**Project: Saving money!**

“An investment in knowledge pays the best interest.” (Benjamin Franklin)

Ah, money...we need to deal with it in our everyday lives, don’t we? Those little dollars and cents represent so much: food, clothes, shelter...in many cases, basic needs. So – how do we make sure we have enough?

In class we’ve touched on exponential growth in various contexts, and we’ve even talked about financial exponential growth a little. Exponential growth makes sense with saving money, because the money often grows as a percent of what’s actually in the account at a given time. However, basic compounding interest (the kind of stuff we’ve been doing with inflation and savings accounts and such) only takes you so far – and the purpose of this project is to go farther!

Here’s an example of what I mean: an example from a former student’s experience: This student had $7500 in school loans. Their loans carried an interest rate of 8%, compounded annually (once at the end of each year). This student, being diligent, started saving $45 each month in a non-interest bearing checking account, and then, at the end of each year, paid $540 (12 times $45) toward their student loan.

So, you can see right away that this kind of situation doesn’t lend itself to our basic compounding interest formula...the amount of money in the account is changing, and not just due to the interest-he’s also paying a bit of money toward the debt each year, to aid in paying it off.

Let’s analyze, shall we?

<table>
<thead>
<tr>
<th>Year</th>
<th>Amount my Student Paid Toward the Debt Amount</th>
<th>Debt Amount At the End of the Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$0</td>
<td>$7,500</td>
</tr>
<tr>
<td>1</td>
<td>$540</td>
<td>$7,560</td>
</tr>
<tr>
<td>2</td>
<td>$540</td>
<td>$7,625</td>
</tr>
<tr>
<td>3</td>
<td>$540</td>
<td>$7,695</td>
</tr>
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<td>4</td>
<td>$540</td>
<td>$7,770</td>
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<td>$540</td>
<td>$7,852</td>
</tr>
<tr>
<td>6</td>
<td>$540</td>
<td>$7,940</td>
</tr>
<tr>
<td>7</td>
<td>$540</td>
<td>$8,035</td>
</tr>
<tr>
<td>8</td>
<td>$540</td>
<td>$8,138</td>
</tr>
<tr>
<td>9</td>
<td>$540</td>
<td>$8,249</td>
</tr>
<tr>
<td>10</td>
<td>$540</td>
<td>$8,369</td>
</tr>
</tbody>
</table>

If you take a look at the formula bar in that screen capture, you’ll see how the right hand column (that is, the amount of debt left in the account) is calculated:

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*An intentionally loaded question: I can’t answer this for you, since everyone is different. But, there are a few mathematical rules that apply to savings, and that’s what this project is all about.*
Excel multiplies 1.08 by the previous debt remainder (that is, it carries the debt forward and adds 8% to it), and then subtracts the $540 that my student have saved for the year from that total. That formula is then copied down for ten years, applied the same way each year.

See how the amount of the debt keeps getting bigger? You may have heard the phrase (often applied to credit card debt) of “not covering even the interest on a loan”. That’s what was going to happen to my student! You see, if you have a debt that’s accruing interest, there’s one of a number of things you can do:

- Ignore it. Personally, *I* wouldn’t do this. 😊
- Pay too little on the debt. That’s not covering the interest. That means that, sure, you’re paying money towards it – but you’re not actually decreasing it; it’s just increasing more *slowly*.
- Pay off only the interest. I call this the “break even” point of payment. The problem, of course, is you never pay off your original debt – but it also doesn’t increase past that original amount.

To help my student figure his debt more effectively, we calculated his “break even” point. That is, we figured out what he would have to save each month (and pay off at the end of each year) to cover *just* the interest for the year.

1. **(2 points)** Calculate the *monthly* amount he’d have to save to “break even” *after year 1*. In other words, assume he just took out the loan for $7500, and wants to save money over the first year to pay off *just the interest* for the first year. Remember that he was saving $45 per month, and that wasn’t enough. You can use the spreadsheet I took a screen capture of above: it’s linked with the project. If you change the amount in cell B3, it’ll autofill into each year.

   Thankfully, we caught it after year 3, so he had to pay a bit more each month (since the interest had accrued over the previous 3 years). **BUT!** There’s an even better way to approach debt that accrues interest! Pay off *more* than just the interest! Let’s do that next!

   In the previous example, we were paying against a loan that was (well, *should* have been) going down over time. Mortgages are another example of this; you pay a bit each month, and, eventually, you pay off the entire house value debt. But it works the other way, too!

   You can *invest* a certain sum of money at a certain interest rate, and then, instead of just letting it sit there, you can regularly contribute *more* to it! In other words, the account is earning interest on the principal, and that principal is getting bigger due to the interest, but it’s also getting bigger due to the fact that you’re putting extra money into it! These special kinds of accounts are called “annuities”. Here’s an example of one that my dad (“Pop Pop”) started for our son:


<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>=1.08</td>
<td>C2-B3</td>
</tr>
</tbody>
</table>

**Amount my Student Paid** - **Debt Amount At the End**
Suppose Pop-Pop invests $500 in a savings account paying 6% interest per year (this is back when you could get an interest rate like that. 😊). Because it’s an annuity, he must also deposit $500 each year for the life of the annuity. He plans on leaving that money in that account for 20 years. How much will that account be worth, at the end of 20 years? Assume the account only compounds once per year, for simplicity.

There are a couple of ways to attack this! I’ll let you pick which way you like: either with Excel, or with algebra! It’s like a Choose Your Own Adventure!

- To use Excel, please watch this video (link if you need it: https://youtu.be/fzIoXkFwiQI). You’ll also want to open the spreadsheet I’ve started for you (right next to the project link on the schedule page).

- To do it with algebra, watch these videos (link if needed: https://www.youtube.com/watch?v=d61o1gnZPOE&list=PLT9lV-YdYZj3SVl63r8gzIgzjLMH2bpY1)

2. (3 points) (w) How much would be in Pop Pop’s account after the 20 years? Round your answer off to the nearest dollar. For the “w” here, if you used Excel, please take a screenshot of your spreadsheet and include it, and if you did algebra, show me the math you did!

How badass is that! He invested a total of $10,000 ($500, 20 times), and got, essentially, twice that out! Just to compare...if you had placed that $500 into a compound savings account paying 6% yearly and left it there, you’d have the paltry sum of $1600 after 20 years.

Note: some formulas of this type call the interest rate “r”. In that case, the formula would look a little different (I’ve been using “1.06” as the “r”). Just pay attention to what’s being used! #readingisfundamental

Let’s try two more!

3. (5 points) (w) You figure, in 5 years, you’ll need to have a sum of $10,000 for expenses. How much should you regularly deposit into an annuity paying 5% per year (compounded annually) so that you’ll have the necessary $10,000 after 5 years? For the “work for this one, either supply a screenshot of the spreadsheet you used, or show me your arithmetic! Either way’s fine – whichever you like better!

4. (5 points) (w) Same situation, but now you do a little more future planning. Suppose you need $10,000 in ten years. How much should you regularly deposit into an annuity paying 5% per year (compounded annually) so that you’ll have the necessary $10,000 after 10 years? Same notes about (w)ork. b

5. (5 points) Please explain why you chose the method you chose (that is, either algebra or Excel) in at least one sentence!

b The point of this last problem is more than just an exercise in math...I want you to think about how important it is for you to start saving NOW for future events. Even if you only put a little bit of money aside, start doing it. Keep it up. Get in the habit of saving, for it will help you in the future.
6. **(5 points) (w)** Suppose Pop Pop wanted to invest in not an annuity, but a compounding bank account (that is, he puts $500 in when Max is born, and then forgets about it). What interest rate would he need to earn over the 20 years to get $500 to grow to $19,500?

Not an unheard-of rate...but certainly not attainable from a bank account. You’d need mutual funds, or stocks...and that’s beyond the scope of this project. 😊 The only downside of an annuity is that you have to **make** the annuity payment each year. That’s where responsible saving comes in.

You may have heard the term “annuity” before. The most important, I think, are home mortgages. This is where you borrow a lump sum of money, and then pay it back in installment (usually monthly) until the mortgage is paid off. As you hold the mortgage, though, the debt accrues interest. However, the math behind mortgages is really, really complex. Here are a couple of reasons:

- You can “refinance” your mortgage, meaning you can pay to change the interest rate charged against your account.
- When you make a mortgage payment, your debt is lessened. However, there’s “principal” debt and “interest” debt. It seems a little voodoo, but basically, at first, you’re only paying off the interest.
- You can save more than the annuity amount, which makes the repayment schedule change.

Because of these (and even more) reasons, we’ll leave annuities as they are for now and call it good. You’ve got the big idea!