A few years ago, I was observing a colleague’s writing class at COCC. Near the end of the class, she told her students a story from her youth: How, as a senior year in high school, her dad had told her that he would buy her a car for graduation if she got straight A’s her senior year. She began to weave a beautiful tale: of late nights studying for finals, practicing speeches, notecards...the works. I was riveted, as was every student in the room. Then, at the end, she said, “And, in the end, I got all A’s...and a B+.” And all the students in the room groaned. “No! That sucks!!”

After the grinding and gnashing of teeth had subsided, I called from the back, “Did he buy you the car?”

A student turned around and said, “Didn’t you hear? She just said her dad would buy her the car if she got straight A’s.”

To which I replied, “Yes, but he didn’t say what he would do if she didn’t get straight A’s.”

At this point, the students in the room all looked at me like I was this ignorant, unfeeling jerk. But my colleague smiled at me, because she knew I had a point (that she wished she had thought of during her senior year).

But my point was just applying one of the laws of logic. Let’s dive in!

**Part 1: Logical Statements!**

My colleague’s dad had made a **logical statement** when he said, “If you get straight A’s, then I’ll buy you a car.” A logical statement is simply any sentence that’s either true or false, but not both. Here are some examples:

<table>
<thead>
<tr>
<th>Logical Statements!</th>
<th>Not Logical Statements!</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>True Ones!</strong></td>
<td><strong>False Ones!</strong></td>
</tr>
<tr>
<td>• Putting gas into a diesel engine will ruin it.</td>
<td>• Cats are better than fish.</td>
</tr>
<tr>
<td>• Metolius Hall is a building at COCC.</td>
<td>• There are too many humans.</td>
</tr>
<tr>
<td>• If you play piano, then you play a percussion instrument.</td>
<td>• If my car’s engine isn’t running, then the car is out of gas.</td>
</tr>
<tr>
<td></td>
<td>• That engine in your car is pretty loud.</td>
</tr>
<tr>
<td></td>
<td>• I paid too much for that LP.</td>
</tr>
</tbody>
</table>

At the foundations of logic lie, quite simply, logical statements\(^1\). There’s no room for opinion in logic – only truth or falsehood.

Got it? Good! Let’s **do** stuff with this stuff!

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\(^1\) Please realize that what we sometimes conversationally call “logical” isn’t necessarily universally, nor mathematically logical. For example, when I climb mountains, my partners and I agree on logical anchor systems – while many people think we’re completely and utterly illogical for traveling tens of thousands of feet up exfoliating slag heaps.
Part 2: Truth or Falsehood of Statements!

Part 2a: Falsehood!

Any logical statement, by definition, is either true or false. As we will see, showing that a statement is true can be slightly involved (just like in life!). However, showing a statement is false is straightforward: you simply must provide one example that shows it doesn’t hold – and that example is called a counterexample. Let’s illustrate by finding counterexamples for some of the false logical statements from page 1!

Example: Cats are members of the fish family.

Our family has a cat. His name is Mercury. He lacks gills, fins, and lives on land. Therefore, by the definition of what makes fish fish, he can’t be a fish. Therefore, we call this statement false, since I found an exception.

Example: If my car’s engine isn’t running, then the car is out of gas.

Well, that’s certainly one explanation. But here’s another: suppose the car is turned off. Then, the engine wouldn’t be running. So, therefore, we found a counterexample to show that this is a false statement!

The thing to keep in mind here is that you only need one counterexample to prove a statement false. Think of it like this: when you hear someone say something like, “Well, I had an uncle who smoked every day until he was 103…smoking can’t be bad for you!” All I need to say is, “Welp, I had a friend who died at 22 from smoking – related emphysema. So yeah…your theory isn’t always true.”

“True for some” isn’t good enough to prove logical statements…you must achieve “true for all”. More on that next!

Part 2b: Truth of Logical Statements!

Formal logic is often assembled symbolically; this makes it easier to see patterns when you’re working within the rules of logic, which’ll make it WAY easier to get at this idea of proving logical statements true. For example, let’s create a few more statements – these’ll be of the “if/then” flavor, as that’s where we’ll spend most of our time:

- If we pay our mortgage each month, then our bank will stay happy.
- If I continue to bike commute, then I’m doing a little bit extra to be a good environmental steward.
- If my fishing flies make it through TSA from the US to Mexico, then they’ll make it back through TSA from Mexico to the US².

In each of these, I’ve color–coded the conditions (that is, the part that follows “if”) in green and the conclusions (the parts that follow “then”) in red. Each of those color coded parts is either true or false (for example, I either will or won’t continue to bike commute, we either will or won’t continue to pay our mortgage, etc.).

Now, what’s rad about talking logic formally is that, in logic’s eyes, all of those statements are the same! To logic, it’s all about structure, not content – and all of those statements are of the form “if condition, then conclusion.” Many times, folks who study logic formally call the condition “p” and the conclusion “q”, and that makes it even easier – because then every conditional statement looks like this:

² Seems reasonable to assume…until they take them all from you.
"If $p$, then $q$."

Each of those types of statements ("if $p$, then $q$") is called a **conditional statement**. That means that there’s a condition placed at the beginning, and then a conclusion placed at the end ("if this happens, then that will happen").

Let’s go back to my friend’s dad’s conditional statement (the one that started this whole mess):

"If you get straight A’s, then I’ll buy you a car."

1. **(2 points; 1 point each)** Identify $p$ and $q$ in this conditional!

   Now, each of $p$ and $q$ could be either true or false. So, in other words, we have 4 total potential outcomes for this statement!

<table>
<thead>
<tr>
<th>If $p$ is...</th>
<th>...and $q$ is...</th>
</tr>
</thead>
<tbody>
<tr>
<td>True</td>
<td>True</td>
</tr>
<tr>
<td>True</td>
<td>False</td>
</tr>
<tr>
<td>False</td>
<td>True</td>
</tr>
<tr>
<td>False</td>
<td>False</td>
</tr>
</tbody>
</table>

   Now – let’s look at all 4 possible outcomes in the context of my friend’s dad’s statement, and discuss them in turn!

   - **Possibility 1 ($p$ and $q$ both true):** if she got straight A’s, and then he bought her a car. This is good, right? I mean, he told her, straight up, “If you get straight A’s, then I’ll buy you a car.” If this scenario had happened, she satisfied the condition, he satisfied the conclusion, end of story. Since this is a “fair” outcome of the conditional, we call it, logically, **true**.

   - **Possibility 2 ($p$ true but $q$ false):** If she got straight A’s, and then he **didn’t** buy her a car. This is bad! He told her that, if she did get straight A’s, he’d buy her a car. And then he didn’t! If this scenario had happened, she satisfied her end of the deal, he failed to satisfy his, and, most likely, she’d be upset. You would, too! Since this one doesn’t seem fair, we call this outcome logically **false**.

   - **Possibility 3 ($p$ false but $q$ true):** If she **didn’t** get straight A’s, and then he **did** buy her a car. This outcome, although potentially confusing, can’t be deemed “unfair”. He told her what he would do if she got straight A’s…not what he would do if she **didn’t**. Since this one isn’t unfair, we have no logical choice to but to call it **true**!³

   - **Possibility 4 ($p$ and $q$ both false):** If she **didn’t** get straight A’s, and then he **didn’t** buy her a car. This outcome is, essentially, what the students had automatically assigned in their minds when they heard my colleague **hadn’t** gotten straight A’s. This one feels fair, only because it (most likely) makes sense. So, we’ll call it **true**.

   The best part is that **any** conditional statement can be analyzed in this way, and any conditional statement, logically, has to have the same set of outcomes! Once you hear an “if, then” statement, identify the condition and conclusion, and then you can synopsize all possible outcomes of the statement in what’s called a **truth table**:

³ This might seem confusing, but the only condition placed on this whole agreement was “If you get straight A’s.” He didn’t say “If AND ONLY IF you get straight A’s”… we’ll look at **that** in a little bit!
If the condition is... ...and the conclusion is... ...then the overall statement is...

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>TRUE</td>
<td>TRUE</td>
<td>TRUE</td>
</tr>
<tr>
<td>TRUE</td>
<td>FALSE</td>
<td>FALSE</td>
</tr>
<tr>
<td>FALSE</td>
<td>TRUE</td>
<td>TRUE</td>
</tr>
<tr>
<td>FALSE</td>
<td>FALSE</td>
<td>TRUE</td>
</tr>
</tbody>
</table>

Here’s one you might hear around your house!

**Example:** Take the statement “If you do the dishes, then I’ll take out the trash.”

- **Possibility 1:** You did the dishes, and I took out the trash. Yep – that seems fair. And, logic tells us that it’s a true application of the conditional, so yay! The conclusion follows from the hypothesis.

- **Possibility 2:** You did the dishes, but I didn’t take out the trash. Totally lame! And, as we saw above, a false application of the conditional, as the true condition leads to a false conclusion.

- **Possibility 3:** You didn’t do the dishes, but I did take out the trash. No one can complain, right? I only specified what I would do if you did the dishes...not what I would (or wouldn’t) do if you didn’t. Gotta call it true.

- **Possibility 4:** You didn’t do the dishes, and I didn’t take out the trash. Again, this one’s an “all bets are off” type, since the hypothesis wasn’t met. So, again, we have to call it true.

Since all conditionals follow the same line of logic, their truth tables all look the same. And, since they get used so much, the condition (also sometimes called a “hypothesis”) is generally called “p”, the conclusion “q”, and the table ends up looking like this:

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>If p, then q?</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
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<td>F</td>
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<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

So in examining the truth of any conditional statement, we must always conclude that it is true – unless the hypothesis is true and the conclusion is false! Let’s analyze one from Oregon’s constitution – our state constitution has a tax rebate built into it (commonly known as a “kicker”).

2. **(3 points)** In a sentence, tell me what the kicker does. Google it up!

3. **(3 points)** Rewrite the above sentence as a conditional statement (since the kicker needs to satisfy a condition to be met, it seems the perfect candidate!).

4. **(3 points)** What is the only time the above condition would be a false statement (and, most likely, really aggravate those in Oregon)?
Part 3: Other types of Logical Operators!

The “if/then” conditional; is often called a logical operator, as it does something to the variables inserted into it (in the case of “if \( p \), then \( q \)”, it inputs all the possible cases of the variables and outputs the truth of the overall statement:

\[
\begin{array}{c|c|c}
   p & q & \text{If } p, \text{ then } q? \\
   \hline
   T & T & T \\
   T & F & F \\
   F & T & T \\
   F & F & T \\
\end{array}
\]

But in the world of logic, there are other operators, as well! For example, you may have felt a little perplexed when you read possibility 3 above:

Now – let’s look at all 4 possible outcomes in the context of my friend’s dad’s statement!

- **Possibility 1 (\( p \) and \( q \) both true):** She got straight A’s, and then he bought her a car. This is good, right? I mean, he told her, straight up, “If you get straight A’s, then I’ll buy you a car.” If this scenario had happened, she satisfied the condition, he satisfied the conclusion, end of story. Since this is a “fair” outcome of the conditional, we call it, logically, true.

- **Possibility 2 (\( p \) true but \( q \) false):** She got straight A’s, and then he didn’t buy her a car. This is bad! He told her that, if she did get straight A’s, he’d buy her a car. And then he didn’t! If this scenario had happened, she satisfied her end of the deal, he failed to satisfy his, and, most likely, she’d be upset. You would, too! Since this one doesn’t seem like we call this outcome logically false.

- **Possibility 3 (\( p \) false but \( q \) true):** She didn’t get straight A’s, and then he did buy her a car. This outcome, although potentially confusing, can’t be deemed “unfair”. He told her what he would do if she did get straight A’s, not what he would do if she didn’t. Since this one isn’t unfair, we have no logical choice but to call it true!

- **Possibility 4 (\( p \) and \( q \) both false):** She didn’t get straight A’s, and then he didn’t buy her a car. This outcome is, essentially, what the students had automatically assigned in their minds when they heard my colleague hadn’t gotten straight A’s. This one feels fair, only because it (most likely) makes sense. So, we’ll call it true.

What made you uncomfortable, most likely, was that Possibility 3 seemed...off. The other three likely made sense to you, but that one might have seemed weird. I mean, why would he buy her a car if she didn’t do what he asked her to?

To which I would answer, “Well, he doesn’t have to buy her a car if she failed to get straight A’s...but if he did, it wouldn’t be logically inconsistent with his original deal”.

But I think the real reason is more subtle – it’s because we, as humans, want to make “if” statements stronger than they actually are. We want them to be a stricter version, called **If And Only If**! If her dad’s statement had been an if and only if statement, it would read like this:

“\( \text{If – and ONLY IF } – \) you get straight A’s, I’ll buy you a car.”
And its truth table would look like this:

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>If AND ONLY IF p, then q?</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
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<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

See that third line? See how it’s false now? Yep – that’s because “if and only if” is stronger than just “if”!

So now you’ve seen “if \( p \), then \( q \)” and “if and only if \( p \), then \( q \)”. We’ll continue more with them in the next project, but, for now, let’s learn about some other commonly used logical operators – and use Excel to do it! Start by watching this video: [https://youtu.be/iu2QCm8wm48](https://youtu.be/iu2QCm8wm48)

5. **(2 points)** Please take a screenshot of what you did in Excel and give it as your answer to #5 here!

For these last three questions, I’ll have you look at the far – right columns of each of the truth tables you created. In particular, I want to know what must be true about the individual values of \( p \) and \( q \) in order for each of the three operators to be true overall.

6. **(4 points)** What must be true about the values of \( p \) and \( q \) in order for \( p \) AND \( q \) to be true? Be as specific as you can!

7. **(4 points)** What must be true about the values of \( p \) and \( q \) in order for \( p \) OR \( q \) to be true? Again, be as specific as you can!

8. **(4 points)** What must be true about the values of \( p \) and \( q \) in order for \( p \) XOR \( q \) to be true? #specificasalways