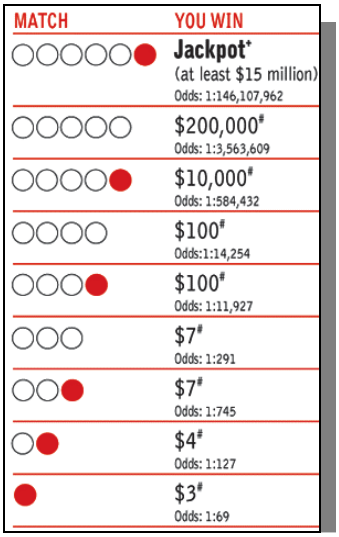
# An analysis of PowerBall[1]

"I consider lotteries to be a tax on the mathematically challenged." – *anon.*

PowerBall is a pretty infectious game all across the country. I agree with the mantra that, "If you don't play, you won't win." However, I think this little breakdown will convince you to put your money elsewhere.




At right is a listing of the payouts for playing basic PowerBall[2]. In case you've never played before, you give the clerk $1, and they give you a ticket. You select 5 numbers, without replacement, from a field of 55, and then one (the "PowerBall") from a field of 42. At right, the white circles represent matching the first 5 numbers, and the red the PowerBall. So, for example, if you match 3 of the first five numbers and the PowerBall, you win $100.

Pay close attention, however, to the "odds[3]" under each outcome. I want to show you how to calculate each one below.

| Outcome of ticket | Probability | Decimal Equivalent |
|---|---|---|
| 5 Numbers plus PowerBall | $\frac{\binom{5}{5}\binom{1}{1}\binom{50}{0}\binom{41}{0}}{\binom{55}{5}\binom{42}{1}} = \frac{1}{146107962}$ | 0.00000007 |
| 5 Numbers w/o PowerBall | $\frac{\binom{5}{5}\binom{1}{0}\binom{50}{0}\binom{41}{1}}{\binom{55}{5}\binom{42}{1}} \approx \frac{1}{3563609}$ | 0.0000003 |
| 4 Numbers plus PowerBall | $\frac{\binom{5}{4}\binom{1}{1}\binom{50}{1}\binom{41}{0}}{\binom{55}{5}\binom{42}{1}} \approx \frac{1}{584432}$ | 0.000002 |
| 4 Numbers w/o PowerBall | $\frac{\binom{5}{4}\binom{1}{0}\binom{50}{1}\binom{41}{1}}{\binom{55}{5}\binom{42}{1}} \approx \frac{1}{14254}$ | 0.00007 |
| 3 Numbers plus PowerBall | $\frac{\binom{5}{3}\binom{1}{1}\binom{50}{2}\binom{41}{0}}{\binom{55}{5}\binom{42}{1}} \approx \frac{1}{11927}$ | 0.00008 |

[1] Graphic taken from http://www.oregonlottery.org/power/p_howto.shtml#howto ; also, the rules of play change slightly over time, so your ticket might not exactly resemble the one I've described.
[2] Not using their "multiplier".
[3] They're actually probabilities.

| | | |
|---|---|---|
| 3 Numbers w/o PowerBall | $\frac{\binom{5}{3}\binom{1}{0}\binom{50}{2}\binom{41}{1}}{\binom{55}{5}\binom{42}{1}} \approx \frac{1}{291}$ | 0.003 |
| 2 Numbers plus PowerBall | $\frac{\binom{5}{2}\binom{1}{1}\binom{50}{3}\binom{41}{0}}{\binom{55}{5}\binom{42}{1}} \approx \frac{1}{745}$ | 0.001 |
| 1 Number plus PowerBall | $\frac{\binom{5}{1}\binom{1}{1}\binom{50}{4}\binom{41}{0}}{\binom{55}{5}\binom{42}{1}} \approx \frac{1}{127}$ | 0.008 |
| Just PowerBall | $\frac{\binom{5}{0}\binom{1}{1}\binom{50}{5}\binom{41}{0}}{\binom{55}{5}\binom{42}{1}} \approx \frac{1}{69}$ | 0.01 |
| 1 Number (wins nothing) | $\frac{\binom{5}{1}\binom{1}{0}\binom{50}{4}\binom{41}{1}}{\binom{55}{5}\binom{42}{1}} \approx \frac{1}{3}$ | 0.32 |
| Nothing at all (wins nothing) | $\frac{\binom{5}{0}\binom{1}{0}\binom{50}{5}\binom{41}{1}}{\binom{55}{5}\binom{42}{1}} \approx \frac{1}{69}$ | 0.59 |

So now that we have all of the probabilities, we can calculate the expected value of playing PowerBall. Of course, this is dependent on the Jackpot's size. Let's be generous and make it one billion dollars:

$$\begin{aligned} E &= 0.00000007*999999999 + 0.0000003*199999 + 0.000002*9999 \\ &\quad + 0.00007*99 + 0.00008*99 + 0.003*6 + 0.001*6 + 0.008*3 \\ &\quad + 0.01*2 - 0.32*1 - 0.59*1 \\ &= \$69.25 \end{aligned}$$

Heck, that's not too shabby! In other words, if we play over and over and over again, we can expect, on average, to win almost $70 each time we play. Wow!

I know what you're thinking...you're thinking, "Man, if I can just convince a bunch of friends to buy a whole bunch of PowerBall tickets, that'd increase our chances of winning even more!" Trust me...that's ***exactly*** what the PowerBall people ***want*** you to think. But also consider this:

1) The jackpot never gets close to a billion dollars. The highest it's ever gotten is around a third of a billion dollars. The median jackpot is around $36 ***million***.

2) Remember that the government takes about 40% of your prize money (anything over $600) due to "taxes"[4].

3) Don't forget state "income" tax! Oregon's is 8%.

4) Remember also that, historically in PowerBall (as of 2006), 176 jackpots were won by 205 parties. That means that you'll only get, on average, about 85.9% of the jackpot.

5) Speaking of the "jackpot"...it's horribly inflated. For example, for the February 11, 2006 jackpot, the reported value was "$250 million"...in little type under that, it says "$120.1 million cash value". The reported jackpot is misleading, due to financial reasons I can't understand[5]. Basically, when you see the jackpot, cut it in half. ***That's*** the true cash value.

Taking these points into account, the new PowerBall expectation is

$$\begin{aligned} E = {} & 0.00000007 * 0.859 * 0.6 * 0.92 * 35999999 \\ & + 0.0000003 * 0.6 * 0.92 * 199999 + 0.000002 * 0.6 * 0.92 * 9999 \\ & + 0.00007 * 99 + 0.00008 * 99 + 0.003 * 6 + 0.001 * 6 \\ & + 0.008 * 3 + 0.01 * 2 - 0.32 * 1 - 0.59 * 1 \\ = {} & \$0.41 \end{aligned}$$

Ouch. But heck, it's still positive, so you still average ***something***, right?

Sure...however, this is mathematical ***expectation***, not ***guarantee***. If you put that dollar in a bank account, however, not only would you get to keep it, you'd also get interest on it. Sure you're taxed on the interest. But your overall earnings are still over a dollar. And that's no ***average***; it's a guarantee[6].

Consider this: If you play the PowerBall every day for 50 years, averaging 41 cents a day, you'll have earned, on average, around $7500. Oops...wait a minute. PowerBall drawings are only held two days a week. That means you'll probably have around $2100. But, if you put 1 dollar in a bank account averaging 1% compounded annually, every day for 50 years, and the end of 50 years you'll have around $20,000...an order of magnitude more, even with a low interest rate. And that's guaranteed. Even if you take out half of it for taxes...you're still making money. Imagine what you could do with a higher interest rate.

Some of you will say, "Yeah, but I'll probably win the jackpot of I play every day for 50 years!" Well, no. Remember, you'd have to play for 146 million days to average a jackpot win[7].

The PowerBall game is designed so that the probability of winning ***something*** is pretty darn good. That's what keeps people coming back. However, after reading this, hopefully you realize it's not all it's cracked up to be. Play if you want...but play smart!

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[4] I've often considered this to be one of the most bogus acts in government. The $1 you played with was already taxed. I can't believe they can hit you again, because they're considering the lottery win an "investment". Geez...

[5] The same reason I can't do my own taxes, I'm sure.

[6] Thanks to this little thing called the FDIC.

[7] Or, just buy all 146 million possible tickets. Good luck with *that*.