Activity Non-numeric Quadratics *Franz Helfenstein* Name

Often we want to apply the quadratic formula to solve an equation with non-numeric parameters.

Process

- Shift all the terms to one side
- Rewrite in the form $ax^2 + bx + c = 0$
- Identify [a], [b], [c]
- Apply the Quadratic Formula

When $ax^2 + bx + c = 0$ then,

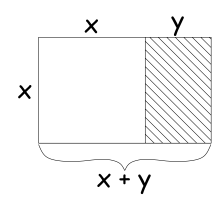
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1) The **Falling Body Equation** $H = -\frac{g}{2}t^2 + v_0 t + h_0$ predicts the height of a free-falling object as a function of time where, H = height at time t with t = sec. g = acceleration due to gravity, v_0 = initial velocity, h_0 = initial height.

(a) Solve this equation for t.

(b) Determine the time it takes a ball to drop 1600′ when g = 32 ft/sec^2 and $v_0 = 0$.

2) A rectangle is a **Golden Rectangle** if after a square is cut away, the remaining rectangle has the same length to width ratio. That is, is obeys the *proportion* $\frac{x+y}{x} = \frac{x}{y}$. Solve this equation for x.

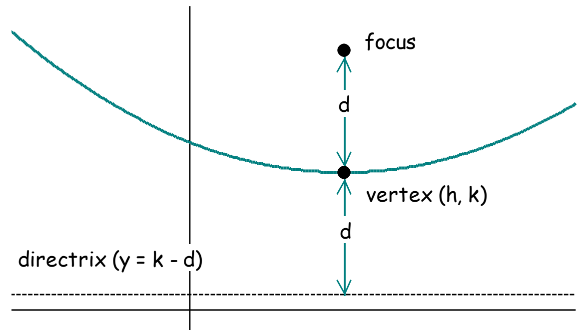


3) As far back as 2000 BC, Babylonian mathematicians were interested in solving the 2 × 2 system of equations $x + y = p,\ xy = q$. Use substitution to eliminate y and obtain a quadratic in x. Solve this equation for x.

4) The geometry of a parabola is special in that it focuses parallel beams of light or radiation toward a single point. The form of this parabola is given by

$$y = \frac{1}{4d}(x - h)^2 + k$$

Solve this equation for x

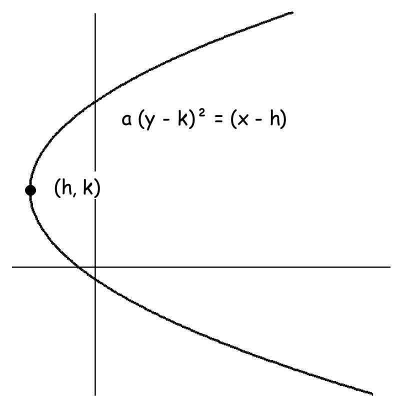


5) The geometry of a parabola is special in that it focuses parallel beams of light or radiation toward a single point. The form of this parabola is given by

$$a(y - k)^2 = (x - h)$$

(a) Solve this equation for y

(b) Enter your solutions into Y_1 & Y_2. Then set a = 5, (h, k) = (-5, 4).
Graph in the standard window and sketch your result.



6) Perhaps you saw some of the damage caused by Hurricane Sandy. Roofs/cars/boats are 'lifted' off due to severe pressure gradients much like an airplane gets its lift. Assuming the pressure (P) depends upon the wind velocity (v) as a quadratic gives us: $P = av^2 + bv + c$. Use the following data and Quadratic Regression to find the function P. Then determine the value of v which will yield a pressure of 25 lb/sq-ft.

Wind Speed (mph)	Pressure (psf)
10	2
20	3
30	4.5
40	6.5
50	10

7) $x^2 - 4xy + 4y^2 + 10x = 30$ is a rotated parabola. Solve for y and enter into Y_1 & Y_2 to see its graph.

Activity Non-numeric Quadratics *Franz Helfenstein* Name **ANSWER KEY**

Often we want to apply the quadratic formula to solve an equation with non-numeric parameters.

Process

- Shift all the terms to one side
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1) The **Falling Body Equation** $\boxed{H = -\frac{g}{2}t^2 + v_0 t + h_0}$ predicts the height of a free-falling object as a function of time where, H = height at time t with t = sec. g = acceleration due to gravity, v_0 = initial velocity, h_0 = initial height.

(a) Solve this equation for t.

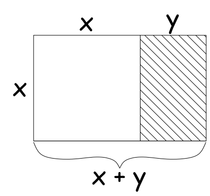
$[-g]\,t^2 + [2v_0]\,t + [2h_0 - 2H] = 0$ $A = -g,\ B = 2v_0,\ C = 2h_0 - 2H$

$$t = \frac{-2v_0 \pm \sqrt{(2v_0)^2 - (4)(-g)(2h_0 - 2H)}}{(2)(-g)} = \frac{v_0 \pm \sqrt{v_0^2 + g(2h_0 - 2H)}}{g}$$

(b) Determine the time it takes a ball to drop 1600′ when $g = 32$ ft/sec^2 and $v_0 = 0$.

$$t = \frac{\sqrt{32(2(1600))}}{32} = 10 \text{ sec}$$

2) A rectangle is a **Golden Rectangle** if after a square is cut away, the remaining rectangle has the same length to width ratio. That is, is obeys the *proportion* $\frac{x+y}{x} = \frac{x}{y}$. Solve this equation for x.



$xy + y^2 = x^2$ $[1]\,x^2 + [-y]\,x + [-y^2] = 0$ $A = 1,\ B = -y,\ C = -y^2$

$$a = \frac{- -y \pm \sqrt{(-y)^2 - (4)(1)(-y^2)}}{(2)(1)} = \frac{y \pm \sqrt{5y^2}}{2} = \frac{1 + \sqrt{5}}{2}\,y \text{ only since } y > 0!$$

3) As far back as 2000 BC, Babylonian mathematicians were interested in solving the 2 × 2 system of equations $\boxed{x + y = p,\ xy = q}$. Use substitution to eliminate y and obtain a quadratic in x. Solve this equation for x.

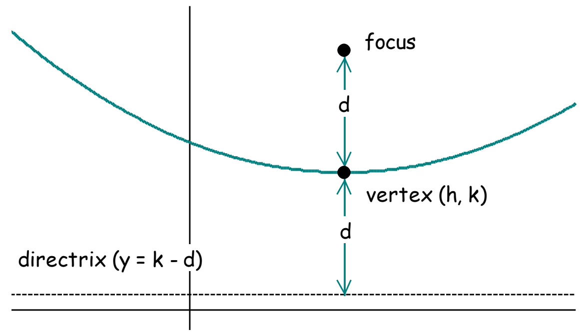
$y = p - x \quad xy = x(p - x) = q$ $[1]\,x^2 + [-p]\,x + [q] = 0$ $A = 1,\ B = -p,\ C = q$

$$x = \frac{- -p \pm \sqrt{(-p)^2 - (4)(1)(q)}}{(2)(1)} = \frac{p \pm \sqrt{p^2 - 4q}}{2}$$

4) The geometry of a parabola is special in that it focuses parallel beams of light or radiation toward a single point. The form of this parabola is given by

$$y = \frac{1}{4d}(x - h)^2 + k$$

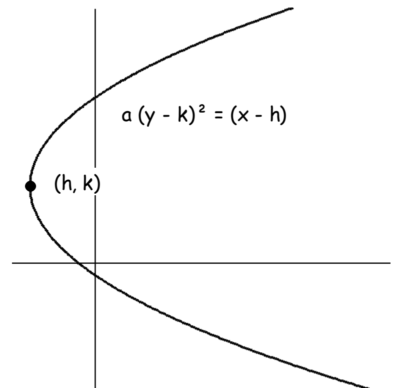
Solve this equation for x



$4d(y - k) = (x - h)^2$ $x = h \pm \sqrt{4d(y - k)}$

5) The geometry of a parabola is special in that it focuses parallel beams of light or radiation toward a single point. The form of this parabola is given by

$$a(y - k)^2 = (x - h)$$

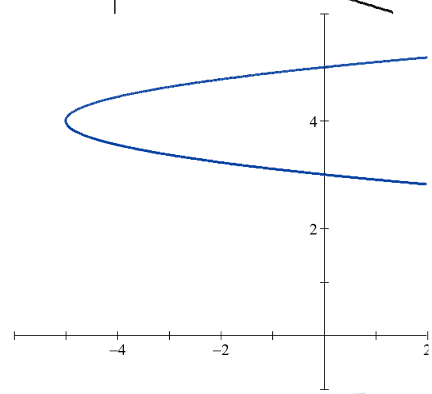


(a) Solve this equation for y. $y = k \pm \sqrt{\frac{x - h}{a}}$

(b) Enter your solutions into Y_1 & Y_2. Then set a = 5, (h, k) = (-5, 4).
Graph in the standard window and sketch your result.

$Y1 = 4 + \sqrt{\frac{x + 5}{5}}$

$Y2 = 4 - \sqrt{\frac{x + 5}{5}}$



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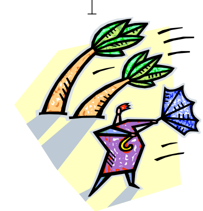

$P \approx 0.00393v^2 - 0.0407v + 2.1$ P(82 mph) ≈ 25 psi

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7) $x^2 - 4xy + 4y^2 + 10x = 30$ is a rotated parabola. Solve for y and enter into Y_1 & Y_2 to see its graph.

$[4]\,y^2 + [-4x]\,y + [x^2 + 10x - 30] = 0 \qquad A = 4,\ B = -4x,\ C = x^2 + 10x - 30$

$$y = \frac{- -4x \pm \sqrt{(-4x)^2 - (4)(4)(x^2 + 10x - 30)}}{(2)(4)} = \frac{x \pm \sqrt{-10x + 30}}{2}$$

$$Y1 = \frac{x + \sqrt{-10x + 30}}{2}$$

$$Y2 = \frac{x - \sqrt{-10x + 30}}{2}$$