Grade Exercises

Let’s think back to some examples like we started with in class:

E1. If a road rises 200 feet over 2 miles of run, what is its grade (or “slope”)?

E2. What angle is formed by the road in E1, on average?

E3. If a hill has a grade of 3.4%, and it rises 1500 feet, how much does it run?

E4. What angle is formed by this road, on average?

E5. If a road forms a 5.8 degree angle, what’s its approximate grade?

E6. Look back to E3. If you hike up this hill, from bottom to top, how far do you walk? Assume, for simplicity, that the hill rises in a uniform manner; in reality, hills don’t often! (hint: you need to use the wonderful Pythagorean Theorem on this one. If $a$ and $b$ are the legs of a right triangle (the “legs” are the ones that form the right angle; they’re the “rise” and the “run”), and $c$ is the hypotenuse (or longest side...the one across from the right angle. We’ve been calling it the “road”), then $a^2 + b^2 = c^2$ (or $\text{rise}^2 + \text{run}^2 = \text{road}^2$). If you can’t stand doing algebra, you can always use the Excel Calculator.

E7. Now, some folks might forget to use the hypotenuse when they perform the previous problem, and, instead, just say, “Well, you walked the run, which was 8.4 miles!”. They’re obviously in error…but by how much? Here’s a helpful formula for percent error:

$$\text{Percent error} = \frac{\text{amount used incorrectly} - \text{correct amount}}{\text{correct amount}}$$

So...what’s their percent error?

E8. Make sure you understand why that formula makes sense.

E9. Suppose that the hill actually rises 15,000 feet (with the same grade). Re-answer E6 two ways: in one, use the road (hypotenuse – the “wrong way”). In the other, use the rise (the “right way”).

E10. Recompute E17 (percent error) now. What do you notice?
Consider the following graphic (a computer generated image from a hill workout I do):

On this graph, you see elevation as a function of distance traveled (in meters and kilometers; sorry, I can’t, for the life of me, figure out how to change the default distance setting. Not to worry; it’ll do us good to pretend we’re European for a moment or two). For the first 6 km, I ride my bike to the trailhead, then from 6 km to about 19.5 km, run up and down a hill 5 times, then ride home. If you’d like a conversion, you can use 3.1 miles \(\approx\) 5 km.

**E11.** How long, in miles, is this workout?

**E12.** For how many miles do I run up and down the hill?

Consider, for a moment, just this running part.

**E13.** How many feet in elevation do I gain over the 5 hill repeats? (Pessimists will say, “Why even bother? You come back down each time. Save yourself the energy!”)

Consider, for this moment, just 1 hill repeat (it looks like the first one starts at 6 km and goes to about 8.5 km).

**E14.** What’s the average grade on that repeat?

**E15.** What’s the average grade on its uphill section?

**E16.** There are some points along that trail, however, that are steeper than the average grade. How can that be?

**E17.** What’s the average grade on its downhill section?
E18. I made a flier for a show my band played in Bend a few years ago. Since I’m a *Raising Arizona* fan, I decided to “customize” the image below and to the left to create the flier at right:

The trick was that I needed to create the sign HI’s holding separately, and then drop it into the original image (well, that and detach and then reattach his thumb…but that was pretty easy). I couldn’t make the sign tilted in the software I was using, so I needed to create it horizontally, and then adjust it by the correct angle until it matched. About which angle is that, to the nearest degree? Also, I couldn’t find the protractor in our house…which would have made this problem much easier.

“E” Answers.

E1. Grade is $\frac{\text{rise}}{\text{run}}$, so $\frac{200 \text{ feet}}{2 \times 5280 \text{ feet}} = \frac{200}{10560}$ (units can cancel), or about 0.0189 (1.89%, or about 2%). This assumes that you’re going uphill. If you were going downhill, the grade’s sometimes expressed as a negative quantity – but only in math classes. ☺.

E2. For these, there are a number of ways. In class, we probably used a tangent table that we looked up online (here’s one that’s been available for a while now: [http://quest.arc.nasa.gov/space/teachers/rockets/act9ws4.html](http://quest.arc.nasa.gov/space/teachers/rockets/act9ws4.html)). But, by now, I’ll bet that you like the Excel calculator even more!
So, you can see the angle’s just over 1 degree (and you also get the grade, which is kinda why I built this thing in the first place – to save you time. 😊)

If you need more precision in your answer (that is, a more exact answer than “2%”), you can use the “increase decimal” buttons in Excel (shown at right). Every time you left-click that button, you’ll get one more decimal place of accuracy. First, left-click in the cell in which you’d like to get more precision (in this case, J3). Then left – click that increase decimal button twice, and you’ll get this:

E3. Let’s use the equation \( \text{grade} = \frac{\text{rise}}{\text{run}} \) to get \( 0.034 = \frac{1500 \text{ feet}}{\text{run}} \). Then, we can multiply to find that \( 0.034(\text{run}) = 1500 \text{ feet} \). Dividing by 0.034 gives us \( \text{run} = \frac{1500}{0.034} \), or just around 8.4 miles (about 44000 feet).

E4. I get just shy of 2 degrees.

E5. I never coded the spreadsheet to work in reverse (that is, let you input an angle and get a grade out). But if you just use the above linked table, you can look up the angle and then reference the cell to the left, which tells you the grade. I get just about 10%.

E6. The “rise” and “run” of the hill are the legs of the right triangle, so we can say that \( a = 1500 \text{ feet} \) and \( b \approx 44117 \text{ feet} \). The hill, therefore, would look like this (horribly not to scale!):
How much you have to walk (let's call it $c$, OK?)

$a = 1500'$

$b = 44,117'$

So, we’d have $(1500')^2 + (44,117')^2 = c^2$, or $c^2 = 1,948,559,689$ square feet. Taking square roots, we have that $c \approx 44,142'$ (see how distorted that triangle I drew was? In fact, let’s have some fun: without using a ruler, approximate the grade of that distorted triangle).

**E7.** Percent error = \[
\frac{\text{amount used incorrectly} - \text{correct amount}}{\text{correct amount}} = \frac{44117' - 44142'}{44142'} \approx -0.00057, \text{ or about 0%}. \text{ No big deal!}
\]
We’ll talk more and more about this in class, if we haven’t already – the “right way” versus “wrong way” calculations of grade, and how, if the grades aren’t “too steep”, how it doesn’t; matter which way you do them).

**E8.** The numerator gives you an idea of how far off (in feet) your erroneous estimate is. The denominator then weights that by the entire distance over which you were in error. You can see that we were only off by around 25 feet...over 44142. That’d be like shooting a rifle at a bullseye a thousand feet away from you and missing by a little less than 6 inches. Not too shabby!

**E9.** So, repeating the math from E3, I get a run of about 83.56 miles. Correctly using Pythagoras, the hill that you walk on would be 83.6 miles.

**E10: **Percent error = \[
\frac{\text{amount used incorrectly} - \text{correct amount}}{\text{correct amount}} = \frac{83.56 \text{ miles} - 83.6 \text{ miles}}{83.6 \text{ miles}} \approx -0.00047, \text{ or about 0%}. \text{ Again...no big deal!}
\]
Sure, the error will stretch out over long distances (so, those of you planning rocket trips to the moon – use Pythagoras!) but for those of us on Earth doing things like hiking Mount Hood, no prob.

**E11.** According to the graphic, it was 25.7866 km, so, converting, I get $25.7866 \text{ km} \cdot \frac{3.1 \text{ miles}}{5 \text{ km}} \approx 16 \text{ miles}$. (if you want more of these types of conversions, just you wait! Your first project is ALL about them!)

**E12.** Let’s be a tad clever: since I biked to and from this workout, let’s figure the distance I biked in, double that, then take it away from 16, shall we? Looks to be about 6 km each way, so $12 \text{ km} \cdot \frac{3.1 \text{ miles}}{5 \text{ km}} \approx 7.5 \text{ miles} \text{ biked}$. The rest (16 miles – 7.5 miles, or about 8.5 miles) must be how much I ran.

**E13.** Each hill repeat looks to be about 190 meters of elevation gain, so I do 950 meters, or about 3100 feet of elevation gain uphill (and, conversely, the same amount running back down).

**E14.** Ha! It’s 0%. See why?

**E15.** OK! Here’s where we start to sub in the “wrong way” grades!
What you see in the graph isn’t the rise and the run – it’s the rise and the **road**! These types of elevation apps never actually give the run – because that’s not what you’re actually traveling. They, for sure, track your up-and-down (“rise”) changes, but the distance you cover, “horizontally”, is actually the **road**. So, I do about 190 meters of rise, and about 1.25 km (or 1250 meters) of road.

Because this happens so frequently (not just in our class, but also if you use these apps), I coded the Excel Calculator to handle it:

![Insert the rise and either the run or road at right! Make sure units agree!](image)

<table>
<thead>
<tr>
<th>Rise</th>
<th>Run</th>
<th>Road</th>
</tr>
</thead>
<tbody>
<tr>
<td>190</td>
<td></td>
<td>1250</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Right Way Grade!</th>
<th>Wrong Way Grade!</th>
<th>Angle Formed!</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.38%</td>
<td>15%</td>
<td>8.74°</td>
</tr>
</tbody>
</table>

Here’s how this works:

- You place the 190 in the rise, and the 1250 in the road.
- You get a “right way” and a “wrong way” grade (in this case, 15.38% and 15%, respectively).
  - In the “right way”, the Calculator figures out the run (using Pythagoras) from the road and the rise, and then calculates the grade as \( \frac{\text{rise}}{\text{run}} \).
  - In the “wrong way”, the calculator simply calculates \( \frac{\text{rise}}{\text{road}} \).

See how **close** they are in value? While we’ll spend more time on this in class and on assessments, I’ll give you a preview now: so long as the hill “isn’t too steep”, they’ll be very close in value (I encourage you to mess around with the calculator to find roads that are too steep to see how off then can get!). Something else to think about – the “right way” is always larger number than the “wrong way”. Can you think of why? #**geometry**

**E16.** Does the hill always have to go **up** on the uphill? Look at the graphic again!

A quick word on “average grade”, in case you forget what we discussed in class: Average grade “smoothes out” the road into a perfectly straight line – it makes the assumption that the ups and downs along the way don’t exist, and just looks at the total rise divided by the run (or road). By “total rise”, I just mean the difference between the starting and ending elevations; in-between elevation changes “don’t count”.

**E17.** Same exact one! I guess you could say -15%, if you’re feelin’ saucy. 😊

**E18.** So here’s a picture of just the sign bottom:
Notice that the rise is about 1/12(ish) the run (does it matter which units you use? Why or why not?). Therefore, the grade (or tangent) of the angle formed is about 1/12, or, roughly, 0.083. Consulting my handy – dandy tangent table link from above, I see that a tangent of 8.3% corresponds to an angle of about 5 degrees.
Quiz 1.

My house is at roughly 3400 feet above sea level. My 6.5 – mile commute to COCC each day ends at 4000 feet above sea level (this is a slightly different route than the one we talked about in class).

1. (2 points) I’m going to list three components of the “grade triangle” we’ve been using in class below, and I’d like you to tell me what two of their three values are (one of them is unknown from the data above – you can calculate it, but I don’t want you to):

<table>
<thead>
<tr>
<th>Rise:</th>
<th>Run:</th>
<th>Road:</th>
</tr>
</thead>
</table>

2. What’s the average grade of my commute? Give two measurements (remember, you can use the Excel Calculator if you want!):

   a. (2 points) The “wrong” way.

   b. (2 points) The “right” way.

   Note: When I say “do it two ways”, all you need to do is enter the two measurement you have into the Excel Calculator. Just make sure you put the measurements in the correct places! Email if you get stuck!

3. (4 points) In this case, do you think that it matters whether we do grade the “wrong way” or “right way”? Why or why not? Write at least two sentences explaining your answer!
Quiz 2.

Burma Road is a fairly steep section of double track in the BLM land right outside of Smith Rock state Park in Terrebonne Oregon. Some of you may have been on or near it before. In the picture at below left, it’s the jagged–looking diagonal gash running across the center of the image (I colored it red, so you can see it a little easier). It actually switchbacks on itself, as you can see in the picture at right below (taken atop the large rock formation you can see at center left, called the Wombat).

A student asked me a great question in class once: “What’s the grade on Burma Road?” Fantastic! Let’s find out!

Remember that **Average Grade** = \( \frac{\text{Total Rise}}{\text{Total Run}} \) …so all we have to do is get both of those, and we’re set!

I went to this site and used this topo (“topological”) map to zoom in and capture the image below – it’s a look, down from above, at an elevation map of this area of Smith Rock:

Now let’s use this image to get our rise and run. We’ll start with the rise:

\[ \text{Total Rise} \]

\[ \text{Total Run} \]

* I know that, sometimes, we’ve been swapping out “road” for “run”, and this’ll also be one of those cases!
Do you see those little lines on the map? They’re called “contour lines”, and they let you know how steep certain sections of land are. Notice how certain numbers are indicated? I see a “3600’” up near the top of Burma Road – that means that, if you’re standing anywhere along that line, you’re standing at 3600 feet above sea level. See the one that says “3800’”? Same thing, but that one’s 200 feet higher. Now, there are also lines in between the darker, bolded ones – I count 4, which means each line represents a change of 40 feet in elevation.

The closer together contour lines are, the steeper the terrain – conversely, the farther apart they are, the flatter the terrain. Pretty rad, eh? OK, back to Burma Road…

1. **(1 point)** At what elevation does Burma Road start (the lowest part of the red line)? Approximate it – I know the map isn’t perfect.

2. **(1 point)** At what elevation is the end of Burma Road (the highest part of the red line)?

3. **(1 point)** So, if you walk from the bottom to the top, how much elevation have you gained overall?

OK...there’s your rise! Now let’s work on the “run” (actually, “road”...remember why?) – this map has a scale of about 1” to 0.2 miles (that is, for every inch you “walk”, you cover about 1/5 of a mile). Estimate how long, in miles, Burma Road is\(^b\). Make sure to keep the “zoom” on your web browser is at about 100% (not more nor less). If you’re doing this on a mobile device, it’s most likely going to distort your numbers, so shoot me an email and we’ll figure it out!

4. **(1 point)** Convert your answer to feet, and find your run (er, road) in feet.

5. **(1 point)** *(w)* What’s the average grade of Burma Road to the nearest percent? “Right way”, wrong way”...either’s fine. 😊

6. **(1 point)** What’s the average angle formed by Burma Road?

OK, one more...a few years ago, I hauled my son up the ridgeline colored yellow at right below (we started at the top of Burma Road, and finished on top of the butte shown).

7. **(1 point)** What’s my rise over this journey?

8. **(1 point)** What’s the road? Again, don’t zoom in!

9. **(1 point)** What’s the average grade of this ridgeline?

10. **(1 point)** What’s the average angle formed?

11. **(extra 1 point)** Why was I so insistent about you not zooming in on these?

\(^b\) You might find it easier to find a piece of string and lay it along Burma Road on the map. Then, you can straighten the string along a ruler to see how long it is.

\(^c\) We’ll talk more, later, about why this assumption isn’t *always* legit.
Quiz 3.

Remember when you all were so busily running around campus measuring the grades of staircases? A few terms ago, I diligently measured the ADA ramp outside of Modoc Hall. In case you don’t remember it, here’s a picture:

“Why do such a thing?”, you might ask. Well, as it turns out, the ADA ramps are bound by (you guessed) the actual ADA (Americans with Disabilities Act). As such, there are very strict requirements as to how the ramp is to be constructed (note: “slope” and “grade” are used interchangeably):

4.8.2* Slope and Rise
The least possible slope shall be used for any ramp. The maximum slope of a ramp in new construction shall be 1:12. The maximum rise for any run shall be 30 in (760 mm).

OK – here are the measurements I took (I only did the ramp at the bottom, behind the studious student in the picture…I assumed they were consistent for the top one):

25 feet, 1 inch
2 feet, 1/2 inch

1. (1 point) Is this ramp’s rise within the code’s restrictions?

2. (2 points) Calculate the grade (slope) of the ramp the “wrong” way…you figure, “It’s good enough for government work (nyuck, nyuck, nyuck)”. To the nearest thousandths’ place, please! And not as a percent – as a decimal.

Notice how they gave us the maximum allowable slope as a “1:12”? Let’s compare ours to that!

3. (1 point) Convert “1:12” (which is actually \( \frac{1}{12} \)) to a decimal. Use all decimals, please!
4. **(1 point)** Does our ramp’s grade comply with the ADA code?

5. **(5 points)** Now, suppose that COCC was audited for this ramp. The government might say, “well, we notice that you calculated your grade using the \( \frac{\text{rise}}{\text{hypotenuse}} \) shortcut, which, as you know, is not the same as the correct method of calculating grade.” How would you answer their charge? **Hint:** you’ll need to calculate the grade the **right** way to answer this satisfactorily.
Many of you have either hiked, biked (or both) and encountered switchbacks on your road or trail. **Switchbacks** are where the trail or road curves back and forth along itself to make climbing easier. I’ve encountered them on MacKenzie Pass, Farewell Trail, and plenty of other places in and around Central Oregon. Here’s a shot of some switchbacks from the mountains near Asheville, NC (“Bend of the East Coast”):

“Man!”, you might think, “How does this make climbing easier? You’re traveling MORE!!!” Let’s analyze that! Here’s a picture of one of those switchbacks, taken from the curve in the road:

In red, I’ve outlined the path that you would take if you stayed on the road. In yellow, I’ve outlined the “fall line” (that is, the path you would take if you were to climb directly up a mountain – it’s called the “fall line” because that’s where an object would “fall” if left under only the force of gravity).
What I’d like to do is mathematically generally analyze a switchback by comparing it to the fall line route. But first...

1. **(1 point)** If Road A has a larger average grade than Road B, which would be easier to walk/bike/drive up? Define “easier” in this case as “less energy expended at the hardest point”, and assume that a higher grade equals more energy expenditure\(^d\). Also assume that their lengths are irrelevant.

   Hopefully this make sense! I notice this all the time when I ride up Iowa Street – the steepest section is only about 200 feet long – but I’m whooped when I get to the top of it – way more than I would be if I simply took Archie Briggs around the side of Awbrey Butte.

   So let’s see what’s going on, grade – wise, between these two routes. Complete the following with either a “<”, “>”, or an “=”:

2. **(3 points)** Rise of the switchback route \[\boxed{\_}\] Rise of the Fall Line Route

3. **(3 points)** Run\(^e\) of the switchback route \[\boxed{\_}\] Run of the Fall Line Route

Now, put those ideas together into grade!

4. **(3 points)** \(\frac{\text{Rise}}{\text{Run}}\) of the switchback route \[\boxed{\_}\] \(\frac{\text{Rise}}{\text{Run}}\) of the Fall Line Route

   So there you have it! You now understand why exhausted cyclists (i.e., my son!) weave back and forth on steep hills, why Misery ridge zig – zags, and why mountaineers rarely head straight up a mountain all day. Wahoo!

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\(^d\) Road cyclists understand what I’m talking about here.

\(^e\) Feel free to substitute “hypotenuse” for “run” here.
Quiz 5.

As you might remember, “grade” is just another word for “slope” or “tangent of an angle”. Well, as it turns out, there are plenty of other terms that are also used! Let’s explore some here!

Roof pitches – carpenters and builders often refer to the “pitch” (of slope) of a roof by the fractional equivalent of the grade with 12 in the denominator (I believe this is because there are 12 inches in a foot). So, for example, if a roof has a “6:12 pitch”:

\[
\text{6:12 pitch} = \frac{6 \text{ inches rise}}{12 \text{ inches run}} = 0.5 = 50\% \text{ grade (which translates to about a 26.5 degree angle)}
\]

a. (2 points for each line) Please complete the following chart for some commonly used roof pitches!

<table>
<thead>
<tr>
<th>Pitch</th>
<th>Grade (as a decimal)</th>
<th>Grade (as a percentage)</th>
<th>Angle (in approximate degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4:12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6:12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12:12</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In extreme weather situations (like in high snow areas) pitches/grades often exceed 100% so that the snow loads aren’t too great. There’s an example of one at right, based on an equilateral triangle.

b. (2 points) Using a measuring device of your choosing, approximate the pitch of the roof in that set of plans.

c. (1 extra point) Knowing that the cabin’s side elevation was based on an equilateral triangle, how could you have gotten your answer in part b directly without measurement? Please explain fully!

The same pitch idea is used in plumbing – for example, waste pipes in Bend have to have \( \frac{1}{8} \)” of fall per foot of run. So, therefore,

d. (2 points) If a pipe has to run 20 feet, how far must it fall?

As mentioned in class, pitch is used in other applications (high heeled shoes, extreme skiing), but I can’t seem to find any rigorous treatment of the measurements in these situations (outside of “Yo, you really shredded the gnar on that pitch, brah!” or “Wowsers! Can you actually walk on heels that pitched?”).
This quiz is a follow-up to the activity we did on the first day of class (when we analyzed card decks for uniqueness). That day, we had some fun learning something (officially) called the “Fundamental Counting Principle” (don’t worry about memorizing that phrase – it’ll show up again and again in our class. And, if there’s one thing I’ve learned after all the decades I’ve taught math, it’s that I’d much rather you understand “how to use something” than “what it’s named”).

The basic idea is that, when constructing things like sequences of items, you multiply possibilities. Here’s what we did in class that day!

<table>
<thead>
<tr>
<th>Cards in deck</th>
<th>Number of possible ways to shuffle them all out...</th>
<th>...thought about as multiplication!</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2*1</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>3<em>2</em>1</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
<td>4<em>3</em>2*1</td>
</tr>
<tr>
<td>5</td>
<td>120</td>
<td>5<em>4</em>3<em>2</em>1</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>52</td>
<td>8 x 10^67</td>
<td>52<em>51</em>51*...<em>3</em>2*1</td>
</tr>
</tbody>
</table>

(we also learned that this kind of thing is called a “factorial”, but that isn’t crucial to remember right now)

Let’s see how we can use this in context!

From time to time, when we come into our classroom, the little alarm that guards the class will be alarming. On days like that, my first job is to disarm it. The way to disarm it is to correctly enter the 4 – digit PIN. Let’s start with analyzing that!

Now, you might say, “How are passwords related to cards in a deck?” To that question, I’d answer, “Both are unique sequences of characters whose orders matter!” If you move some (or all) of the characters around, you get a different shuffle, or PIN.

So, let’s think of this as a deck of cards! Each number you use in the combination is like a card dealt out of the deck. However, there’s one small (but important!) difference...complete the following sentence (by selecting the right words in gray) to see why:

1. **(2 points)** When we dealt out our decks of cards, we **could/could not** reuse the exact same card in two different places in the shuffle order. With the PIN on the wall alarm, you **could/could not** have the same number be in more than one place in the PIN.

Knowing that, we can construct a similar table to the one we constructed above!

<table>
<thead>
<tr>
<th>Digits in PIN (each digit has 10 possibilities...0 through 9)</th>
<th>Number of possible PINs...</th>
<th>...thought about as multiplication!</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>10*10</td>
</tr>
<tr>
<td>3</td>
<td>1000</td>
<td>10<em>10</em>10</td>
</tr>
</tbody>
</table>

† You might not. That’s OK, too. 😊
2. **(2 points)** Continuing this pattern, how many different possible PINs could the wall alarm have?

3. **(2 points)** Assuming it takes me about 5 seconds to check each PIN, how long would it take me to check every single one? I know I most likely wouldn’t have to – I’d probably stumble upon it somewhere in the middle. But, worst case scenario…how long would it take? To the nearest hour, please.

   So yeah...we’re not gonna go ahead and **guess** at the PIN. 😊 But, hopefully, you can also see why they keep adding characters to the minimum password lengths, can’t you? With each additional digit, it becomes 10 times harder to crack the code. That’s the beauty of the multiplication!

   But, it’s not all ponies and rainbows. There’s a growing industry of folks whose main aim in life is to attempt to crack codes like this: hackers. And, with publicly accessible keypads like this, there’s a security concern that pops up that creates a potential issue: when you use the keypads, you leave finger oils behind. This means that, if the same number is keyed over and over again, oils are left on the PIN’s numbers used. Therefore, if you’re trying to hack into a keypad, you can figure out the numbers by finding the oils! If you can figure out where the oils are, you’ll have a better idea of which numbers are used in the PIN. At right, you can see how they do it (they use a blacklight (or something like it)). So, in this example, even though the keypad looks fine in normal light, under blacklight, you can tell that the numbers 1, 2, 5, and 8 are used over and over. So, most likely, those are the PIN numbers.

4. **(2 points)** Assuming that the PIN has **just** the numbers 1, 2, 5, and 8 in it, and it’s a 4 – digit PIN (with each of those numbers used once), how many possible PINS are there?

5. **(2 points)** List them all!

   And those would take, on average, around a minute to crack. So that’s why, whenever I see one of these little keypads, I touch all the buttons as much as possible…to help keep security at a premium. 😊

6. **(extra 3 points)** Suppose a hacker has identified 3 numbers that are used on our little 4 – digit keypad (so, using the keypad image at right, they’ve established that the digits 1, 2, and 5 are the ones used). How many possible PINs are there now?

   *(remember – “extra point” problems are just that: extra! You can try them and still get a perfect 10 on the quiz, even if you get them wrong)*

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8 Or is it puppies and rainbows? Puppies and kittens? Unicorns? I can never keep these cutey phrases straight.

h If you’ve seen the movie National Treasure, this is exactly how Nicholas Cage breaks in to get the Declaration of Independence. And, in more recent (and realistic) applications, [https://www.extremetech.com/mobile/83690-hacking-the-smudges-on-touch-screens](https://www.extremetech.com/mobile/83690-hacking-the-smudges-on-touch-screens).
Quiz 7.

This quiz is a follow-up to the last quiz (which was a follow-up on what we did on the first day of class 😊).

What I’d like to do is to apply this concept to another related idea: combination locks! Let me paint ya a picture: my buddy HB forgot the combination on her bike lock one day. She has one of those little wheelie-type ones. There’s a picture of it at right. The way it works (in case you haven’t used one before) is that you have to spin the dials until the correct 4-digit sequence is between the little arrows (indicated by the red circles in the picture).

1. **(1 point)** Throwback question: how many different possible combinations could my buddy HB’s lock have?

So this is numerically identical to the PIN problem from the last quiz! And, at this point of that quiz, we decided not to test every single PIN, because it would take too long (even though I adore my buddy HB, neither of us have the time to sit around that long trying every possible one of those). So then I asked her, “Do you remember anything at all about any of the numbers in the combo?” And the first thing she told me was…”You know, I do. I know it has a 9 in it!”

Now, here’s where it gets interesting: She told me that she knew it had a 9…but she couldn’t remember how many 9’s it had, nor where they were in the pattern. So, I decided to attack it like this: figure out every possible combo that could have a nine in it, in each of the 4 possible places, and then add those together. Here we go!

**If the 9 is in the...**

| ...first space, then there are 1*10*10*10 = 1000 possible combinations | ...second space, then there are 10*1*10*10 = 1000 possible combinations | ...third space, then there are 10*10*1*10 = 1000 possible combinations | ...fourth space, then there are 10*10*10*1 = 1000 possible combinations |

(you might wonder why there are 10 choices for the other 3 spaces...remember, HB told me that she knew there was at least one nine, so I know, for a fact, that one of the 4 spaces has exactly one character – a 9 – in it. The other three might, as well, so I allow for them with the 10 possibilities)

2. **(2 points)** So, how many total combos would we have to check to be sure we got her lock opened? Worst case scenario here: chances are, it won’t be the last one that you would try.

But, 4000 still seemed like too many to check (c’mon! We’ve got families and stuff!). So, I asked her again, “Can you remember anything else about the lock’s combo?” And she said, “Oh wait! I know it starts with an 8, and it has a 9, too! And I said, “Cool! Are you sure?” And she said, “Yep! I know it! It might have more 8’s and 9’s, but I know it starts with an 8, and has a 9 in it somewhere!”

3. **(2 points)** Which of the following ways would be the correct method for figuring out the number of combinations, knowing this new information?

   a. 8*9*10*10 + 8*10*9*10 + 8*10*10*9
   b. 1*1*10*10 + 1*10*1*10 + 1*10*10*1
   c. 2*1*10*10 + 2*10*1*10 + 2*10*10*1

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1 This actually over counts the total number of passwords, but you’ll deal with that in later stat classes!
After I told her about the above number (and fretted that I still didn’t want to check all of them…even though there were only 300 of them!...*hint, hint!*), the following discussion ensued:

HB: “Wait! Dude! It’s all 8’s and 9’s!
Me: “Oh, cool! So it starts with an 8, and has only 8’s and 9’s?”
HB: “Well, I’m not really sure about the whole ‘starting with the 8’ bit. But I know, for a fact, that it only has 8’s and 9’s in it!!”
Me: “Rad!”

4. **(4 points)** List out all possible 4-digit combinations that have only 8’s and 9’s in them (and, even though it’s not explicitly stated, make sure that both of the numbers 8 and 9 occur at least once in each arrangement...I’m assuming that’s what HB meant by “it only has 8’s and 9’s in it”).

5. **(1 point)** Whaddaya think? Could we check all of those in a reasonable period of time? Just “Yes” or “No” for this one. 😊

And we did – and one worked. 😊

Now, as inane as this may sound, there’s actually quite a bit of historical precedent for using this kind of math that we introduced on day 1. If you’ve ever watched the movie “The Imitation Game”, this is precisely the math that the US (led by Alan Turing) used to crack Nazi codes during WWII (if you haven’t seen the movie, it’s pretty fascinating). Because the total number of passwords is so large, if you can figure out something about the position of one (or more) of the characters, you decrease the total number of passwords exponentially. Of course, this is also how hackers gain access to your secured accounts. Just ask Equifax.

\(^1\) Again, over counted.
\(^k\) Of course, this is also how hackers gain access to your secured accounts. Just ask Equifax.
Quiz 8.

1. Suppose a road rises 7 feet for every 50 that it runs.
   a. (1 point) what is its grade?

   b. (2 points) Use the Pythagorean Theorem to tell me what the road length would be in this case. Round this one to the nearest tenth. Please show me what you do! If you use some kind of online computation tool, just take a screenshot of it and include it (if you need help with screenshots, here ya go!)

2. Suppose a road rises 21 feet for every 150 that it runs.
   a. (1 point) What is its grade?

   b. (1 point) What’s the road length in this case? Same rounding as before (no need for work on this one; I assume you’re doing the same thing as before).

3. One more! Suppose a road rises 49 feet for every 350 feet it runs.
   a. (1 point) Grade, please!

   b. (1 point) And, yep...road, in that case?

Seeing anything going on after those last three situations? If so, great! If not – go back and look again. Then give the following a shot!

4. (3 points) Choose the correct phrases:

   If you multiply the rise and run by a constant value, then the grade (is also multiplied by that constant value/remains the same) and the road (is also multiplied by that constant value/remains the same).
Quiz 9.

I may have lied to you a little in class...I told you that Iowa Street in Bend was the road in town with the steepest section of grade. Now? I’m not so sure...see, when I bike in the Iowa route, I also have to go up Palisades Drive, between Rimrock and Glassow). That, my fine feathered friends, is a total PITA:

Now, I don’t know if it’s actually steeper than Iowa, or if it just feels like it because I just survived Iowa...so, I need your help to check.

(10 points) (w) Find about ½ an hour to an hour to do this quiz. Stop by the office and borrow my transit and 200 – foot tape (I’ll teach you how to use both). Then, go out, find Palisades Drive, and figure its grade. Show/describe everything you do. Pictures would be rad!

(5 extra points) That’s right – you get 5 extra credit points for doing this quiz. Why? Because it requires a field trip! And instrumentation! And parking/walking on a crazy hill. 😊
This is another quiz that one of your classmates gave me the idea to create. Remember the game I had us play in class? “Sean’s War?” In case you’ve forgotten, it was the one where you and a partner took turns simultaneously dealing out cards of the top of a poker deck. A “win” happened if the two of you threw the exact same cards down in the exact same place! And, perhaps most surprisingly, wins happened about \( \frac{2}{3} \) of the time!

One of your classmates asked if they could play with three people instead of two. At the time, I said “no”, because I only wanted to adjust one variable at a time. But, as soon as he asked the question, I wanted to make a quiz about it. \( \smile \) So here we go! We’re going to play “3 person Sean War!”

Open up the spreadsheet that accompanies this quiz. In it, you’ll see one similar to the one we used in class, except now, there are 3 people playing!

Start on the tab marked “deck”. Go ahead and press the \( \text{F} \) button, like we did in class. When you do this, you see 3 “decks” of “cards”, “dealt out” in “front of you.” Every time you hit the \( \text{F} \) button again, you get a new game. Go ahead and hold it down, for a good long time (maybe a minute for now).

Did you see any red flashes in the “5” row go by as you were holding the button down? They would have been marking the occurrences of all three people having the same card at the same place! If you didn’t see any flashes, that’s OK! There might not have been any, or they may have happened too fast to notice.

Click over to the tab marked “deck (with %)”. In here, like in class, you see a pie chart showing the frequency of the times we get matches.

1. \( \text{(2 points)} \) What percent of the time do all three players get a match?

2. \( \text{(2 points)} \) How does this compare to the games we played in class (with only two players)?

OK! Now, we’re going to explore two more sheets (like we also did in class) – the “HMM” and “WMO” sheets! “HMM” stands for “How Many Matches” and “WMO” stands for “Where the Match(es) Occurred”.

Before we go any further, you need to generate a lot more game results. Like, a metric butt-ton\(^m\) more. So, go ahead and hold down that \( \text{F} \) for a good long time (what I tend to do is hold it down with a stapler, or a ruler, or something like that. Then I go and grab a snack or take a quick walk around the office: maybe go downstairs and visit with some colleagues. Then come back and take the stapler off). Once you do, answer the following!

3. \( \text{(2 points)} \) What percent of the time did the three players get exactly 1 match? Exactly 2 matches?

4. \( \text{(2 points)} \) So, out of 10000 games, assuming that percentage, in how many games did they get exactly 1 match? Exactly 2 matches?

Way less frequent than the matches we got in class! Hopefully, that makes sense – it’s going to be harder to get three people to match then just two.

5. \( \text{(2 points)} \) Take a look at the “WMO” sheet – does any position seem to be more likely than others to get matches, or do they seem pretty uniformly distributed?

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\(^1\) OK….that was annoying. I’ll try to avoid using that many quotes in a single sentence.


\(^n\) I decided against having you try to get 3 or more matches. I mean, it’ll happen eventually, but much like sensible tax law, we might be dead and gone before it does.
During the summer of 2018, my family took a fun little road trip to b-e-a-utiful Southern Utah. Did some canyoneering, some fishing and some hiking and biking. One day of the trip, we decided to ride our bikes into Bryce National park from our campsite.

Here’s a picture of our little guy, Max, checking out one of the trail maps just outside the park:

And here’s a closer look at the elevation chart he’s looking at:

1. **(1 point)** The “elevation” axis, for sure, will help us get the rise. Is the “distance” axis the “run” or the “road”? 

   (next page for rest!)
2. (3 points) Justify your previous answer with a sentence or two!

3. (3 points) Using the Excel Calculator (for simplicity! #yousrewelcome), approximate the average grade between the Start (0-mile) mark and the “You Are Here” point. This is where Max started complaining about it being too steep. 😊 “Right way” or “wrong way” doesn’t matter – it’s not steep enough to make a real difference.

4. (3 points) If you look carefully at the top photo, you see that the maximum path grade is 5%. Find two mile markers between which that maximum grade occurs! Hint: think small.°

° Coincidentally enough, I’m writing this quiz in a van on tour...on a very steep section of I-5. In Southern Oregon #notdriving 😊