Grade Exercises

Let’s think back to some examples like we started with in class:

**E1.** If a road rises 200 feet over 2 miles of run, what is its grade (or “slope”)?

**E2.** What angle is formed by the road in **E1**, on average?

**E3.** If a hill has a grade of 3.4%, and it rises 1500 feet, how much does it run?

**E4.** What angle is formed by this road, on average?

**E5.** If a road forms a 5.8 degree angle, what’s its approximate grade?

**E6.** Look back to **E3**. If you hike up this hill, from bottom to top, how far do you walk? Assume, for simplicity, that the hill rises in a uniform manner; in reality, hills don’t often! (hint: you need to use the wonderful Pythagorean Theorem on this one. If $a$ and $b$ are the legs of a right triangle (the “legs” are the ones that form the right angle; they’re the “rise” and the “run”), and $c$ is the hypotenuse (or longest side...the one across from the right angle. We’ve been calling it the “road”), then $a^2 + b^2 = c^2$ (or $\text{rise}^2 + \text{run}^2 = \text{road}^2$). If you can’t stand doing algebra, you can always use the Excel Calculator.

**E7.** Now, some folks might forget to use the hypotenuse when they perform the previous problem, and, instead, just say, “Well, you walked the run, which was 8.4 miles!” They’re obviously in error…but by how much? Here’s a helpful formula for percent error:

$$\text{Percent error} = \frac{\text{amount used incorrectly} - \text{correct amount}}{\text{correct amount}}$$

So...what’s their percent error?

**E8.** Make sure you understand why that formula makes sense.

**E9.** Suppose that the hill actually rises 15,000 feet (with the same grade). Re-answer **E6** two ways: in one, use the road (hypotenuse – the “shortcut way”). In the other, use the rise (the “right way”).

**E10.** Recompute **E17** (percent error) now. What do you notice?
Consider the following graphic (a computer generated image from a hill workout I do):

On this graph, you see elevation as a function of distance traveled (in meters and kilometers; sorry, I can’t, for the life of me, figure out how to change the default distance setting. Not to worry; it’ll do us good to pretend we’re European for a moment or two). For the first 6 km, I ride my bike to the trailhead, then from 6 km to about 19.5 km, run up and down a hill 5 times, then ride home. If you’d like a conversion, you can use 3.1 miles \( \approx \) 5 km.

**E11.** How long, in miles, is this workout?

**E12.** For how many miles do I run up and down the hill?

Consider, for a moment, just this running part.

**E13.** How many feet in elevation do I gain over the 5 hill repeats? (Pessimists will say, “Why even bother? You come back down each time. Save yourself the energy!”)

Consider, for this moment, just 1 hill repeat (it looks like the first one starts at 6 km and goes to about 8.5 km).

**E14.** What’s the average grade on that repeat?

**E15.** What’s the average grade on its uphill section?

**E16.** There are some points along that trail, however, that are steeper than the average grade. How can that be?

**E17.** What’s the average grade on its downhill section?
E18. I made a flier for a show my band played in Bend a few years ago. Since I’m a *Raising Arizona* fan, I decided to “customize” the image below and to the left to create the flier at right:

The trick was that I needed to create the sign HI’s holding separately, and then drop it into the original image (well, that and detach and then reattach his thumb...but that was pretty easy). I couldn’t make the sign tilted in the software I was using, so I needed to create it horizontally, and then adjust it by the correct angle until it matched. About which angle is that, to the nearest degree? Also, I couldn’t find the protractor in our house...which would have made this problem much easier.

“E” Answers.

**E1.** Grade is \( \frac{\text{rise}}{\text{run}} \), so \( \frac{200 \text{ feet}}{2 \times 5280 \text{ feet}} = \frac{200}{10560} \) (units can cancel), or about 0.0189 (1.89%, or about 2%). This assumes that you’re going uphill. If you were going downhill, the grade’s sometimes expressed as a negative quantity – but only in math classes. ☺.

**E2.** For these, there are a number of ways. In class, we probably used a tangent table that we looked up online (here’s one that’s been available for a while now: [http://quest.arc.nasa.gov/space/teachers/rockets/act9ws4.html](http://quest.arc.nasa.gov/space/teachers/rockets/act9ws4.html)). But, by now, I’ll bet that you like the Excel calculator even more!
So, you can see the angle’s just over 1 degree (and you also get the grade, which is kinda why I built this thing in the first place – to save you time. 😊)

If you need more precision in your answer (that is, a more exact answer than “2%”), you can use the “increase decimal” buttons in Excel (shown at right). Every time you left-click that button, you’ll get one more decimal place of accuracy. First, left-click in the cell in which you’d like to get more precision (in this case, J3). Then left – click that increase decimal button twice, and you’ll get this:

E3. Let’s use the equation \( \text{grade} = \frac{\text{rise}}{\text{run}} \) to get \( \frac{0.034}{\text{run}} = \frac{1500 \text{ feet}}{\text{run}} \). Then, we can multiply to find that \( 0.034(\text{run}) = 1500 \text{ feet} \). Dividing by 0.034 gives us \( \text{run} = \frac{1500}{0.034} \), or just around 8.4 miles (about 44000 feet).

E4. I get just shy of 2 degrees.

E5. I never coded the spreadsheet to work in reverse (that is, let you input an angle and get a grade out). But if you just use the above linked table, you can look up the angle and then reference the cell to the left, which tells you the grade. I get just about 10%.

E6. The “rise” and “run” of the hill are the legs of the right triangle, so we can say that \( a = 1500 \text{ feet} \) and \( b \approx 44117 \text{ feet} \). The hill, therefore, would look like this (horribly not to scale!):
How much you have to walk (let's call it $c$, OK?)

$$a = 1500'$$
$$b = 44,117'$$

So, we'd have $(1500')^2 + (44,117')^2 = c^2$, or $c^2 = 1,948,559,689$ square feet. Taking square roots, we have that $c \approx 44,142'$ (see how distorted that triangle I drew was? In fact, let's have some fun: without using a ruler, approximate the grade of that distorted triangle).

$E7$. Percent error = \[ \frac{\text{amount used incorrectly} - \text{correct amount}}{\text{correct amount}} = \frac{44117' - 44142'}{44142'} \approx -0.00057, \text{ or about 0%}. \] No big deal! We'll talk more and more about this in class, if we haven't already – the “right way” versus “shortcut way” calculations of grade, and how, if the grades aren’t “too steep”, how it doesn’t matter which way you do them).

$E8$. The numerator gives you an idea of how far off (in feet) your erroneous estimate is. The denominator then weights that by the entire distance over which you were in error. You can see that we were only off by around 25 feet...over 44142. That’d be like shooting a rifle at a bullseye a thousand feet away from you and missing by a little less than 6 inches. Not too shabby!

$E9$. So, repeating the math from E3, I get a run of about 83.56 miles. Correctly using Pythagoras, the hill that you walk on would be 83.6 miles.

$E10$: Percent error = \[ \frac{\text{amount used incorrectly} - \text{correct amount}}{\text{correct amount}} = \frac{83.56 \text{ miles} - 83.6 \text{ miles}}{83.6 \text{ miles}} \approx -0.00047, \text{ or about 0%}. \]

Again...no big deal! Sure, the error will stretch out over long distances (so, those of you planning rocket trips to the moon – use Pythagoras!) but for those of us on Earth doing things like hiking Mount Hood, no prob.

$E11$. According to the graphic, it was 25.7866 km, so, converting, I get $25.7866 \text{ km} \cdot \frac{3.1 \text{ miles}}{5 \text{ km}} \approx 16 \text{ miles}$. (if you want more of these types of conversions, just you wait! Your first project is ALL about them!)

$E12$. Let’s be a tad clever: since I biked to and from this workout, let’s figure the distance I biked in, double that, then take it away from 16, shall we? Looks to be about 6 km each way, so $12 \text{ km} \cdot \frac{3.1 \text{ miles}}{5 \text{ km}} \approx 7.5 \text{ miles biked}$. The rest (16 miles – 7.5 miles, or about 8.5 miles) must be how much I ran.

$E13$. Each hill repeat looks to be about 190 meters of elevation gain, so I do 950 meters, or about 3100 feet of elevation gain uphill (and, conversely, the same amount running back down).

$E14$. Ha! It’s 0%. See why?

$E15$. OK! Here’s where we start to sub in the “shortcut way” grades!
What you see in the graph isn’t the rise and the run – it’s the rise and the road! These types of elevation apps never actually give the run – because that’s not what you’re actually traveling. They, for sure, track your up-and-down (“rise”) changes, but the distance you cover, “horizontally”, is actually the road. So, I do about 190 meters of rise, and about 1.25 km (or 1250 meters) of road.

Because this happens so frequently (not just in our class, but also if you use these apps), I coded the Excel Calculator to handle it:

Here’s how this works:

- You place the 190 in the rise, and the 1250 in the road.
- You get a “right way” and a “shortcut way” grade (in this case, 15.38% and 15%, respectively).
  - In the “right way”, the Calculator figures out the run (using Pythagoras) from the road and the rise, and then calculates the grade as $\frac{\text{rise}}{\text{run}}$.
  - In the “shortcut way”, the calculator simply calculates $\frac{\text{rise}}{\text{road}}$.

See how close they are in value? While we’ll spend more time on this in class and on assessments, I’ll give you a preview now: so long as the hill “isn’t too steep”, they’ll be very close in value (I encourage you to mess around with the calculator to find roads that are too steep to see how off then can get!). Something else to think about – the “right way” is always larger number than the “shortcut way”. Can you think of why? #geometry

E16. Does the hill always have to go up on the uphill? Look at the graphic again!

A quick word on “average grade”, in case you forget what we discussed in class: Average grade “smooths out” the road into a perfectly straight line – it makes the assumption that the ups and downs along the way don’t exist, and just looks at the total rise divided by the run (or road). By “total rise”, I just mean the difference between the starting and ending elevations; in-between elevation changes “don’t count”.

E17. Same exact one! I guess you could say -15%, if you’re feelin’ saucy. 😊

E18. So here’s a picture of just the sign bottom:
Notice that the rise is about 1/12(ish) the run (does it matter which units you use? Why or why not?). Therefore, the grade (or tangent) of the angle formed is about 1/12, or, roughly, 0.083. Consulting my handy – dandy tangent table link from above, I see that a tangent of 8.3% corresponds to an angle of about 5 degrees.
Quiz 1.

My house is at roughly 3400 feet above sea level. My 6.5 – mile commute to COCC each day ends at 4000 feet above sea level (this is a slightly different route than the one we talked about in class).

1. **(2 points)** I’m going to list three components of the “grade triangle” we’ve been using in class below, and I’d like you to tell me what two of their three values are (one of them is unknown from the data above – you can calculate it, but I don’t want you to):

<table>
<thead>
<tr>
<th>Rise:</th>
<th>Run:</th>
<th>Road:</th>
</tr>
</thead>
</table>

2. What’s the average grade of my commute? Give two measurements (remember, you can use the Excel Calculator if you want!):

   a. **(2 points)** The “shortcut” way.

   b. **(2 points)** The “right” way.

   Note: When I say “do it two ways”, all you need to do is enter the two measurement you have into the Excel Calculator. Just make sure you put the measurements in the correct places! Email if you get stuck!

3. **(4 points)** In this case, do you think that it matters whether we do grade the “shortcut way” or “right way”? Why or why not? Write at least two sentences explaining your answer!
Quiz 2.

Burma Road is a fairly steep section of double track in the BLM land right outside of Smith Rock state Park in Terrebonne Oregon. Some of you may have been on or near it before. In the picture at below left, it’s the jagged – looking diagonal gash running across the center of the image (I colored it red, so you can see it a little easier). It actually switchbacks on itself, as you can see in the picture at right below (taken atop the large rock formation you can see at center left, called the Wombat).

A student asked me a great question in class once: “What’s the grade on Burma Road?” Fantastic! Let’s find out!

Remember that \[ \text{Average Grade} = \frac{\text{Total Rise}}{\text{Total Run}} \]

so all we have to do is get both of those, and we’re set!

I found this really cool topo (“topological”) map online (the site’s gone now, alas – but you can still find other on the magical interwebs). This is a look, down from above, at an elevation map of this area of Smith Rock:

Now let’s use this image to get our rise and run. We’ll start with the rise:

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\( ^* \) I know that, sometimes, we’ve been swapping out “road” for “run”, and this’ll also be one of those cases!
Do you see those little lines on the map? They’re called “contour lines”, and they let you know how steep certain sections of land are. Notice how certain numbers are indicated? I see a “3600’” up near the top of Burma Road – that means that, if you’re standing anywhere along that line, you’re standing at 3600 feet above sea level. See the one that says “3800’”? Same thing, but that one’s 200 feet higher. Now, there are also lines in between the darker, bolded ones – I count 4, which means each line represents a change of 40 feet in elevation.

The closer together contour lines are, the steeper the terrain – conversely, the farther apart they are, the flatter the terrain. Pretty rad, eh? OK, back to Burma Road...

1. **(1 point)** At what elevation does Burma Road start (the lowest part of the red line)? Approximate it – I know the map isn’t perfect.

2. **(1 point)** At what elevation is the end of Burma Road (the highest part of the red line)?

3. **(1 point)** So, if you walk from the bottom to the top, how much elevation have you gained overall?

OK...there’s your rise! Now let’s work on the “run” (actually, “road”...remember why?)– this map has a scale of about 1” to 0.2 miles (that is, for every inch you “walk”, you cover about 1/5 of a mile). Estimate how long, in miles, Burma Road is\(^b\). Make sure to keep the “zoom” on your web browser is at about 100% (not more nor less). If you’re doing this on a mobile device, it’s most likely going to distort your numbers, so shoot me an email and we’ll figure it out!

4. **(1 point)** Convert your answer to feet, and find your run (er, road) in feet.

5. **(2 points)** What’s the average grade of Burma Road to the nearest percent? “Right way”, “shortcut way”...either’s fine. ☺

OK, one more...a few years ago, I hauled my son up the ridgeline colored yellow at right below (we started at the top of Burma Road, and finished on top of the butte shown).

6. **(1 point)** What’s my rise over this journey?

7. **(1 point)** What’s the road? Again, don’t zoom in!

8. **(2 points)** What’s the average grade of this ridgeline?

9. **(extra 1 point)** Why was I so insistent about you not zooming in on these?

\(^b\) You might find it easier to find a piece of string and lay it along Burma Road on the map. Then, you can straighten the string along a ruler to see how long it is.

\(^c\) We’ll talk more, later, about why this assumption isn’t *always* legit.
Quiz 3.

Remember when you all were so busily running around campus measuring the grades of staircases? A few terms ago, I diligently measured the ADA ramp outside of Modoc Hall. In case you don’t remember it, here’s a picture:

“Why do such a thing?”, you might ask. Well, as it turns out, the ADA ramps are bound by (you guessed) the actual ADA (Americans with Disabilities Act). As such, there are very strict requirements as to how the ramp is to be constructed (note: “slope” and “grade” are used interchangeably):

4.8.2* Slope and Rise
The least possible slope shall be used for any ramp. The maximum slope of a ramp in new construction shall be 1:12. The maximum rise for any run shall be 30 in (760 mm).

OK – here are the measurements I took (I only did the ramp at the bottom, behind the studious student in the picture… I assumed they were consistent for the top one):

<table>
<thead>
<tr>
<th>Rise</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 ft, 1 in</td>
<td>2 ft, 1/2 in</td>
</tr>
</tbody>
</table>

1. (1 point) Is this ramp’s rise within the code’s restrictions?

2. (2 points) Calculate the grade (slope) of the ramp the “shortcut” way…you figure, “It’s good enough for government work (nyuck, nyuck, nyuck)”. To the nearest thousandths’ place, please! And not as a percent – as a decimal.

Notice how they gave us the maximum allowable slope as a “1:12”? Let’s compare ours to that!

3. (1 point) Convert “1:12” (which is actually $\frac{1}{12}$) to a decimal. Use all decimals, please!
4. **(1 point)** Does our ramp’s grade comply with the ADA code?

5. **(5 points)** Now, suppose that COCC was audited for this ramp. The government might say, “well, we notice that you calculated your grade using the \( \frac{\text{rise}}{\text{hypotenuse}} \) shortcut, which, as you know, is not the same as the correct method of calculating grade.” How would you answer their charge? **Hint:** you’ll need to calculate the grade the **right** way to answer this satisfactorily.
Quiz 4.

Many of you have either hiked, biked (or both) and encountered switchbacks on your road or trail. **Switchbacks** are where the trail or road curves back and forth along itself to make climbing easier. I’ve encountered them on MacKenzie Pass, Farewell Trail, and plenty of other places in and around Central Oregon. Here’s a shot of some switchbacks from the mountains near Asheville, NC (“Bend of the East Coast”):

!”Man!”, you might think, “How does this make climbing easier? You’re traveling MORE!!!” Let’s analyze that! Here’s a picture of one of those switchbacks, taken from the curve in the road:

In red, I’ve outlined the path that you would take if you stayed on the road. In yellow, I’ve outlined the “fall line” (that is, the path you would take if you were to climb directly up a mountain – it’s called the “fall line” because that’s where an object would “fall” if left under only the force of gravity).
What I’d like to do is mathematically generally analyze a switchback by comparing it to the fall line route. But first...

1. (1 point) If Road A has a larger average grade than Road B, which would be easier to walk/bike/drive up? Define “easier” in this case as “less energy expended at the hardest point”, and assume that a higher grade equals more energy expenditure\(^d\). Also assume that their lengths are irrelevant.

    Hopefully this make sense! I notice this all the time when I ride up Iowa Street – the steepest section is only about 200 feet long – but I’m whooped when I get to the top of it – way more than I would be if I simply took Archie Briggs around the side of Awbrey Butte.

    So let’s see what’s going on, grade – wise, between these two routes. Complete the following with either a “<”, “>”, or an “=”:

    2. (3 points) Rise of the switchback route \[ \underline{\text{\text{}} \text{}} \text{Rise of the Fall Line Route} \]

    3. (3 points) Run\(^e\) of the switchback route \[ \underline{\text{\text{}} \text{}} \text{Run of the Fall Line Route} \]

    Now, put those ideas together into grade!

    4. (3 points) \(\frac{\text{Rise}}{\text{Run}}\) of the switchback route \[ \underline{\text{\text{}} \text{}} \frac{\text{Rise}}{\text{Run}} \text{ of the Fall Line Route} \]

    So there you have it! You now understand why exhausted cyclists (i.e., my son!) weave back and forth on steep hills, why Misery ridge zig – zags, and why mountaineers rarely head straight up a mountain all day. Wahoo!

\(^d\) Road cyclists understand what I’m talking about here.

\(^e\) Feel free to substitute “hypotenuse” for “run” here.
Quiz 5.

As you might remember, “grade” is just another word for “slope” or “tangent of an angle”. Well, as it turns out, there are plenty of other terms that are also used! Let’s explore some here!

Roof pitches – carpenters and builders often refer to the “pitch” (of slope) of a roof by the fractional equivalent of the grade with 12 in the denominator (I believe this is because there are 12 inches in a foot). So, for example, if a roof has a “6:12 pitch”:

\[
6:12 \text{ pitch} = \frac{6 \text{ inches rise}}{12 \text{ inches run}} = 0.5 = 50\% \text{ grade (which translates to about a 26.5 degree angle)}
\]

a. **(2 points for each line)** Please complete the following chart for some commonly used roof pitches!

<table>
<thead>
<tr>
<th>Pitch</th>
<th>Grade (as a decimal)</th>
<th>Grade (as a percentage)</th>
<th>Angle (in approximate degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4:12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6:12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12:12</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In extreme weather situations (like in high snow areas) pitches/grades often exceed 100% so that the snow loads aren’t too great. There’s an example of one at right, based on an equilateral triangle.

b. **(2 points)** Using a measuring device of your choosing, approximate the pitch of the roof in that set of plans.

c. **(2 extra points)** Knowing that the cabin’s side elevation was based on an equilateral triangle, how could you have gotten your answer in part b directly without measurement? Please explain fully!

The same pitch idea is used in plumbing – for example, waste pipes in Bend have to have \(\frac{1}{8}\)” of fall per foot of run. So, therefore,

d. **(2 points)** If a pipe has to run 20 feet, how far must it fall?

As mentioned in class, pitch is used in other applications (high heeled shoes, extreme skiing), but I can’t seem to find any rigorous treatment of the measurements in these situations (outside of “Yo, you really shredded the gnar on that pitch, brah!” or “Wowsers! Can you actually walk on heels that pitched?”).
Quiz 6.

1. Suppose a road rises 7 feet for every 50 that it runs.
a. (1 point) what is its grade?

   b. (2 points) (w) Use the Pythagorean Theorem to tell me what the road length would be in this case. Round this one to the nearest tenth. Please show me what you do! If you use some kind of online computation tool, just take a screenshot of it and include it (if you need help with screenshots, here ya go!)

2. Suppose a road rises 21 feet for every 150 that it runs.
a. (1 point) What is its grade?

   b. (1 point) What’s the road length in this case? Same rounding as before (no need for work on this one; I assume you’re doing the same thing as before).

3. One more! Suppose a road rises 49 feet for every 350 feet it runs.
a. (1 point) Grade, please!

   b. (1 point) And, yep...road, in that case?

   Seeing anything going on after those last three situations? If so, great! If not – read on!

4. (3 points) Complete the following sentence by inserting the phrases “is also multiplied by that constant value” and “remains the same” into the blanks. Use each phrase in only one spot!

   If you multiply the rise and run by a constant value, then the grade ____________________________, and the road ____________________________.
Quiz 7.

I may have lied to you a little in class...I told you that Iowa Street in Bend was the road in town with the steepest section of grade. Now? I’m not so sure...see, when I bike in the Iowa route, I also have to go up Palisades Drive, between Rimrock and Glassow). That, my fine feathered friends, is a total PITA:

![Map of Iowa and Palisades Drives](image)

Now, I don’t know if it’s actually steeper than Iowa, or if it just feels like it because I just survived Iowa...so, I need your help to check.

(10 points) (w) Find about ½ an hour to an hour to do this quiz. Stop by the office and borrow my transit and 200 – foot tape (I’ll teach you how to use both). Then, go out, find Palisades Drive, and figure its grade. Show/describe everything you do. Pictures would be rad!

(5 extra points) That’s right – you get 5 extra credit points for doing this quiz. Why? Because it requires a field trip! And instrumentation! And parking/walking on a crazy hill. 😊
Quiz 8.

During the summer of 2018, my family took a fun little road trip to b-e-a-utiful Southern Utah. Did some canyoneering, some fishing and some hiking and biking. One day of the trip, we decided to ride our bikes into Bryce National park from our campsite.

Here’s a picture of our little guy, Max, checking out one of the trail maps just outside the park:

And here’s a closer look at the elevation chart he’s looking at:

1. (1 point) The “elevation” axis, for sure, will help us get the rise. Is the “distance” axis the “run” or the “road”? 
2. **(3 points)** Justify your previous answer with a sentence or two!

3. **(3 points)** Using the Excel Calculator (for simplicity! #yourewelcome), approximate the average grade between the Start (0-mile) mark and the “You Are Here” point. This is where Max started complaining about it being too steep. ☺ “Right way” or “shortcut way” doesn’t matter – it’s not steep enough to make a real difference.

4. **(3 points)** If you look carefully at the top photo, you see that the maximum path grade is 5%. Find two mile markers between which that maximum grade occurs! Hint: think small.

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*Coincidentally enough, I’m writing this quiz in a van on tour...on a very steep section of I-5. In Southern Oregon #notdriving ☺*