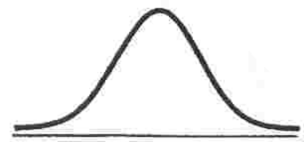
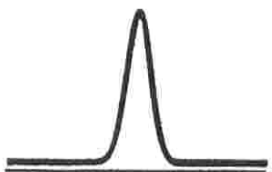
Mighty Central Limit Theorem Exercises

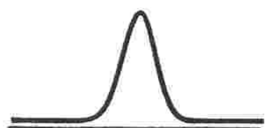
Pages	Suggested Reading
281 – 284	Sections 7.1 and 7.2

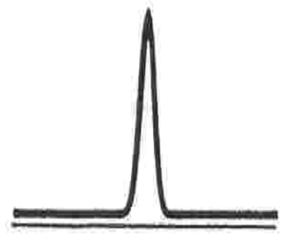
Pages	Problems
297 – 300	(***Section 7.7***) 1 (for this one, give the shape, average, and SE of the sampling distribution. Ignore the rest of what the text asks you), 2, 3(same as 1), 8(a,b,d,e), 10(a,c,e), 14(a,b,d,e)

E1: Samples of different sizes are drawn at random from a population shaped like the one at right. Which sampling distribution below indicates a sample of size 5? Size 10? Size 30?

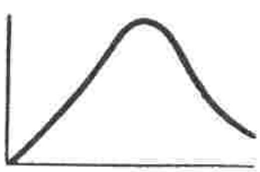


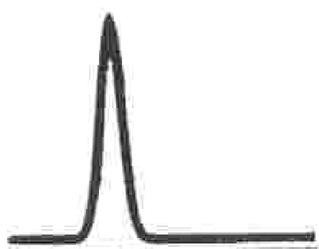
a. 

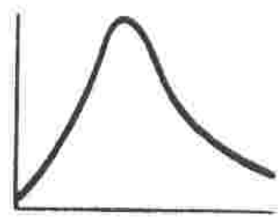
b. 

c. 

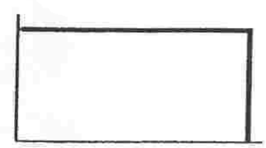
E2: Samples of different sizes are drawn at random from a population shaped like the one at right. Which sampling distribution below indicates a sample of size 5? Size 10? Size 30?

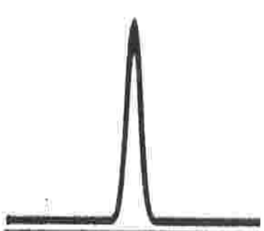
a. 

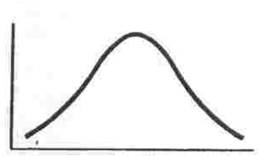
b. 

c. 

E3: Samples of different sizes are drawn at random from a population shaped like the one at right. Which sampling distribution below indicates a sample of size 5? Size 10? Size 30?



a. 

b. 

c. 

E4: Samples of different sizes are drawn at random from a population with the distribution shown at right. What is the mean and standard deviation of this distribution? Which sampling distribution below indicates a sample of size 5? Size 10? Size 30?

x	*P(x)*
0	0.1
1	0.4
2	0.3
3	0.1
4	0.1

a.

$\bar{x}$	$P(\bar{x})$
[0,1)	0.065
[1,2)	0.53
[2,3)	0.36
[3,4)	0.045

b.

$\bar{x}$	$P(\bar{x})$
[0,1)	0.02
[1,2)	0.6
[2,3)	0.37
[3,4)	0.01

c.

$\bar{x}$	$P(\bar{x})$
[0,1)	0
[1,2)	0.8
[2,3)	0.2
[3,4)	0

E5: Suppose a highly right – skewed population has $\mu = 25.2$ and $\sigma = 12$ (ish).

a. If you draw random samples of size 4 from this population, what shape would the sampling distribution of their averages have? What would its approximate average be? Its approximate standard error?
b. If you draw random samples of size 16 from this population, what shape would the sampling distribution of their averages have? What would its approximate average be? Its approximate standard error?
c. If you draw random samples of size 30 from this population, what shape would the sampling distribution of their averages have? What would its approximate average be? Its approximate standard error?

E6: As one final note...it's important to really see (and/or compute) why the CLT is so wonderful. Remember that, in ***any*** normal distribution (including sampling distributions), about 95% of the data collected will lie within 2 standard deviations of the average. Suppose a certain extremely skewed (non – normal) population has $\mu = 14.7$ and $\sigma = 9$ (ish).

a. If you draw random samples of size 10 from this population, what shape would the sampling distribution of their averages have? What would its approximate average be? Its approximate standard error?
b. 95% of the averages in part a. will fall between which two numbers?
c. If you draw random samples of size 100 from this population, what shape would the sampling distribution of their averages have? What would its approximate average be? Its approximate standard error?
d. 95% of the averages in part c. will fall between which two numbers?
e. If you draw random samples of size 1000 from this population, what shape would the sampling distribution of their averages have? What would its approximate average be? Its approximate standard error?
f. 95% of the averages in part e. will fall between which two numbers?
g. Draw the three ranges of values from parts b., d., and f. with a number line, for reference.
h. Based on part g., which sample size is most likely to net you an $\bar{x}$ closest to μ? Why?
i. (**extra credit**) What percentage of the ***data*** would have to be within 2 standard deviations of μ in the original, skewed population?

Answers.

E1: a. ***n*** = 10, b. ***n*** = 5, c. ***n*** = 30
(as sample sizes get larger, standard error goes down, so the curves get narrower)

E2: a. ***n*** = 5, b. ***n*** = 30, c. ***n*** = 10
(notice how a sample of size 5 doesn't quite pull the sampling distribution to symmetry, but 10 does)

E3: a. n = 30, b. n = 5, c. n = 10
(this symmetric distribution is called "uniform"; we'll study it later in MTH 244)

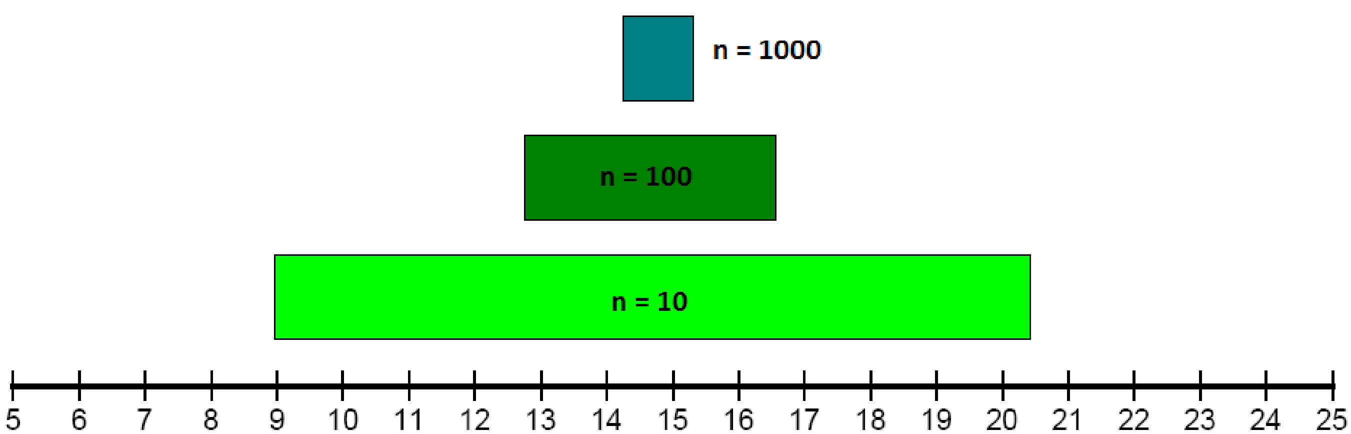
E4: μ = 1.7, σ = 1.1, a. n = 5, b. n = 10, c. n = 30
(a little harder, since you don't see the histogram...you could make the graph, or realize that the standard error is decreasing, as indicated by the "tail" probabilities in the sampling distributions getting rarer and rarer. Also, the category that contains μ, [1,2), is getting more and more of the $\bar{x}$ values in there)

E5: a. bell – shaped (normal), $\bar{x}$ ≈ 25.2, ***SE*** ≈ 6 (since $SE = \frac{\sigma}{\sqrt{n}}$)

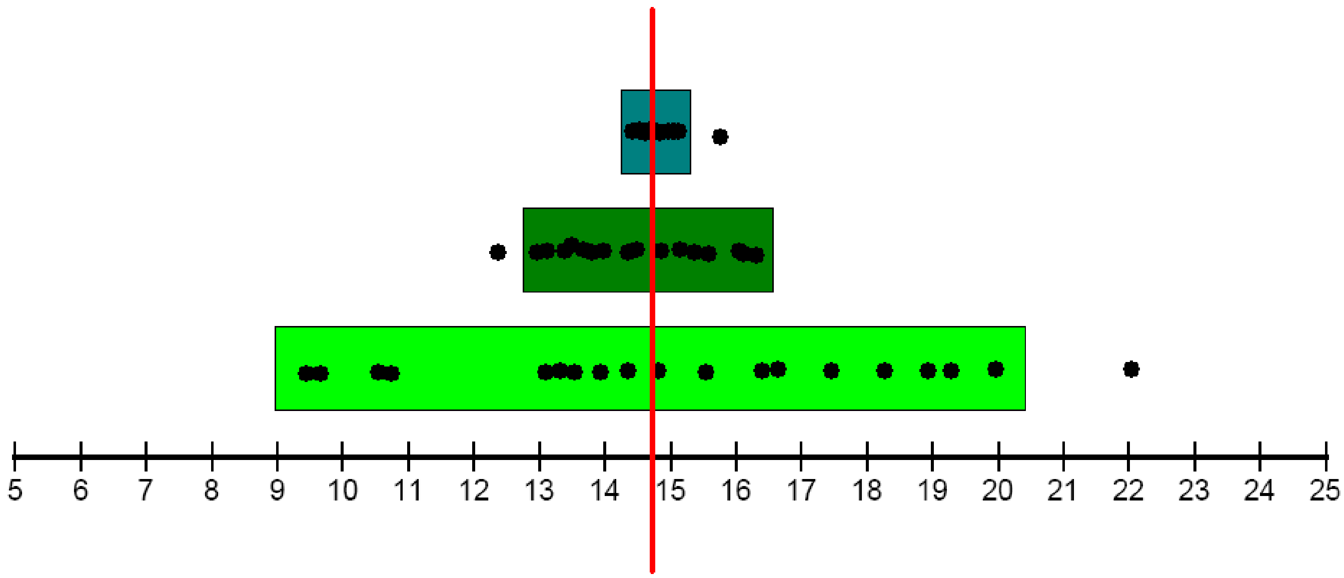
b. bell – shaped, $\bar{x}$ ≈ 25.2, ***SE*** ≈ 3
c. bell – shaped, $\bar{x}$ ≈ 25.2, ***SE*** ≈ 2.2

E6: a. bell – shaped (normal), $\bar{x}$ ≈ 14.7, ***SE*** ≈ 2.85 (since $SE = \frac{\sigma}{\sqrt{n}}$)

b. 9.0 and 20.4 (about 95% of the averages will fall within $\bar{x} \pm 2(\text{SE})$, or $14.7 \pm 2(2.85)$)
c. bell – shaped, $\bar{x}$ ≈ 14.7, ***SE*** ≈ 0.9
d. 12.9 and 16.5
e. bell – shaped, $\bar{x}$ ≈ 14.7, ***SE*** ≈ 0.28
f. 14.1 and 15.3
g.



h. Definitely, the sample size of 1000. As your sample size increases, the CLT tells us that the SE of the average of the samples you draw goes down. Look at part g. again...you see how narrow the bar for n = 1000 is? Remember, however, that 95% (or, about 19 out of 20) of your sample averages are within that bar! Look again, below, to see what I mean:



Each dot represents an $\bar{x}$ gotten from a sample of the three different sizes. Notice how much tighter the $\bar{x}$'s are as your sample sizes go up? That's because the CLT guarantees that the variability in the sample means goes down as the sample sizes go up, which is why the $\bar{x}$'s when ***n*** = 1000 are so clustered around μ (the red line).

i. Harness your inner Russian, and use Chebyshev.

Mighty Central Limit Theorem Quizzes

Quiz 1.

Suppose a highly skewed population has $\mu = 23$ and $\sigma = 4$ (ish). Of course, you don't ***know*** that, so you take a simple random (i.e., representative) sample of size ***n*** = 100 from this population.

a. (**2 points**) Your sample's average is part of a sampling distribution, as you recall. What shape would this sampling distribution of the sample averages have?

b. (**2 points**) What would its approximate average be?

c. (**2 points**) What would its approximate standard error be?

d. (**2 points**) (**w**) Based on your previous three answers, approximately 95% of the averages from the sampling distribution will fall between which two values?

e. (**2 points**) (**w**) Based on your answers to a, b, and c, approximately 99.7% of the averages from the sampling distribution will fall between which two values?