1. Usain Bolt (our favorite Jamaican sprinter, mon) set a world record in the 100 meters at the IAAF World Championships in Berlin, Germany, on August 16\textsuperscript{th}, 2009 with a time of 9.58 seconds (a record that still stands). Here are his 10 meter splits as a function of the times at those splits:

<table>
<thead>
<tr>
<th>time (s)</th>
<th>0</th>
<th>1.89</th>
<th>2.88</th>
<th>3.78</th>
<th>4.64</th>
<th>5.47</th>
<th>6.29</th>
<th>7.1</th>
<th>7.92</th>
<th>8.75</th>
<th>9.58</th>
</tr>
</thead>
<tbody>
<tr>
<td>distance (m)</td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
<td>90</td>
<td>100</td>
</tr>
</tbody>
</table>

You’ll need to open the spreadsheet “\textit{Usain.xlsx}” that accompanies this exam to assist you with these questions.

For starters, click on the “Data and Fit Exploration” tab in the Excel sheet (“tabs” are near the bottom of the sheets – you left click them to access them).

You might notice that the data tends to “curve” a smidge – this is because Bolt (like most sprinters) accelerates off of the blocks at the start. So, I think that a linear model is ruled out right there! Please find a \textit{quadratic} trendcurve that passes through the origin for this data (if you need help, here’s a quick little video!).

a. \textbf{(5 points)} Does this quadratic model do a pretty good job of fitting the data? “Yes” or “no”, and then please explain! Be sure to talk about the data points, and how they’re interacting with the curve!

Now, after watching the video of this race, it sure looks like Bolt’s increasing his speed (i.e., “accelerating”) more and more with each additional second that passes by...but the data don’t show that, do they? Let’s see if you can see what I’m talking about...look at the quadratic model fit again (at right)...

Do you see how the curve starts above the data points (i.e., it’s overestimating how much distance Bolt is covering initially), but then, at about 5 seconds, it begins to underestimar? Then, again, it switches to overestimating (around the end of the race). Fascinating...in mathematics, this is called an “inflection point”...a spot on the curve where the acceleration is changing (you might remember this from one or two of the exponential quizzes a while back). Quadratics don’t have an inflection point – their acceleration is constant (much like my acceleration down the hill on my bike). We need our model to increase by one degree to see our first inflection point – so let’s do \textit{that}! Go ahead and delete the quadratic one – and add the CUBIC one!
Hells YEAH! That fits b-e-a-utifully! It represents the fact that his acceleration is slowing past second 5 or 6 (very hard to see in the video, though). Now...let's figure out how fast he was going at the end of the race...click on the “IROC at end of race” tab. In this sheet, you’ll see (besides Bolt’s goofy grin) a similar workup to what we did when we tracked my speed on my bike (and, if you did it, the penny drop quiz). Find the following AROC’s in meters per second, rounded to one decimal (in order to make the point move, you need to either use the scroll arrows, or drag the scroll bar...shown in the image above). WARNING: this apparently won’t work on (some) MacIntoshes! The “AROC” is given near the top in the yellow cell.

b. (1 point) Between 7 seconds and 9.58 seconds.

c. (1 point) Between 8 seconds and 9.58 seconds.

d. (1 point) Between 9 seconds and 9.58 seconds.

e. (1 point) Now...find his “IROC” (that is, how fast he was moving at the end of the race). Well, sort of...we’ll do what the police radar detectors do, and use the AROC between 9.57 and 9.58 seconds, and call that good enough.

f. (1 point) How fast is that previous speed in miles per hour? You can go ahead and round the nearest whole MPH.

WOW! Amazing! But here’s something to think about: Bolt was actually averaging a higher speed than that before the finish line – remember, he slowed down a touch! Can you see where, according to your model’s graph, he was at his absolute highest speed?

g. (2 points) Go find it! Click on the “Fastest IROC” tab, and explore (moving both points connected to the secant line). As you move the points, the A(I)ROC cell above computes his average (or “instantaneous”, depending on how close your points are). What’s his fastest IROC, to the nearest tenth of a meter per second?

h. (1 point) What’s that fastest IROC, to the nearest mile per hour?

Pretty badass, huh? It’s hard to bike that fast on flat ground, sometimes. 😊
2. As I was developing one of your warmups this term, I stumbled upon this little ditty:

![Average Tuition Costs Meme]

a. (5 points) At first glance, what do you think the point of this meme is/was? A sentence or two, please!

b. (5 points) Are the amounts claimed as “average tuition costs” valid in this meme? Do a little online research and check. Please supply at least one source that you find, and the amounts that they give for these years (note: “present day” appears to have been 2013). Also realize that the meme’s “tuition” claim doesn’t specify which kind of tuition (community college, 4 – year school, graduate school, etc.), so be sure to comment on that, as well.

c. (5 points) I checked these numbers myself, and what I found was that many of the sources used median tuition costs, not average tuition costs. We talked about these idea in class; please explain to me the mathematical process by which the median tuition cost is found, and also the average tuition cost (you don’t actually have to calculate them; just explain how, given a set of numbers, you would!).

d. (5 points) (w) Let’s assume that the data in the meme is valid. To the nearest tenth of a percent, what has been the average rate of tuition inflation from 1985 to 2013? Show me everything you do to arrive at this!

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1 Is it weird that I’m always trying to measure the veracity of memes? Or are they simply there for a quick laugh, and then intended to disappear? #feellold
e. (5 points) (w) What was the average rate of inflation in the US over that same time period? Let’s go from June 1985 to June 2013, since June’s the month when this thing is due. 😊

f. (2 points) Which has grown faster over that time period: economic inflation or the cost of college tuition?

g. (5 points) Look back at your answer to a…has the meme succeeded in what you thought their purpose was? Why or why not?

3. Here’s a graphic that appeared in the Central Oregon Regional Health Profile (2017 update):

![Figure 34. Age-adjusted gonorrhea incidence rate per 100,000 population, OPHAT 2011-2015](image)

**Significantly lower than Oregon rate.

a. (5 points) What’s your first reaction upon seeing the data in this chart? No right or wrong answer here, as long as you answer.

“Rate per 100,000” is an interesting measure that is often used to discuss percentages, when they don’t want to (for whatever reason) use the term “percent”. So, looking at the above graphic, 54.5 out of every 100,000 (or, more logically, 545 out of every million) Oregonians had gonorrhea.

b. (2 points) What percent would that be, for Oregon?

c. (3 points...1 for each) Convert the rates for Crook, Deschutes and Jefferson Counties to percentages, as well!

Personally? I think that graph’s a tad misleading…the population of Oregon is just about 4 million (a quick Google search yields that). If I use their “54.5 per 100,000 people” have gonorrhea, that means that about 2180 people had it when surveyed in 2017.
d. (8 points – 3 for the populations, 5 for the calculations!) (w) Repeat what I just did, but use the corresponding populations (and rates) for Crook, Deschutes, and Jefferson Counties (Google them up and make sure you give them to me!) to figure out the number (not rate) of gonorrhea cases in each in 2017. Show me everything you do!

e. (5 points) Create another bar graph – but this time, graph the number of gonorrhea cases in each of Crook, Deschutes, and Jefferson counties. Make sure you title the graph properly, and take a screen shot and include it in your document.

f. (5 points) Do you get a different gut feeling when looking at this graph versus the original one? Why or why not?

I play music with some wonderful fellas out here on the Left Coast, and, during this term, we released a 12 – song LP. Let’s use our little group to look at another application of the math we’ve been doing!

4. When we were planning the album release, we tried to find the absolute best possible song order\(^2\). Suppose I load these 12 songs into an mp\(_3\) player, so I can listen to them as I travel from place to place.

a. (5 points) (w) If these are the only 12 songs on my mp\(_3\) player, and I play exactly 12 songs, with “shuffle off” but “random on”...how many different song orders could I hear? Assume that songs can be repeated more than once within these 12 (or, possibly, not heard at all). If you want, you can answer in “-illion” form (just answer correctly). Also – show me the math you used to arrive at this answer!

b. (5 points) (w) The 12 songs have an average length of 1 minute, 55 seconds...this makes a playlist about 23 minutes long. How long would it take to listen to all of the possible arrangements in a? Remember to show me how you did this!\(^3\)

Ok – so that’s insane. Truly. I mean, stars would form and die in that time\(^4\). So let’s answer it again, but more realistically!

c. (5 points) (w) If these are the only 12 songs on my mp\(_3\) player, and I play exactly 12 songs, how many different song orders could I hear...this time, assuming that songs can not be played more than once?

Still a ridiculously huge number – but orders of magnitude smaller than the first!

Let’s wrap up this question with an extension of the last question that also addresses something that happens with digital media quite a bit. To give you motivation, I recently received this email from a student:

“At the gym I work at we have a cardio cinema which is basically a cardio room with the lights off (and cool color-changing LED strips) and movies playing 24/7. We have somewhere around 100 movies on a random playlist back there, and today I had a complaint that The Hobbit was the only movie that was playing on repeat. It just so happens that it played twice in a row. I thought, "What are the odds that a movie would play twice in a row?" And then I realized I have way too much to get done to be thinking about that so instead of trying to solve it I got back to work (yes, I SHOULD be working right now...). Anyways, I’m not asking you to solve it, I’m simply sending a fun little math problem experience I had today!”

\(^2\) Words like “LP” and “album” might seem foreign to you. They’re archaic ways that old people use to collect music. 😊

\(^3\) To give you an idea of scale, round to the nearest hundred million years. 😊

\(^4\) Not stars like Cyndi Lauper or Taylor Swift. Like, actual galactic stars.
This happens to me quite a bit. I’ve had students email me from vacation, complaining that their digital song players are playing “too much of this artist” or “not enough of that artist”. Apple itself had to address this a few years ago when its customers complained about its iPod Shuffle. As we saw in class, randomness clumps…but how to measure that clumpiness?

So let’s say that we have our 12 songs on our mp3 player. What is the average number of songs you need to randomize until you hear each song at least once (that is, allowing for repeats before hearing each song once)? Follow the steps below to answer:

If you did the “Max and his Lego minifigures” quiz, you’ve actually done this once already! If you didn’t (or even if you did – you’ll probably need a refresher), follow the steps below to answer:

• Start by going to this page! http://www.randomservices.org/random/apps/CouponCollectorExperiment.html

• Set the screen up like you did for the Max Lego quiz – but use 12 instead of 16. 😊 If you need the link: http://coccweb.cocc.edu/srule/MTH105/homework/4randomness.pdf#page=4

• Press the “fast forward” button (like on the quiz) and let the randomizer work 1000 times! It’ll take a minute or two (maybe even less). Technology amazes me, to this day. 😊

• See the bottom right rectangle? That’s where (as you might remember from the Lego quiz) the percentages of the outcomes are listed. If you scroll to the bottom of that rectangle, the average (“mean”) and standard deviation are given. Make a note of them! Note: make sure to use the one from the “Data” column, even though both columns are pretty darned close.

This means that we can be fairly sure that the number of songs we’ll have to listen to will be “within one standard deviation of the mean” (more on what that means – and why – in MTH 243!). This is the range of values formed by adding and subtracting the standard deviation from the mean.

d. (5 points) So, we can be reasonably sure that we’ll have to listen to between which two numbers of songs?

Sweet! Still a bunch of listening, but a more realistic number to deal with! To put it in perspective – listening to all 1000 of those playlists would have only taken about 50 days (assuming that we split the listening up amongst the four of us, less than two weeks each!). Compare that to the number you got in b. Wow!

e. (2 points) Take a screenshot of your results from the randomizer website and paste it into your document as your answer to this one!