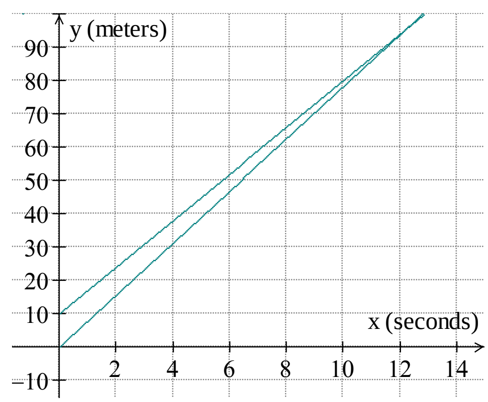
Example 1

Suppose two friends decide to have a race. Jen can run 100m in 12.8 sec and Beth can run 100m in 14.3 sec. Thus, Jen's speed is m_J = 100m/12.8sec ≈ 7.81 mps and Beth's speed is m_B = 100m/14.3sec ≈ 6.99 mps.

$m = \frac{\text{y-units}}{\text{x-units}} = \frac{\text{meters}}{\text{seconds}}$ so x = seconds and y = meters.

Since Jen is faster, she gives Beth a 10m head start. Thus b for Jen is 0m and b for Beth is +10m.

Hence: J = 7.81x, B = 6.99x + 10



We plot the results to view the outcome of the race. Visually, it's too close to call. We must use mathematics to find the actual winner.

If the lines were graphed on a TI we could use the CALC menu to see if the lines intersect prior to 100m.

On the other hand, we can use mathematics to solve for x when y is 100m.

100 = 7.81x ➔ x = 12.80 sec. 100 = 6.99x + 10 ➔ 12.88 sec. Thus, Jen wins by a nose.

Example 2

Suppose per unit cost for souvenirs is $2.75 and overhead is fixed at $200. Then, m = $2.75 = $\frac{\text{y-units}}{\text{x-units}} = \frac{\$}{\text{units}}$. So x = units produced and y = total cost of production.

y = 2.75x + 200