## ANOVA Exercises

| Pages | Suggested Reading |
| :---: | :---: |
| $571-578$ | Section $13.1,13.2,13.3,13.4$ |


| Pages | Problems |
| :---: | :---: |
| $584-585$ | (Section 13.8) $1,3,5,7,8$ |

ANOVA, like GOF, is inconclusive when you get a small (less than 5\%) $\mathbf{P}$ - value; it doesn't tell you which means are different...just that at least one is. You should always do post hoc testing once you're gotten a small $\mathbf{P}$ - value after an ANOVA (STAT $\Delta \triangle$ ENTER), just like you did with $\chi^{2}$ GOF tests.

Just like last time, I want to make sure you had the steps for ANOVA post hoc down well. Let's take a look at this question:
Random samples of 2009 pickup trucks with 4 -cylinder,
5 -cylinder, 6 -cylinder, and 8-cylinder engines were
obtained. Each pickup truck was tested for miles per
gallons in highway driving.

| 4 (yl (H) | 5 (yl (H) | 6 (yl (H) | 8 (yl (H) |  |
| :--- | :--- | :--- | :--- | :--- |
| 24 | 21 | 19 | 20 |  |
| 23 | 21 | 19 | 19 |  |
| 22 | 23 | 19 | 19 |  |
| 24 | 21 | 18 | 20 |  |
| 24 | 18 | 21 | 16 |  |
| 23 | 22 | 20 | 18 |  |
| 23 | 23 | 19 | 15 |  |
| 24 | 18 | 20 | 21 |  |
| 24 | 20 | 19 |  |  |
| 23 | 20 | 19 |  |  |

Is there significant evidence to show that the MPG for
pickup trucks is not the same for all four engine sizes?

When I ran an ANOVA on these data, I got $\mathbf{F}>21, \mathbf{P} \approx 0.00000005$. At least one of the averages is different. Let's find which one(s) by running 2 - sample $T$ tests on each pair of means. But how many pairs are there?

For any ANOVA post hoc, you need to start by finding the number of ways you can pair off the data categories two at a time. Although this sounds confusing, it's pretty straightforward. Here are the matchups:

| 4 cylinder vs. 5 cylinder | 4 cylinder vs. 6 cylinder | 4 cylinder vs. 8 cylinder |
| :--- | :--- | :--- |
| 5 cylinder vs. 6 cylinder | 5 cylinder vs. 8 cylinder | 6 cylinder vs. 8 cylinder |

So there are 6 ways to pair off 4 categories, two at a time ${ }^{1}$. So that means that we need to do a $2-$ Sample $T$ Test for each of these pairs. Here we go ( 2 - tailed in each):


Now, to test for significance we need to compare each of those $P$ - values to the adjusted significance level:
Our ANOVA adjusted post hoc $\alpha-$ level $=\frac{0.05}{\text { number of pairs }}=\frac{0.05}{6} \approx 0.0 .0083$

[^0]4 cylinder vs. 5 cylinder


So, 4 cylinders get better mileage than 5 cylinders (how can I tell it's not the other way around?)

4 cylinder vs. 6 cylinder


4 cylinders get better mileage than 6 cylinders.

4 cylinder vs. 8 cylinder


4 cylinders get better mileage than 8 cylinders.

At this point, we know that 4 cylinder engines are superior in mileage, but we're not sure how the others rank. So...


Ah ha! So, 4 cylinders get the best mileage, but the other 3 are not different enough to statistically notice. This set of boxplots might help you see it better:


Now, some of you might look at that and say, "Sean! It looks like 5 cylinders actually DO get better mileage than 6 cylinders!" Well, they might, right? Explain, if you like, for 3 extra points, why your statement in quotes might be right based on the adjusted $P$-value. Feel free to turn in your answer next class.

Now, that method of post hoc analysis isn't always the one that's used after ANOVA (there are actually about $1 / 2$ a dozen different ones!). Very often, something called "Tukey's Honest Significant Difference (HSD) Test" is (it's basically the same as what you did above). The good news? All that analysis you just did will be done for you in software packages.

Here's an example: a researcher tested pectoral fin bone growth in fish in three groups: a control group, a group exposed to low levels of water contamination, and a group exposed to high levels of water contamination. The original ANOVA yielded a low $P$ - value, so SPSS (a software package some of you will use eventually) yielded the following Tukey HSD test results.

| Multiple Comparisons |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Tukey HSD |  |  |  |  |  |  |  |
|  |  |  |  |  |  | 95\% Confide | nce Interval |
| Dependent Variable | (1) 3 groups | (J) 3 groups | Mean Difference <br> (I-J) | Std. Error | Sig. | Lower Bound | Upper Bound |
| Pectoral-7 | Control | Low | -.450 | . 440 | , 570 | -1,55 | , 65 |
|  |  | High | -2.028 | , 451 | , 000 | -3,15 | -.90 |
|  | Low | Control | . 450 | . 440 | . 570 | -.65 | 1,55 |
|  |  | High | -1.578 | . 426 | ,003 | -2,64 | -,51 |
|  | High | Control | $2.028^{\circ}$ | , 451 | ,000 | . 90 | 3,15 |
|  |  | Low | $1.578^{\circ}$ | . 426 | ,003 | . 51 | 2,64 |

E1. Using this SPSS readout, identify which means ( $\mu_{\text {control }} \mu_{\text {low }}$, and $\mu_{\text {high }}$ ) are different from each other. I know I'm not giving you a whole lot to go on here, but I know you can figure this out.

E2. Of the ones you marked as different in the previous exercise, can you identify which in each pair is larger?

## Answers.

Remember the flowchart to help you with post hoc testing (http://coccweb.cocc.edu/srule/MTH244/homework/PHflowchart.xlsx)! Here's an additional sheet (http://coccweb.cocc.edu/srule/MTH244/homework/boxplots.xlsx) for making box plots in Excel (good for those times you have more than 3 data sets).

E1. The "sig" column is the same thing as a " $\mathbf{P}$ - value" column. Also, SPSS gives you twice as many post hoc's as you need (there need only be 3), which is why you get some repetition.

E2. Look at those Cl's!

## ANOVA Quizzes

## Quiz 1.

I'm ADD. You probably figured that out in the first 5 minutes of your first class with me. Because of this, I was immediately drawn to the study, "A Comparison of Ritalin and Adderall: Efficacy and Time-Course in Children with ADHD" (Pediatrics, Vol. 103,No.4). In the study, ADHD children were placed into one of 5 groups: a placebo group, or one of 4 groups that received varying amounts of either Ritalin or Adderall. After the dosage, the children were given instructions to accomplish a task, and were given a score corresponding to how well they followed instructions (a higher score indicated better instruction following). The results are below. Is there any difference in "instruction score" among the different dosages of drugs? Include the "placebo" group as a dosage.

| placebo | 10 mg Ritalin | 17.5 mg Ritalin | 7.5 mg Adderall | 12.5 mg Adderall |
| :---: | :---: | :---: | :---: | :---: |
| 47 | 96 | 37 | 57 | 84 |
| 20 | 75 | 83 | 71 | 105 |
| 34 | 57 | 76 | 85 | 83 |
| 60 | 44 | 43 | 92 | 92 |
| 28 | 83 | 87 | 75 | 74 |
| 72 | 69 | 62 | 55 | 92 |
| 25 | 57 | 99 | 73 | 64 |


[^0]:    ${ }^{1}$ You might remember this as a combination: $\binom{4}{2}=6$. Then again, you might not, and that's fine!

